
SOCIETY OF ACTUARIES
Quantitative Finance and Investment Advanced Exam

Exam QFIADV

MORNING SESSION

Date: Thursday, November 2, 2017

Time: 8:30 a.m. – 11:45 a.m.

INSTRUCTIONS TO CANDIDATES

General Instructions

1. This examination has a total of 100 points. It consists of a morning session (worth 60 points) and an afternoon session (worth 40 points).
 - a) The morning session consists of 10 questions numbered 1 through 10.
 - b) The afternoon session consists of 6 questions numbered 11 through 16.The points for each question are indicated at the beginning of the question.
2. Failure to stop writing after time is called will result in the disqualification of your answers or further disciplinary action.
3. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions on the exam booklet.

Written-Answer Instructions

1. Write your candidate number at the top of each sheet. Your name must not appear.
2. Write on only one side of a sheet. Start each question on a fresh sheet. On each sheet, write the number of the question that you are answering. Do not answer more than one question on a single sheet.
3. The answer should be confined to the question as set.
4. When you are asked to calculate, show all your work including any applicable formulas. When you are asked to recommend, provide proper justification supporting your recommendation.
5. When you finish, insert all your written-answer sheets into the Essay Answer Envelope. Be sure to hand in all your answer sheets because they cannot be accepted later. Seal the envelope and write your candidate number in the space provided on the outside of the envelope. Check the appropriate box to indicate morning or afternoon session for Exam QFIADV.
6. Be sure your written-answer envelope is signed because if it is not, your examination will not be graded.

Tournez le cahier d'examen pour la version française.

****BEGINNING OF EXAMINATION***
Morning Session

1. (5 points)

You are a member of the portfolio management team of a bond fund. The investment guidelines for the portfolio allow for investing in various bond sectors from different countries and economic regions. Your team is interested in exploring and utilizing duration times spread (DTS) analysis.

- (a) (1 point) Identify two key hypotheses supporting DTS as a primary measure of credit exposure.
- (b) (1 point) Outline the steps that develop a theoretical basis for DTS based on a simple variant of the Merton Model.

Your team is undertaking research to identify the shape of the relation between systematic spread volatility and spread level. Your team is testing the hypothesis that for each sector, the spread changes of the component bonds within that sector is normally distributed with a mean of zero and a volatility that is not constant over time, but rather is proportional to the current level of the spread.

The team is using the following specification to test the relation between spread and volatility in month t and the spread level at the end of the previous month:

$$\sigma_t(\Delta s) = \alpha + \beta \hat{s}_{t-1} + \gamma \hat{s}_{t-1}^2$$

where

- $\sigma_t(\Delta s)$ is the volatility of the spread during period t
- \hat{s}_{t-1} is the beginning of period spread level.
- α, β and γ are constants.

Parameters are estimated using the method of maximum likelihood.

The team believes the linear factor (β) to be 10%.

1. Continued

The research has yielded the following results and your team is using the accompanying t Table for determining significance:

	Average Number of Bonds	α	t -statistic	β	t -statistic	γ	t -statistic
Sector 1	286	3.1E-05	0.2	8.1%	4.1	-2.5	0.5
Sector 2	775	2.4E-05	0.3	12.0%	4.6	0.8	0.5
Sector 3	225	5.1E-05	-0.2	8.9%	3.4	-0.2	0.1
Sector 4	118	4.5E-05	0.3	11.8%	4.4	0.5	-0.9
Sector 5	157	6.6E-05	0.3	12.2%	2.8	-5.1	0.6
Sector 6	236	1.0E-04	0.1	13.3%	1.6	0.1	0.4
Sector 7	106	3.9E-05	0.4	11.9%	3.5	-3.1	0.0
Sector 8	206	5.7E-05	0.3	5.8%	1.5	24	2.0

t Table

	Cumulative probability	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$
	one-tail	0.50	0.25	0.20	0.15	0.10	0.05	0.025	0.01
	two-tails	1.00	0.50	0.40	0.30	0.20	0.10	0.050	0.02
Degrees of Freedom									
60		0.000	0.679	0.848	1.045	1.296	1.671	2.000	2.390
80		0.000	0.678	0.846	1.043	1.292	1.664	1.990	2.374
100		0.000	0.677	0.845	1.042	1.290	1.660	1.984	2.364
1000		0.000	0.675	0.842	1.037	1.282	1.646	1.962	2.330

- (c) (2 points) Critique your teams' hypothesis in light of the research results. Provide support for your conclusions.

You take a special interest in the results of sector 8.

- (d) (1 point) Explain how spread volatility reacts for bonds from sector 8 in both high and low spread environments.

2. (6 points) ABC is a 100 million US-based pension fund and currently has a portfolio asset allocation of 60% domestic large cap equities, and 40% domestic bonds (both government and corporate). Its Investment Officer has recently surveyed alternative asset classes and has also read “Infrastructure as an Asset Class” by Greg Inderst. As a result, ABC is now seeking to invest 5 million in an unlisted infrastructure fund.

ABC’s investment objectives are to diversify portfolio risk with moderate to high returns but not take any active or dominant role in control of any investment. It understands that the new investment would typically require a multiyear time horizon.

- (a) (1 point) Describe other ways an investor can obtain exposure to infrastructure assets.
- (b) (2 points) Explain why infrastructure exposure may be unable to reduce total portfolio risk for ABC.

After deliberation, ABC’s Investment Officer decides to further research several infrastructure funds.

- (c) (1 point) Describe how an infrastructure fund might invest across the infrastructure asset class in order to maximize portfolio diversification.

ABC is now considering three alternative investment opportunities:

- An infrastructure fund which invests exclusively in a few infrastructure projects at any given time and has a 7% historical IRR and 15% standard deviation of returns
 - Seed investment in a new web app recently developed by a small software company
 - A buyout fund which targets a specific industry and has a 10% historical IRR and 21% standard deviation of returns
- (d) (2 points) Recommend which of the three investments is most likely to satisfy the goals for the pension fund’s 5 million investment.

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3. (6 points) You work for the asset management department of an insurance company and have recently attended a meeting regarding counterparty risks.

- (a) (1 point) Identify and explain if the following creates right-way or wrong-way exposures to the buyer:
- (i) An oil producer selling oil in a swap
 - (ii) A company writing put options on its own stock

In a discussion regarding counterparty risks, your colleague makes the following statements.

- Collateral agreements can eliminate all counterparty risks.
- Liquidity puts reduce credit exposures by altering the effective maturities of trades.

(b) (1 point) Explain whether you agree or disagree with him.

A portfolio has the following positions with a single counterparty. Note that positive values of collateral are collateral that the counterparty has posted with the portfolio, and negative values of collateral are collateral the portfolio has posted with the counterparty.

Netting Node	Position Value	Collateral Value
Node 1	1.0m	0.5m
Node 2	-0.8m	-0.3m
Node 3	2.1m	0.6m
Node 4	-0.9m	0.8m
Node 5	1.3m	-0.9m

- (c) (1 point) Calculate the net exposure to the counterparty.
- (d) (1 point) Describe in four steps how to determine the market value of default risks at the t -th future time period when only one of the two counterparties has a credit exposure. (Ignore the effect of correlation between exposure, defaults, and interest rates.)

3. Continued

Insurance company A (the payer) enters a plain-vanilla interest rate swap with Bank B (the receiver). Assume the two counterparties are of equal credit quality. The swap is a 5-year (annual coupon) US dollar LIBOR swap with a fixed rate that would be at-market if the counterparties were default-free.

The table below contains values of:

PV(X) - The expected present value of the exposure to the payer in the event of default by the receiver at each coupon date.

PV(Y) - The expected present value of the exposure to the receiver in the event of default by the payer at each coupon date.

Assume that the two parties default independently of each other and of changes in interest rates, and that the loss rate is the same for both parties.

Time	PV(X)	PV(Y)	Loss rate (bps)
1	38,300	40,500	24.6
2	42,000	45,000	23.5
3	36,000	39,500	22.6
4	22,000	25,500	21.5

- (e) (2 points) Calculate:
- (i) the total present value of default losses to B due to first default by A
 - (ii) the credit adjustment to B

4. (9 Points) Your company uses the Black-Scholes model to price a Guaranteed Lifetime Withdrawal Benefit (GLWB) rider attached to a single premium variable annuity contract. The variable funds mimic a tradable equity index (“the index”) whose value at time t is S_t . Your company is evaluating the following two hedge portfolios to reduce its GLWB risk:

Hedge Portfolio I: $\Delta_{S_t} S_t + \Delta_{B_t} B_t$

Hedge Portfolio II: $\Delta_{S_t} S_t + \Delta_{B_t} B_t + \Delta_{X_t} X_t$

Where: $\Delta_{B_t} = (\psi_t - \Delta_{S_t} S_t - \Delta_{X_t} X_t) / B_t$

ψ_t = Fair value of GLWB at time t as determined by the Black-Scholes model with volatility parameter σ

$\Delta_{S_t}, \Delta_{B_t}$ and Δ_{X_t} are the quantities invested in the index S_t , money market B_t and 1-year ATMF straddle option X_t on the index, respectively

The following table gives their values of Δ_{S_t} and Δ_{X_t} :

	Δ_{S_t}	Δ_{X_t}
Hedge Portfolio I	$\frac{\partial \psi_t}{\partial S_t}$	0
Hedge Portfolio II	$\frac{\partial \psi_t}{\partial S_t} - \Delta_{X_t} * \frac{\partial X_t}{\partial S_t}$	$\frac{\partial \psi_t}{\partial \sigma} / \frac{\partial X_t}{\partial \sigma}$

Assume that the true volatility is not constant, but instead follows a mean reverting stochastic process.

- (a) (1 point) Describe the Hedge Portfolio I including its impact on the company’s hedge profit/loss.
- (b) (1 point) Identify potential problems with Hedge Portfolio II.
- (c) (1 point) Recommend improvements to Hedge Portfolio II.

4. Continued

In order to encourage policyholders not to make any withdrawals from the contract before the end the 5th policy year, your company has decided to add a Guaranteed Minimum Accumulation Benefit (GMAB) to supplement the current GLWB as below:

- If and only if the policyholder does not take any withdrawal before the end of the 5th policy year, the account value at the end of the 5th policy year is reset to the initial premium amount if it exceeds the then-account value.
- This GMAB is available to new sales only.

Your manager was just hired from outside of your company. He asked you to measure the GMAB delta based on the following assumptions:

- The dividend yield of the index is 0%
- The implied volatility of an option on the index at time t is given by

$$\sigma_{t,T} = 0.2 - 0.1 * T * \ln \frac{K}{S_t}$$

Where T = remaining time to maturity (measured in years)

K = the option's strike price

S_t = the index value at time t (same as in Hedge Portfolio I and II above)

- The continuously compounded risk free interest rate = 2% for all maturities
- Mortality rate = 0% for the first 5 policy years
- Lapse rate = 3% at the end of each policy year for ALL policy years
- 70% of the inforce policyholders will not make any withdrawals before the end of the 5th policy year

- (d) (3 points) Calculate the GMAB delta immediately after the contract issue.
- (e) (2 points) Determine if the implied volatility assumption is consistent with the empirically observed shape of smiles for short and long equity option maturities.
- (f) (1 point) Critique the static lapse rate assumption.

5. (6 points) This question concerns credit risk models.

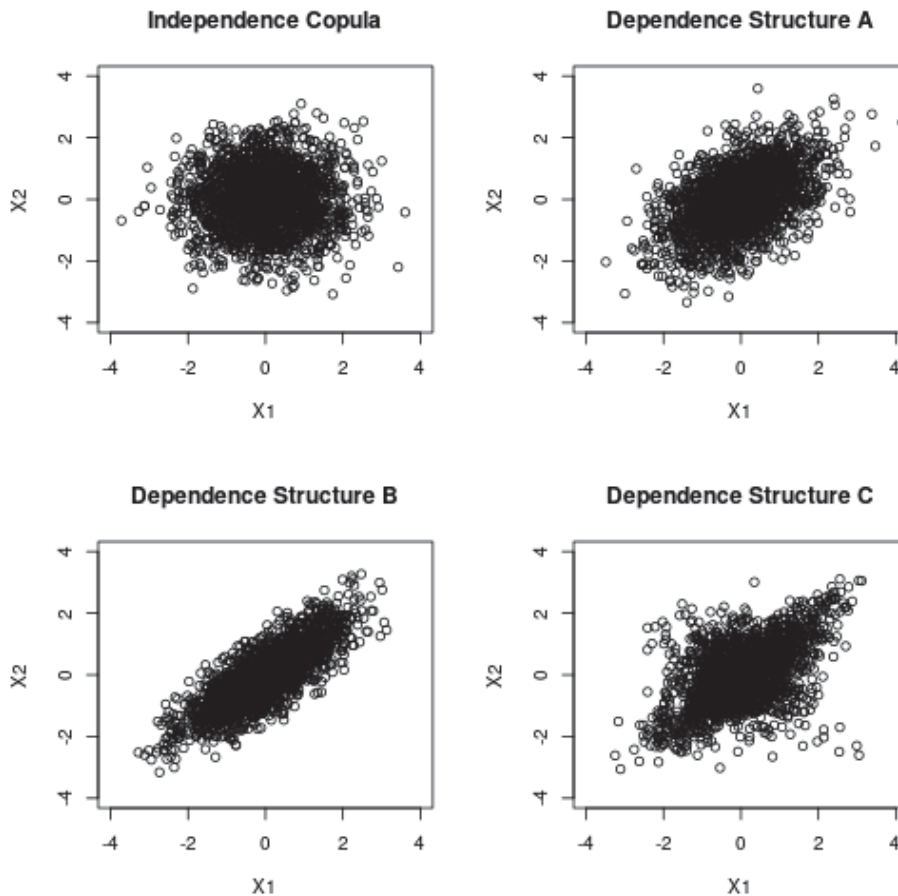
There are four categories of models with different Risk Drivers and Correlations.

Categories of models	Asset Value Model	Macroeconomic Models	Actuarial Models	Intensity Models
Model	KMV-Model (Portfolio Manager by KMV)	B	C	Jarrow/Lando/Turnbull-Model (Kamakura) Duffie/Singleton-Model
Risk Driver	Asset Value Process	Macro-economic Factors	Default Intensity	D
Correlations	A	Implicit by Macroeconomy	Implicit by Sectors	Correlated Intensity

- (a) (1 point) Identify A, B, C, D in the table above.
- (b) (1 point) Describe how you can introduce dependencies between the default times of the counterparties in a portfolio if intensity models are used.

5. Continued

You are given the following scatter plots describing different dependency structures:



You are also given the following descriptions relating to the scatter plots above:

- (1) T-Copula with correlation 0.5 and degree of freedom 2
- (2) Gaussian Copula with correlation 0.5
- (3) Gaussian Copula with correlation 0.8

All the marginals are standard normal.

- (c) (1.5 points) Identify for each of the dependence structures A, B, C pictured above the corresponding description (1), (2), (3) above. Justify your answers.
- (d) (0.5 points) Describe an easy way to increase extreme portfolio losses when using the T-Copula.

Question 5 continued on next page.

5. Continued

The CreditPortfolioView (CPV) model assumes that there are several risk segments reacting to the overall economic conditions.

- (e) (1 point) Describe a three-step algorithm that can generate the conditional migration matrices for different risk segments based on the average migration matrix.
- (f) (1 point) Interpret the state of economy and potential for downgrades or upgrades seen in light of segments in the CPV model, for the following two cases:
 - (i) risk index $r_s < 1$
 - (ii) risk index $r_s = 1$

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6. (6 points) You are working with both structural and reduced form models to evaluate Company A's credit risks.

(a) (1 point) Compare and contrast structural models and reduced form models for modeling default risk.

You consider credit default swaps (CDS) that give the holder the right to buy protection on a name, between time 0 and time T , for a spread of K per annum with notional principal L .

You are using the Hull and White model for credit default swaps (CDS):

- R is the expected risk-neutral recovery rate
- $A(t)$ is the accrued interest per dollar of principal on the reference bond
- $v(t)$ is the present value of one received at time t
- $u(t)$ is the present value of payments at the rate of one on CDS payment dates
- $q(t)$ is the risk neutral default probability density
- $\pi(t)$ is the probability that a company will survive at time t
- $e(t)$ is the present value of the final accrual payment that would be required on the CDS at time t in the event of a default at time t if payments were made at the rate of \$1 per year

(b) (1 point) Describe the steps to value a CDS using the Hull and White approach.

6. Continued

- (c) (0.5 points) Express the CDS payoff (in terms of L , $A(t)$ and R) in the event of default at time t ($0 < t < T$).
- (d) (0.5 points) Express the present value of future payments on the CDS in terms of L , K , and $u(T)$, given that there is no default prior to time T .
- (e) (1.5 points) Show that CDS Spread satisfies the following equation:

$$\text{Spread} = \frac{\int_{t=0}^T [1 - R - A(t) * R] q(t) v(t) dt}{\int_{t=0}^T q(t) * \{u(t) + e(t)\} dt + \pi(T) * u(T)}$$

You are given the following additional information under the Hull and White model for CDS:

- The buyer of protection will find that the cheapest-to-deliver bond is the one for which the accrued interest is least.
- You are provided the following 3 bonds.

	Bond 1	Bond 2	Bond 3
Accrued interest	0%	1%	2%

- Expected risk-neutral recovery rate is 40%.
 - $q(t) = 0.005e^{-0.005t}$
 - $v(t) = e^{-0.01t}$
 - Maturity $T = 10$
 - Assume $\int_{t=0}^T q(t) * \{u(t) + e(t)\} dt + \pi(T) * u(T) = 2$
- (f) (1.5 points) Calculate the CDS Spread under the Hull and White model.

7. (5 points) Justin is the chair of ABC Investment Firm. Justin stops at a local DEF Donuts store every day on his way to work and has developed a strong liking for the restaurant and its employees. He recently recommended investing in DEF Donuts, a national chain, at an Investment Committee meeting.

(a) (1 point) Describe a possible behavioral bias with Justin's recommendation.

Michael owns DEF Donuts stock and is making a decision on whether to sell his stock or not. Michael originally bought the stock for \$45 and it is currently trading at \$50. He believes the stock will either decrease to \$45 or increase to \$55, each with equal 50% probability.

(b) (1 point) Assess Michael's decision to sell his stock according to prospect theory.

The DEF Donuts Chief Financial Officer (CFO) is evaluating whether to release equity back to shareholders through dividends or share repurchases. She has observed that the company's shareholders are predominantly tax-sheltered investors with a strong behavioral tendency to segregate gains from losses.

(c) (2 points) Recommend which equity distribution strategy the CFO should take and justify your answer.

The CFO decides to implement a dividend strategy with a targeted dividend payout rate based on the portion of earnings she believes is fair to return to shareholders. The company will increase future dividends with increases in earnings, but only if it is confident it will not have to reduce dividends in the future.

(d) (1 point) Critique the CFO's dividend strategy.

8. (5 points) You are an investment actuary modeling volatility smiles across multiple asset classes.

- (a) (1 point) Describe two historical facts which have been observed of interest-rate smiles.

You are reviewing the historical behaviors of equity smiles in normal and exceptional market conditions.

- (b) (2 points) Compare and contrast the behaviors, in normal versus exceptional market conditions, of the level, the skew and the convexity of equity smiles as a function of the maturity of the options.

You are now reviewing the shape of volatility smiles of various assets.

- (c) (1 point) Compare the shape of FX smiles for mature-market/mature-market currency pairs and the shape of FX smiles for mature-market/emerging-market currency pairs to the shape of volatility smiles of other asset classes.

Your manager asks you to develop a model for the dynamics of the S&P 500 equity index using your expertise about equity smiles. He believes that the call option price movement captured by the model should always be in the same direction as the underlying equity movement.

- (d) (1 point) Evaluate your manager's statement.

9. (6 points) MAB Inc. is a publicly traded stock company. It recently issued a zero-coupon bond to raise money and XYZ Life is the sole buyer of the bond.

You are given the following information about MAB Inc:

- MAB's total asset value is estimated at \$100 million. Its total asset value is estimated as the sum of the market value of its equity and the value of the zero coupon bond.
- The zero-coupon bond is the only liability of MAB. The maturity value of this zero-coupon bond is \$100 million and the remaining time-to-maturity is 10 years.
- MAB's total asset annual volatility (standard deviation) is 10%.
- MAB has 2 million outstanding shares.

Consider the Merton's Asset Value Model.

- (a) (1 point) Describe the position of the debtholder XYZ Life and the equity holders, using relevant derivative instruments and associated parameters.
- (b) (3 points) Calculate the total value of MAB equity, given the risk-neutral probability of default to be 21.5%.

Consider a call option on MAB's stock with the strike price of \$12.

- (c) (1 point) Determine if this call option is currently in the money, at the money, or out of the money.
- (d) (1 point) Calculate the volatility of MAB equity.

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