
SOCIETY OF ACTUARIES
Quantitative Finance and Investment Advanced Exam

Exam QFIADV

AFTERNOON SESSION

Date: Thursday, November 2, 2017

Time: 1:30 p.m. – 3:45 p.m.

INSTRUCTIONS TO CANDIDATES

General Instructions

1. This afternoon session consists of 6 questions numbered 11 through 16 for a total of 40 points. The points for each question are indicated at the beginning of the question.
2. Failure to stop writing after time is called will result in the disqualification of your answers or further disciplinary action.
3. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions on the exam booklet.

Written-Answer Instructions

1. Write your candidate number at the top of each sheet. Your name must not appear.
2. Write on only one side of a sheet. Start each question on a fresh sheet. On each sheet, write the number of the question that you are answering. Do not answer more than one question on a single sheet.
3. The answer should be confined to the question as set.
4. When you are asked to calculate, show all your work including any applicable formulas. When you are asked to recommend, provide proper justification supporting your recommendation.
5. When you finish, insert all your written-answer sheets into the Essay Answer Envelope. Be sure to hand in all your answer sheets because they cannot be accepted later. Seal the envelope and write your candidate number in the space provided on the outside of the envelope. Check the appropriate box to indicate morning or afternoon session for Exam QFIADV.
6. Be sure your written-answer envelope is signed because if it is not, your examination will not be graded.

Tournez le cahier d'examen pour la version française.

****BEGINNING OF EXAMINATION****

**Afternoon Session
Beginning with Question 11**

11. (7 Points) PQR Investments is a high-yield fixed income investment manager. Its fund returns have tracked very closely to its defined benchmark. The portfolio manager for PQR Investments has the mandate to track the benchmark index, with the latitude to underweight or overweight portfolio allocations based on his market view in order to enhance portfolio return. However portfolio allocations must stay within 5% of the benchmark allocations.

- (a) (1 point) List and define broad categories of a typical scenario-based return decomposition.

You are given the following information from a performance attribution over a 3 month time period. All returns are denominated in the same currency. You assume that there are no yield-curve returns from rates other than the key rates, and that there is no impact from convexity.

	Annual Yield				Yield-Curve-Matched Market Weight (%)		Option Adjusted Duration (yrs)	
	Level (%)		Change					
	Portfolio	Benchmark	Portfolio	Benchmark	Portfolio	Benchmark	Portfolio	Benchmark
Total					100.0%	100.0%	7.38	8.18
Average	1.53	1.48	0.30	0.30				
Key Rates								
1yr	0.56		0.03		20.0%	20.0%	0.09	0.09
5y	1.11		0.06		36.0%	40.0%	1.46	1.51
10y	2.32		0.69		44.0%	40.0%	5.83	6.58

- (b) (3 points) Calculate the portfolio outperformance due to the change in the yield curve.
- (c) (2 points) Critique the performance of the portfolio manager over this time period, identifying:
- (i) what was done well
 - (ii) what could have been better

The internal portfolio attribution model used by PQR Investments indicates that the portfolio outperformance residual is 958 bps.

- (d) (1 point) Describe areas that warrant further investigation given the above residual.

12. (6 points) You are a pricing actuary helping to design a new variable annuity (VA). Your company wants the new product to be very competitive in the market.

- (a) (1 point) Describe how target volatility funds are designed to achieve their objective.

You constructed a target volatility fund using fixed income and equity assets. You are given the following information:

Asset type	Volatility (annualized)	Expected Return (annualized)	Correlation
Fixed Income	2%	4%	0.3
Equity	16%	6%	

The target volatility is 10%. No short-selling is allowed. The fund will rebalance only when any of the above parameters change.

- (b) (2 points) Calculate the expected return of this fund based on the above information.
- (c) (1 point) Explain how a target volatility fund can be used to design a competitive product.

Currently, your company's hedging model does not include dynamic lapses.

- (d) (1.5 points) Describe how dynamic lapses will impact the insurance company as the moneyness of the guarantee changes.
- (e) (0.5 points) Discuss how a company can mitigate the impact of dynamic lapses on its VA business.

13. (6 points)

- (a) (1 point) Explain how principal component analysis (PCA) can help overcome problems of multicollinearity in the data.

You are planning to use a statistical factor model to analyze changes in the US zero-coupon yield curve.

You have decided to use a principal component method. You have obtained the correlation matrix of changes in zero-coupon yields across various maturities. The highest five eigenvalues and their corresponding eigenvectors for the correlation matrix are shown below.

Eigenvalues					
	λ_1	λ_2	λ_3	λ_4	λ_5
	8.7860	0.8434	0.2161	0.0805	0.0543
Eigenvectors					
Maturity	e_1	e_2	e_3	e_4	e_5
3 month	0.2653	0.5814	0.6284	0.1790	0.4005
6 month	0.2973	0.4652	0.0226	-0.3582	-0.7476
1 year	0.3024	0.3293	-0.6091	-0.3870	0.4818
2 year	0.3306	0.0217	-0.3253	0.3719	-0.0003
3 year	0.3342	-0.0510	-0.1422	0.3532	-0.1084
5 year	0.3336	-0.1128	-0.0417	0.3284	-0.0625
7 year	0.3331	-0.1560	0.0219	0.1727	-0.0806
10 year	0.3287	-0.2309	0.1311	0.0046	-0.0123
20 year	0.3220	-0.2938	0.1669	-0.2956	0.0286
30 year	0.3081	-0.3966	0.2455	-0.4491	0.1596

- (b) (1 point) Calculate the percentage of variation explained by each of the five components above.
- (c) (2 points) Recommend the factors that should be included in your model, including an interpretation of each factor.
- (d) (2 points) Calculate the communality for the 1-year zero-coupon yield based on your recommended model.

14. (7 points)

ABC Company is introducing a new Guaranteed Minimum Death Benefit (GMDB) product with the following product features:

Mortality and Expense Fee	110 bps
Surrender Charge	8, 7, 6, 5, 4, 3, 2, 0% of account value in policy years 1 – 7, and 8 and beyond, respectively
Free withdrawal amount	10% of account value
Return of premium death benefit	Death benefit equals the greater of premiums paid and the account value. Partial withdrawal reduces the death benefit on a pro-rata basis (i.e. for a \$1 withdrawal, benefit base is reduced by the ratio of benefit base over account value)
Fund Options	1) 25% equity 75% bond 2) 75% equity 25% bond

The following asset portfolio is created for backing the GMDB liabilities:

Asset Class	Allocation	Expected Return for Asset Class	Cash Flows under Stress Scenario for Specified Allocation
Equities	30%	8%	24
10 year A Corporate bonds	45%	5%	50
15 year Government bonds	25%	4%	40

- (a) (1 point) Explain which product features impact the company’s net amount at risk associated with the GMDB.
- (b) (1 point) Evaluate the asset portfolio above to support the GMDB liability with respect to credit, diversification and liquidity.

The Chief Risk Officer (CRO) for ABC Company is looking to establish a liquidity risk management program.

- (c) (1.5 points) Describe the key considerations in creating deterministic scenarios for stress testing liquidity risk.

14. Continued

The CRO requires a cash flow coverage ratio of 1.05 for all time periods and stress scenarios. Under the current portfolio, the cash flow coverage ratio over one month for the stress scenario is 1.14. Liability cash flows expected over one month for the stress scenario is 100.

ABC Company is considering reducing the allocation to Equities from 30% to 10% and reallocating that amount to either Real Estate or Private Equity. You have been provided the below information:

Asset Class	Expected Return for Asset Class	Cash Flows under Stress Scenario for 20% Allocation
Real Estate	10%	8
Private Equity	11%	0

- (d) (2 points) Calculate the cash flow coverage ratio over one month for the stress scenario for:
- (i) the reallocation into Real Estate
 - (ii) the reallocation into Private Equity
- (e) (1.5 points) Recommend any changes to the portfolio allocation given the above analysis.

- 15.** (7 points) FGH Bank is a rapidly growing regional bank. In recent years FGH has taken advantage of the recovering housing market by increasing its mortgage loan operations. To finance these loans, FGH has begun securitizing parts of its growing mortgage asset base into collateralized notes with long-term maturities. FGH has also begun utilizing short-term institutional investor borrowing to cover any additional financing needs.

The Chief Risk Officer (CRO) is concerned about how FGH's activities could affect its performance in a future financial crisis similar to that of 2007-2008. She has assigned you to write a memorandum on bank runs during the financial crisis and how a similar crisis could affect the bank.

- (a) (1 point) Explain how FGH Bank's activities are similar to those that led to the "run on the bank" at Northern Rock.

Below is a summary of FGH Bank's balance sheet over the last 3 years.

Year	2014	2015	2016
Asset Class:			
Mortgages	250	500	750
Corporate Bonds	50	200	250

Year	2014	2015	2016
Liability Type:			
Short Term Debt	0	330	475
Securitized Notes	85	100	200
Retail Deposits	200	250	300
Equity	15	20	25

Current financial and capital market conditions are considered normal. During normal conditions, an average Repurchase Agreement (Repo) haircut – the difference between the current market price of a security and the price at which it is sold – is 4%. Banks adjust their leverage based on prevailing haircuts.

- (b) (1 point) Calculate the impact on leverage assuming a stress scenario where the average Repo haircut increases from 4% to 20%.
- (c) (1 point) Explain the risks the CRO should be concerned about under the stress scenario in part b).

15. Continued

You are considering the courses of action FGH Bank should follow going forward to reduce the risk of a liquidity crisis in the future.

- (d) *(1 point)* Recommend changes to FGH Bank's activities that decrease its liquidity risk.

FGH Bank follows the "capital buffer" requirement of the financial regulation for banks, where the key determinant of the size of the buffer is the riskiness of the assets.

- (e) *(1 point)* Critique the "capital buffer" approach for financial regulation of banks.
- (f) *(2 points)* Describe two alternative approaches for liquidity regulations that overcome the shortfalls of the traditional approach of a capital buffer.

- 16.** (7 points) You were recently hired as an investment actuary at XYZ financial. Your company is currently using the Hull – White Model for pricing interest rate derivatives. You propose the following G2++ two-factor interest rate model:

$$\begin{aligned}
 r(t) &= x(t) + y(t) + \phi(t) \\
 dx(t) &= -ax(t)dt + \sigma dW_1(t) \\
 dy(t) &= -by(t)dt + \eta dW_2(t) \\
 dW_1(t)dW_2(t) &= \rho dt
 \end{aligned}$$

- (a) (1 point) Justify the use of this model.

You have been given the following:

T	1/12	1/2	1
$\mu_r(t)$	0.0025	0.015	0.03
$\sigma_r^2(t)$	2.88E-6	1.22E-5	1.71E-5

- (b) (2 points) Calculate the probability that the short rate at 1-month is negative.

A quantitative analyst provided two sets of calibrated model parameters from a set of available investments:

	Set 1	Set 2
a	0.53	0.9
b	0.38	0.78
σ	1.20%	1.15%
η	0.83%	0.90%
ρ	0.51	-0.85

- (c) (1 point) Discuss the shape of the volatility term structure of instantaneous forward rates with Set 1 parameters in light of the shape that usually prevails in the market.

16. Continued

The analyst calculated the model values below, in millionths, using Set 2 parameters.

(t, T)	(0,1)	(0,1/12)	(0,1/2)	(1/12,1)	(1/2,1)
V	6.6	0.0068	1.1	5.34	1.1

Current prices for zero-coupon-bonds with maturity of one month and one year are the following:

$$P(0,1/12) = 0.99$$

$$P(0,1) = 0.95$$

The values of the processes at time $t = 1/12$ are:

$$x(1/12) = 0.015$$

$$y(1/12) = 0.008$$

- (d) (2.5 points) Calculate the price of the zero-coupon-bond at 1-month with maturity at 1 year, $P(1/12, 1)$.
- (e) (0.5 points) Describe how caplet volatilities are defined in the market model for caps.

****END OF EXAMINATION****
Afternoon Session

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