Society of Actuaries

Canadian Institute of Actuaries

Exam MLC
Models for Life Contingencies

Friday, October 27, 2017
8:30 a.m. – 12:45 p.m.

MLC

INSTRUCTIONS TO CANDIDATES

General Instructions

1. Write your candidate number here _____. Your name must not appear.

2. Do not break the seal of this book until the supervisor tells you to do so.

3. Tables and numerical values necessary for solving some of the questions on this examination will be distributed by the Supervisor.

4. This examination has a total of 96 points. It consists of:

   Section A: 20 multiple-choice questions, each worth 2 points for a total of 40 points, and

   Section B: 6 written-answer questions, worth a total of 56 points. The point value for each written-answer question is indicated at the beginning of the question.

   You may divide your time between the two sections of the examination (written-answer, and multiple-choice) as you choose. You should keep in mind the relative weight of the two sections.

   Your written-answer paper will be graded only if your multiple-choice score is at or above a threshold set after the examination is administered.

5. Failure to stop writing or coding after time is called will result in the disqualification of your answers or further disciplinary action.

6. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions on the exam booklet.

Multiple-Choice Instructions

1. A separate answer sheet for the multiple-choice questions is inside the front cover of this book. During the time allotted for this examination, record all your answers on the back of the answer sheet. NO ADDITIONAL TIME WILL BE ALLOWED FOR THIS PURPOSE.

   No credit will be given for anything indicated in the examination book but not transferred to the answer sheet.

2. On the front of the answer sheet, space is provided to write and code candidate information. Complete the information requested by printing in the squares and blackening the circles (one in each column) corresponding to the letters or numbers printed. For each empty box blacken the small circle immediately above the "A" circle. Fill out the boxes titled:

   (a) Name
   (include last name, first name and middle initial)

   (b) Candidate Number
   (Candidate/Eligibility Number, use leading zeros if needed to make it a five digit number)

   (c) Test Site Code
   (The supervisor will supply the number.)

   (d) Examination Part
   (Code the examination that you are taking by blackening the circle to the left of "Exam MLC")

   (e) Booklet Number
   (The booklet number can be found in the upper right-hand corner of this examination book. Use leading zeros if needed to make it a four digit number.)

   In box titled "Complete this section only if instructed to do so," fill in the circle to indicate if you are using a calculator and write in the make and model number.

   In the box titled "Signature and Date" sign your name and write today's date. **If the answer sheet is not signed, it will not be graded**.

   Leave the boxes titled "Test Code" and "Form Code" blank.

   On the back of the answer sheet fill in the Booklet Number in the space provided.

CONTINUED ON INSIDE FRONT COVER

475 N. Martingale Road
Schaumburg, IL 60173-2226

Printed in the U.S.A.
Exam MLC - Front Cover

3. Your score will be based on the number of questions which you answer correctly. No credit will be given for omitted answers and no credit will be lost for wrong answers: hence, you should answer all questions even those for which you have to guess.

4. Five answer choices are given with each multiple-choice question, each answer choice being identified by a key letter (A to E). Answer choices for some questions have been rounded. For each question, blacken the circle on the answer sheet which corresponds to the key letter of the answer choice that you select.

5. Use a soft-lead pencil to mark the answer sheet. To facilitate correct mechanical scoring, be sure that, for each question, your pencil mark is dark and completely fills only the intended circle. Make no stray marks on the answer sheet. If you have to erase, do so completely.

6. Do not spend too much time on any one question. If a question seems too difficult, leave it and go on.

7. Clearly indicated answer choices in the test book can be an aid in grading examinations in the unlikely event of a lost answer sheet.

8. After the examination, the supervisor will collect this book and the answer sheet separately. DO NOT ENCLOSE THE ANSWER SHEET IN THE BOOK OR IN THE ESSAY ANSWER ENVELOPE. All books and answer sheets must be returned. THE QUESTIONS ARE CONFIDENTIAL AND MAY NOT BE TAKEN FROM THE EXAMINATION ROOM.

Written-Answer Instructions

1. Write your candidate number at the top of each sheet. Your name must not appear.

2. Write on only one side of a sheet. Start each question on a fresh sheet. On each sheet, write the number of the question you are answering. Do not answer more than one question on a single sheet.

3. The answer should be confined to the question as set.

4. When you are asked to calculate, show all your work including any applicable formulas.

5. When you finish, insert all your written-answer sheets into the Essay Answer Envelope. Be sure to hand in all your answer sheets because they cannot be accepted later. Seal the envelope and write your candidate number in the space provided on the outside of the envelope. Check the appropriate box to indicate Exam MLC.

6. Be sure your essay answer envelope is signed because if it is not, your examination will not be graded.

7. For all parts of all problems, to maximize the credit earned, candidates should show as much work as possible, considering the time allotted for the question. Answers lacking justification will receive no credit. Answers should be organized so that the methods, logic, and formulas used are readily apparent. Candidates should not round their answers excessively; enough precision should be provided so that their answers can be accurately graded.

   In some cases, candidates are asked to show that a calculation results in a particular number. Typically the answer given will be rounded; candidates should provide a greater level of accuracy than the number given in the question. This structure of question is intended to assist the candidate by giving an indication when the calculation has been done incorrectly, providing an opportunity to explore an alternative approach. It also allows a candidate who cannot obtain the correct answer to use the answer given to proceed with subsequent parts of the problem. (Candidates who are able to solve the problem should use their exact answer for subsequent parts.)

   For questions requiring candidates to derive or write down a formula or equation, the resulting expression should be simplified as far as possible, and where numerical values are provided in the problem, they should be used.

Exam MLC

SECTION A – Multiple-Choice

****BEGINNING OF EXAMINATION****

1. A life is subject to the following 3-year select and ultimate table:

$[x]$	$\ell_{[x]}$	$\ell_{[x]+1}$	$\ell_{[x]+2}$	ℓ_{x+3}	$x+3$
55	10,000	9,493	8,533	7,664	58
56	8,547	8,028	6,889	5,630	59
57	7,011	6,443	5,395	3,904	60
58	5,853	4,846	3,548	2,210	61

You are also given:

(i) $e_{60} = 1$

(ii) Deaths are uniformly distributed over each year of age.

Calculate $\overset{o}{e}_{[58]+2}$.

(A) 1.5

(B) 1.6

(C) 1.7

(D) 1.8

(E) 1.9

USE THIS PAGE FOR YOUR SCRATCH WORK

EXTRA BLANK PAPER IS PROVIDED AT THE END OF THE EXAM BOOK

2. You are given the following excerpt from a double decrement table:

x	$\ell_x^{(\tau)}$	$q_x^{(1)}$	$q_x^{(2)}$
53	---	0.025	0.030
54	5000	---	0.040
55	4625	0.055	0.050

Calculate ${}_2q_{53}^{(1)}$.

(A) 0.056

(B) 0.057

(C) 0.058

(D) 0.059

(E) 0.060

USE THIS PAGE FOR YOUR SCRATCH WORK

EXTRA BLANK PAPER IS PROVIDED AT THE END OF THE EXAM BOOK

3. You are given the following Markov chain model:

(i) Annual transition probabilities between the states Healthy, Sick and Dead, of an organism are as follows:

	Healthy	Sick	Dead
Healthy	0.64	0.16	0.20
Sick	0.36	0.24	0.40
Dead	0	0	1

(ii) Transitions occur at the end of the year.

A population of 1,000 organisms starts in the Healthy state. Their future states are independent.

Using the normal approximation without the continuity correction, calculate the probability that there will be at least 625 organisms alive (Healthy or Sick) at the beginning of the third year.

(A) 13.6%

(B) 14.6%

(C) 15.6%

(D) 16.6%

(E) 17.6%

USE THIS PAGE FOR YOUR SCRATCH WORK

EXTRA BLANK PAPER IS PROVIDED AT THE END OF THE EXAM BOOK

4. For an annuity-due that pays 100 at the beginning of each year that (45) is alive, you are given:

(i) Mortality for standard lives follows the Illustrative Life Table.

(ii) The force of mortality for standard lives age $45+t$ is represented as μ_{45+t}^{ILT}.

(iii) The force of mortality for substandard lives age $45+t$, μ_{45+t}^{S}, is defined as:

$$\mu_{45+t}^{S} = \begin{cases} \mu_{45+t}^{ILT} + 0.05, & \text{for } 0 \le t < 1 \\ \mu_{45+t}^{ILT}, & \text{for } t \ge 1 \end{cases}$$

(iv) $i = 0.06$

Calculate the actuarial present value of this annuity for a substandard life age 45.

(A) 1347

(B) 1357

(C) 1367

(D) 1377

(E) 1387

USE THIS PAGE FOR YOUR SCRATCH WORK

EXTRA BLANK PAPER IS PROVIDED AT THE END OF THE EXAM BOOK

5. Ming, age 80, purchases a whole life insurance policy of 100,000.

You are given:

(i) The policy is priced with a select period of one year.

(ii) The select mortality rate equals 80% of the mortality rate from the Illustrative Life Table.

(iii) Ultimate mortality follows the Illustrative Life Table.

(iv) $i = 0.06$

Calculate the actuarial present value of the death benefits for this insurance.

(A) 66,000

(B) 66,100

(C) 66,200

(D) 66,300

(E) 66,400

USE THIS PAGE FOR YOUR SCRATCH WORK

EXTRA BLANK PAPER IS PROVIDED AT THE END OF THE EXAM BOOK

6. For a 25-year pure endowment of 1 on (x), you are given:

(i) Z is the present value random variable at issue of the benefit payment.

(ii) $Var(Z) = 0.10E[Z]$

(iii) ${}_{25}p_x = 0.57$

Calculate the annual effective interest rate.

(A) 5.8%

(B) 6.0%

(C) 6.2%

(D) 6.4%

(E) 6.6%

USE THIS PAGE FOR YOUR SCRATCH WORK

EXTRA BLANK PAPER IS PROVIDED AT THE END OF THE EXAM BOOK

7. For a special 10-year deferred whole life annuity-due of 300 per year issued to (55), you are given:

(i) Annual premiums are payable for 10 years.

(ii) If death occurs during the deferral period, all premiums paid are returned without interest at the end of the year of death.

(iii) $\ddot{a}_{55} = 12.2758$

(iv) $\ddot{a}_{55:\overline{10}|} = 7.4575$

(v) $(IA)^{1}_{55:\overline{10}|} = 0.51213$

Calculate the level net premium.

(A) 195

(B) 198

(C) 201

(D) 204

(E) 208

USE THIS PAGE FOR YOUR SCRATCH WORK

EXTRA BLANK PAPER IS PROVIDED AT THE END OF THE EXAM BOOK

8. For a 10-year deferred whole life annuity-due with payments of 100,000 per year on (70), you are given:

(i) Annual gross premiums of G are payable for 10 years.

(ii) First year expenses are 75% of premium.

(iii) Renewal expenses for years 2 and later are 5% of premium during the premium paying period.

(iv) Mortality follows the Illustrative Life Table.

(v) $i = 0.06$

Calculate G using the equivalence principle.

(A) 33,400

(B) 33,900

(C) 34,400

(D) 34,900

(E) 35,400

USE THIS PAGE FOR YOUR SCRATCH WORK

EXTRA BLANK PAPER IS PROVIDED AT THE END OF THE EXAM BOOK

9. For a special fully discrete 5-year deferred 3-year term insurance of 100,000 on (x) you are given:

(i) There are two premium payments, each equal to P. The first is paid at the beginning of the first year and the second is paid at the end of the 5-year deferral period.

(ii) The following probabilities:

$${}_5p_x = 0.95$$

$$q_{x+5} = 0.02, \quad q_{x+6} = 0.03, \quad q_{x+7} = 0.04$$

(iii) $i = 0.06$

Calculate P using the equivalence principle.

(A) 3195

(B) 3345

(C) 3495

(D) 3645

(E) 3895

USE THIS PAGE FOR YOUR SCRATCH WORK

EXTRA BLANK PAPER IS PROVIDED AT THE END OF THE EXAM BOOK

10. For a special whole life insurance of 100,000 on (40), you are given:

(i) The death benefit is payable at the moment of death.

(ii) Level gross premiums are payable monthly for a maximum of 20 years.

(iii) Mortality follows the Illustrative Life Table.

(iv) $i = 0.06$

(v) Deaths are uniformly distributed over each year of age.

(vi) Initial expenses are 200.

(vii) Renewal expenses are 4% of each premium including the first.

(viii) Gross premiums are calculated using the equivalence principle.

Calculate the monthly gross premium.

(A) 108

(B) 118

(C) 128

(D) 138

(E) 148

USE THIS PAGE FOR YOUR SCRATCH WORK

EXTRA BLANK PAPER IS PROVIDED AT THE END OF THE EXAM BOOK

11. For a 20-year term life insurance of 100,000 on (45), you are given:

(i) The death benefit is payable at the moment of death.

(ii) Mortality follows the Illustrative Life Table.

(iii) $\delta = 0.05$

(iv) Deaths are uniformly distributed over each year of age.

Calculate the 95th percentile of the present value of benefits random variable for this insurance.

(A) 60,500

(B) 62,000

(C) 63,500

(D) 65,000

(E) 66,500

USE THIS PAGE FOR YOUR SCRATCH WORK

EXTRA BLANK PAPER IS PROVIDED AT THE END OF THE EXAM BOOK

12. For a fully discrete whole life insurance of 200,000 on (45), you are given:

(i) Mortality follows the Illustrative Life Table.

(ii) $i = 0.06$

(iii) The annual premium is determined using the equivalence principle.

Calculate the standard deviation of L_0, the present value random variable for the loss at issue.

(A) 41,550

(B) 46,550

(C) 51,550

(D) 56,550

(E) 61,550

USE THIS PAGE FOR YOUR SCRATCH WORK

EXTRA BLANK PAPER IS PROVIDED AT THE END OF THE EXAM BOOK

13. On July 15, 2017, XYZ Corp buys fully discrete whole life insurance policies of 1,000 on each of its 10,000 workers, all age 35. It uses the death benefits to partially pay the premiums for the following year.

You are given:

(i) Mortality follows the Illustrative Life Table.

(ii) $i = 0.06$

(iii) The insurance is priced using the equivalence principle.

Calculate XYZ Corp's expected net cash flow from these policies during July 2018.

(A) -63,400

(B) -63,500

(C) -63,600

(D) -63,700

(E) -63,800

USE THIS PAGE FOR YOUR SCRATCH WORK

EXTRA BLANK PAPER IS PROVIDED AT THE END OF THE EXAM BOOK

14. Bobby, age 40, buys a fully discrete whole life policy with a face amount of 1,000,000.

Immediately before the second annual premium payment is due, he stops paying premiums and the policy is converted to a paid-up policy with a reduced face amount.

Original and conversion pricing were based on the following:

(i) Mortality follows the Illustrative Life Table.

(ii) $i = 0.06$

(iii) The equivalence principle

The cash value of the policy is equal to the net premium reserve.

Calculate the face amount of the paid-up policy.

(A) 49,000

(B) 50,000

(C) 51,000

(D) 52,000

(E) 53,000

USE THIS PAGE FOR YOUR SCRATCH WORK

EXTRA BLANK PAPER IS PROVIDED AT THE END OF THE EXAM BOOK

15. For a special fully discrete whole life insurance of 1,000 on (45), you are given:

(i) The net premiums for year k are:

$$\begin{cases} P, & k = 1, 2, ..., 20 \\ P + W, & k = 21, 22, ... \end{cases}$$

(ii) Mortality follows the Illustrative Life Table.

(iii) $i = 0.06$

(iv) ${}_{20}V$, the net premium reserve at the end of the 20th year, is 0.

Calculate W.

(A) 25

(B) 29

(C) 33

(D) 37

(E) 41

USE THIS PAGE FOR YOUR SCRATCH WORK

EXTRA BLANK PAPER IS PROVIDED AT THE END OF THE EXAM BOOK

16. For a fully discrete whole life insurance of B on (x), you are given:

(i) Expenses, incurred at the beginning of each year, equal 30 in the first year and 5 in subsequent years.

(ii) The net premium reserve at the end of year 10 is 2290.

(iii) Gross premiums are calculated using the equivalence principle.

(iv) $i = 0.04$

(v) $\ddot{a}_x = 14.8$

(vi) $\ddot{a}_{x+10} = 11.4$

Calculate ${}_{10}V^g$, the gross premium reserve at the end of year 10.

(A) 2190

(B) 2210

(C) 2230

(D) 2250

(E) 2270

USE THIS PAGE FOR YOUR SCRATCH WORK

EXTRA BLANK PAPER IS PROVIDED AT THE END OF THE EXAM BOOK

17. For a Type A Universal Life Policy with a death benefit of 500,000, you are given:

(i) The account value at time 5 is 200,000.

(ii) Premiums of 25,000 are paid annually at the beginning of each year.

(iii) Expense charges are 2% of premium.

(iv) The COI rate per 1,000 in year 6 is 30.

(v) $i^c = i^q = 0.05$

(vi) The policy is subject to a corridor factor of 2.5.

Calculate the Additional Death Benefit for this policy at time 6.

(A) 272,400

(B) 288,900

(C) 305,400

(D) 321,900

(E) 338,400

USE THIS PAGE FOR YOUR SCRATCH WORK

EXTRA BLANK PAPER IS PROVIDED AT THE END OF THE EXAM BOOK

18. Ten years ago Jacob, then age 25, purchased a fully discrete 10-payment whole life policy of 10,000.

All actuarial calculations for this policy were based on the following:

(i) Mortality follows the Illustrative Life Table.

(ii) $i = 0.06$

(iii) The equivalence principle

L_{10} is the present value of future losses random variable at time 10.

At the end of policy year 10, the interest rate used to calculate L_{10} is changed to 0%.

Calculate the increase in $E[L_{10}]$ that results from this change.

(A) 4713

(B) 5713

(C) 6713

(D) 7713

(E) 8713

USE THIS PAGE FOR YOUR SCRATCH WORK

EXTRA BLANK PAPER IS PROVIDED AT THE END OF THE EXAM BOOK

19. You are given:

(i) Mortality follows the Illustrative Life Table.

(ii) Deaths are uniformly distributed over each year of age.

(iii) $i = 0.06$

Calculate $\frac{d}{dt}(\overline{Ia})_{40:\overline{t}|}$ at $t = 10.5$.

(A) 5.1

(B) 5.3

(C) 5.5

(D) 5.7

(E) 5.9

USE THIS PAGE FOR YOUR SCRATCH WORK

EXTRA BLANK PAPER IS PROVIDED AT THE END OF THE EXAM BOOK

20. Julie, age 45, is a participant in a defined benefit pension plan at DMN Widgets. You are given:

(i) She has 15 years of service.

(ii) Her salary for the past year was 120,000.

(iii) She will receive salary increases of 4% annually with the first increase tomorrow.

(iv) Her retirement benefit, payable annually at the start of the year, is 1.5% of her final year's salary, for each year of service.

(v) She can only retire in 20 years, at age 65.

(vi) She will not receive any benefits if she leaves prior to retirement.

(vii) ${}_{20}p_{45}^{(\tau)} = 0.552$

(viii) $i = 0.05$

(ix) $\ddot{a}_{65} = 10.60$

Using the projected unit credit funding method, calculate the normal cost for Julie's retirement benefits for the next year.

(A) 7900

(B) 8100

(C) 8300

(D) 8500

(E) 8700

Exam MLC

SECTION B – Written-Answer

1. (*8 points*) For a fully discrete 10-payment 20-year endowment insurance with a benefit of 100,000 issued to (50), you are given:

(i) Mortality follows the Illustrative Life Table.

(ii) $i = 0.06$

(iii) Reserves are calculated using the Full Preliminary Term (FPT) method.

(iv) Initial expenses are 500 plus 40% of the gross premium.

(v) Renewal expenses at each premium payment date after the first are 50 plus 4% of the gross premium.

(vi) Gross premiums are calculated using the equivalence principle.

(a) (*3 points*)

(i) Show that the gross premium, P^g, is 5340 to the nearest 10. You should calculate the gross premium to the nearest 1.

(ii) Calculate the expense loading, P^e.

(b) (*1 point*) Calculate the expense reserve at time 1.

(c) (*1 point*) Explain why the expense reserve at time 1 is negative.

(d) (*2 points*) Calculate the FPT reserve at times 1, 2 and 19.

(e) (*1 point*) Explain the rationale for the FPT method.

THIS PAGE INTENTIONALLY LEFT BLANK

2. (*8 points*) A life insurance company issues a Type B universal life policy with an Additional Death Benefit of 150,000 to a life age 35.

You are given:

Policy Year	Annual Premium	% Premium Charge	Annual Cost of Insurance Rate Per Thousand	Credited Interest Rate	Account Value at End of Year	Surrender Charge
1	4800	20%	2.31	4%	3647	2500
2	4800	5%	2.46	4%	8166	1500
3	4800	5%	2.62	4%	-	0

(a) (*2 points*) Show that the total death benefit at the end of the third year is 162,800 to the nearest 100. You should calculate the value to the nearest 1.

Now the company wants to carry out a profit test of this contract using the following assumptions:

(i) At the end of the first year, 10% of the in-force policyholders surrender.

(ii) At the end of the second year, 20% of the in-force policyholders surrender.

(iii) At the end of the third year, all remaining in-force policyholders surrender.

(iv) There are no reserves held other than the account value.

(v) The interest earned on all insurer's funds is 8% per year.

(vi) Mortality experience is $q_{35+t} = 0.0015$ for $t = 0, 1$ and 2.

(vii) Pre-contract expenses are 800.

(viii) Percent of premium expenses are 25% in the first year and 7% for years 2 and 3.

(ix) Expenses on death are 500.

(x) The hurdle rate is 10% per year.

2. **Continued**

(b) (*3 points*) Calculate the profit vector.

(c) (*2 points*) Calculate the NPV.

(d) (*1 point*) Calculate the profit margin.

3. (*9 points*) The XYZ Corporation Pension Plan provides the following benefits:

- An annual retirement benefit equal to 2% of the employee's average salary over the final 3 years prior to retirement for every year of service while a member of the plan.
- The Normal Retirement Date is the employee's 65th birthday.
- The pension benefit is payable monthly for life.
- An actuarial reduction factor of 0.5% times the number of months that retirement precedes the employee's 65th birthday is applied to the benefit.

You are given the following information regarding Caroline, a member of the Plan: Caroline was born on January 1, 1958 and became a member of the plan on January 1, 1988. Salary information is as follows:

Salary During Calendar Year		
2015	2016	2017
90,000	92,000	95,000

(a) (*1 point*) Show that the accrued annual pension benefit for Caroline as of December 31, 2017 is 55,000 to the nearest 1,000. You should calculate the value to the nearest 1.

The actuarial valuation is based on the following assumptions:

- A 6% annual effective rate of interest.
- Salary growth of 2% per year with salary increases applied on January 1 of each year.
- There are no exits before retirement.
- Mortality after retirement follows the Illustrative Life Table.
- A two-term Woolhouse approximation is used for life annuities paid more frequently than annually.
- There is a 50% probability that Caroline will retire on January 1, 2018; otherwise she will retire on her Normal Retirement Date.

3. Continued

(b) (*5 points*) Using the Traditional Unit Credit funding method:

(i) Calculate the actuarial accrued liability for Caroline as of December 31, 2017.

(ii) Calculate the gain or loss to the plan if Caroline chooses to retire on her 60^{th} birthday.

(c) (*3 points*) Using the Projected Unit Credit funding method:

(i) Calculate the actuarial accrued liability for Caroline as of December 31, 2017.

(ii) Calculate the gain or loss to the plan if Caroline chooses to retire on her 60^{th} birthday.

4. (*11 points*) For a fully discrete 10-payment whole life insurance of 10,000 on (35), you are given:

(i) The net premium, P, is 166.58.

(ii) Mortality follows the Illustrative Life Table.

(iii) $i = 0.06$

(a) (*1 point*) Write down an expression, in terms of P, K_{40} and interest functions, for the present value of future losses random variable at time 5, L_5.

(b) (*2 points*) Calculate the net premium reserve at time 5.

Let the random variable Z denote the present value, at time 5, of the future death benefits.
Let the random variable Y denote the present value, at time 5, of the future net premiums.

(c) (*3 points*)

(i) Calculate the standard deviation of Z.

(ii) Show that the standard deviation of Y is 42 to the nearest 1. You should calculate the standard deviation to the nearest 0.1.

(d) (*3 points*) Show that the covariance of Y and Z is -30,000 to the nearest 1,000. You should calculate the value to the nearest 10.

(e) (*1 point*) Calculate the standard deviation of L_5.

(f) (*1 point*) Explain why the covariance of Y and Z is negative.

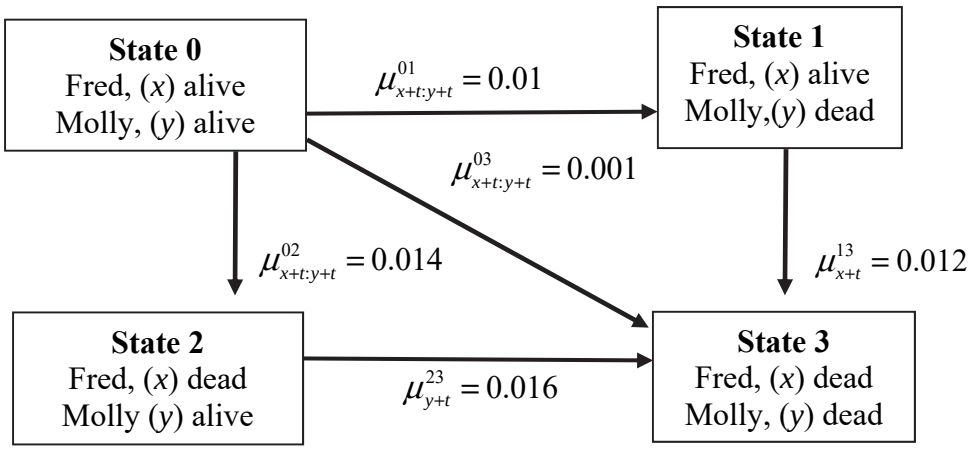
THIS PAGE INTENTIONALLY LEFT BLANK

5. (*11 points*) Fred and Molly are age x and y, respectively. They have just purchased a special joint life 10-year term insurance with a death benefit of 100,000.

The death benefit is payable immediately upon the simultaneous death of Fred and Molly. It is also payable immediately upon the death of Fred if Molly died first. There is no benefit payable if Fred dies first.

Net premiums are payable continuously at an annual rate of P while both Fred and Molly are alive, and at a rate of $0.75P$ after the death of Molly.

The contract is priced using a force of interest of 6% and the following common shock model:



(a) (*2 points*) Show that ${}_t p^{01}_{xy} = \frac{0.010}{0.013}\left(e^{-0.012t} - e^{-0.025t}\right)$.

(b) (*3 points*)

(i) Show that the Expected Present Value (EPV) of the death benefit on simultaneous deaths is 670 to the nearest 10. You should calculate the value to the nearest 1.

(ii) Show that the EPV of the death benefit payable if Fred dies after Molly is 360 to the nearest 10. You should calculate the value to the nearest 1.

(c) (*3 points*) Show that P equals 150 to the nearest 10. You should calculate the value to the nearest 0.1.

(d) (*2 points*) Calculate the net premium reserve at time 5 given that only Fred is alive (State 1).

5. **Continued**

(e) (*1 point*) The actuary realizes that the transition intensity from State 0 to State 2, $\mu_{x+t:y+t}^{02}$, should be lower. Without doing any calculations, state with reasons whether the EPV of the death benefits under the new pricing assumption will be lower, the same, or higher than the values obtained in part (b), assuming that the other transition intensities are correct.

6. (*9 points*)

(a) (*2 points*) Write down the Kolmogorov forward differential equation for ${}_tq_x^{(j)}$ in the case of a general multiple decrement model.

A person age 60 is subject to three decrements. You are given:

(i) The following excerpt from a triple decrement table:

x	$q_x^{(1)}$	$q_x^{(2)}$	$q_x^{(3)}$
60	0.05	0.10	0.08
61	0.00	0.14	0.12

(ii) Decrement 1 occurs exactly one quarter of the way through each year.

(iii) ${}_tq_x^{(2)} = tq_x^{(2)}$ for integer x and $0 \le t \le 1$.

(iv) $${}_tq_x^{(3)} = \begin{cases} 0, & \text{for integer } x \text{ and } 0 \le t < \frac{1}{2} \\ 2\left(t - \frac{1}{2}\right)q_x^{(3)}, & \text{for integer } x \text{ and } \frac{1}{2} \le t < 1 \end{cases}$$

(b) (*3 points*)

(i) Calculate ${}_2p_{60}^{(\tau)}$.

(ii) Calculate ${}_{0.8}p_{60}^{(\tau)}$.

(c) (*2 points*) Sketch the graph of ${}_tp_{60}^{(\tau)}$ as a function of t for $0 \le t \le 1$. Clearly label the axes and show all key values.

(d) (*2 points*) Using the result in part (a) or otherwise, calculate:

(i) $\mu_{60.8}^{(2)}$

(ii) $\mu_{60.8}^{(3)}$

****END OF EXAMINATION****

USE THIS PAGE FOR YOUR SCRATCH WORK

USE THIS PAGE FOR YOUR SCRATCH WORK