GI ADV Model Solutions
Fall 2017

1.

Learning Objectives:

4. The candidate will understand how to apply the fundamental techniques of reinsurance pricing.

Learning Outcomes:

(4c) Calculate the price for a casualty per occurrence excess treaty.

Source:

Basics of Reinsurance Pricing, Clark

Solution:

(a) Explain how the aggregate excess factor is analogous to a Table M charge factor.

The aggregate excess factor is the average amount of loss in excess of the aggregate limit, divided by the expected loss. The Table M charge factor is the average amount of loss in excess of *r* times the expected loss, divided by the expected loss. Hence, they measure the same thing, with a slightly different definition of the point above which the excess is calculated.

(b) Calculate the approximate expected losses in the layer using an exposure rating approach with an expected loss ratio of 60%.

Commentary on Question:

Most candidates did well, although there was often difficulty in correctly applying the aggregate excess factor of 0.10.

For the first item: $1,200,000\dfrac{(2.00-1.80)(0.90)+(1.50-1.00)(0.10)}{(2.00-1.50)(0.90)+1.50(0.10)}=460,000.$

For the second item: $785,000\dfrac{(2.00-1.80)(0.90)+(1.50-1.00)(0.10)}{(2.15-1.50)(0.90)+2.00(0.10)}=230,000.$

The total expected loss is (460,000 + 230,000)(0.6) = 414,000.

(c) Identify the treaty category under which this treaty would fall.

Clash cover

1. Continued

(d) Describe two ways that a loss on this treaty could occur.

Commentary on Question:
Any two of the following were sufficient for full credit.

- Multiple policies involved in a single occurrence
- Extra-contractual obligations
- Rulings awarding damages in excess of policy limits
- ALAE being included with losses and the total exceeding policy limits

2. **Learning Objectives:**

2. The candidate will understand the considerations in selecting a risk margin for unpaid claims.

Learning Outcomes:

(2a) Describe a risk margin analysis framework.

(2b) Identify the sources of uncertainty underlying an estimate of unpaid claims.

(2c) Describe methods to assess this uncertainty.

Source:

A Framework for Assessing Risk Margins, Marshall, et al.

Solution:

(a) Describe each of the following sources of risk:

(i) Parameter selection error for internal systemic risk

(ii) Random component of parameter risk

The parameter selection error for internal systemic risk is the error from the model being unable to adequately measure all predictors of claim cost outcomes or trends.

The random component of parameter risk is the extent to which the randomness associated with the insurance process compromises the ability to select appropriate parameters.

(b) Calculate each of the following:

(i) The independent risk coefficient of variation for both lines combined

(ii) The internal systemic risk coefficient of variation for both lines combined

(iii) The aggregate coefficient of variation for both lines combined

(iv) The amount of the risk margin

The independent risk coefficient of variation for both lines combined is

$$\sqrt{7\%^2\left(\frac{7,500}{11,500}\right)^2+10\%^2\left(\frac{4,000}{11,500}\right)^2}=5.7\%.$$

2. Continued

The internal systemic risk coefficient of variation for both lines combined is

$$\sqrt{6\%^2\left(\frac{7,500}{11,500}\right)^2+9\%^2\left(\frac{4,000}{11,500}\right)^2+2(0.25)(6\%)(9\%)\frac{(7,500)(4,000)}{11,500^2}}=5.6\%.$$

The aggregate coefficient of variation for both lines combined is

$$\sqrt{5.7\%^2+5.6\%^2+3.2\%^2}=8.6\%.$$

The risk margin is 0.086(0.674)(11,500) = 667.

(c) Explain why using the approach of part (b) may not be appropriate.

Commentary on Question:
Any two of the following were sufficient for full credit.

- The normal distribution may not be appropriate for an extreme percentile.
- The internal systemic risk correlation in the extreme tails may be more than 25%.
- There may be other risk drivers than those used.

3. **Learning Objectives:**

4. The candidate will understand how to apply the fundamental techniques of reinsurance pricing.

5. The candidate will understand methodologies for determining an underwriting profit margin.

Learning Outcomes:

(4a) Calculate the price for a proportional treaty.

(5a) Calculate an underwriting profit margin using the target total rate of return model.

(5c) Calculate an underwriting profit margin using the risk adjusted discount technique.

Sources:

Basics of Reinsurance Pricing, Clark

Ratemaking: A Financial Economics Approach, D'Arcy and Dyer

Solution:

(a) Calculate the premium using the Target Total Rate of Return Model.

$$15\% = \left(\frac{P+S-26}{S}\right)1.75\% + \frac{P}{S}UPM$$

$$S = 0.5P$$

$$UPM = \frac{P-26-70}{P}$$

$$0.15 = 0.0175\left(3 - \frac{52}{P}\right) + 2\left(1 - \frac{96}{P}\right)$$

$$0.15P = 0.0525P - 0.91 + 2P - 192$$

$$1.9025P = 192.91$$

$$P = 101.40$$

(b) Determine the implied risk-adjusted rate for losses.

Let r be the risk-adjusted rate. Then,

$$101.40 = \frac{70(1-0.35)}{1+r} + 26 + \frac{(101.40-26)(0.35)}{1.0175} + \frac{(50.70+101.40-26)(0.0175)(0.35)}{1.0175}$$

$$101.40 = \frac{45.5}{1+r} + 26 + 25.94 + 0.76$$

$$r = 45.5 / 48.70 - 1 = -0.066 = -6.6\%.$$

3. Continued

(c) Calculate the Total Rate of Return under each reinsurance offer.

For Reinsurance A:
The original UPM is 1 – [70 + 26]/101.40 = 0.0533 and is unchanged with this reinsurance.
Equity = 101.40(0.4)/2 = 20.28
Investable assets = 20.28 + 101.40 – 26 – 0.6(101.40) + 0.6(26) = 50.44
TRR = (50.44/20.28)(0.0175) + (2)(0.0533) = 15.0%.

For Reinsurance B:
UPM = 1 – 70/101.40 – [26 – 0.3(0.4)(101.40)]/[0.6(101.40)] = 0.0823
Equity = 101.40(0.6)/2 = 30.42
Investable assets = 30.42 + 101.40 – 26 – 0.4(101.40) + 0.3(0.4)(101.40) = 77.43
TRR = (77.43/30.42)(0.0175) + (2)(0.0823) = 20.9%.

4. **Learning Objectives:**

1. The candidate will understand how to use basic loss development models to estimate the standard deviation of an estimator of unpaid claims.

Learning Outcomes:

(1a) Identify the assumptions underlying the chain ladder estimation method.

(1b) Test for the validity of these assumptions.

(1c) Identify alternative models that should be considered depending on the results of the tests.

(1d) Estimate the standard deviation of a chain ladder estimator of unpaid claims.

Sources:

Measuring the Variability of Chain Ladder Reserve Estimates, Mack
Testing the Assumptions of Age-to-Age Factors, Venter

Solution:

(a) Demonstrate that the standard error for accident year 4 was correctly calculated.

$$C_{4,7}^2 \sum_{k=7+1-4=4}^{7-1=6} \frac{\alpha_k^2}{f_k^2}\left(\frac{1}{C_{4,k}}+\frac{1}{\sum_{j=1}^{7-k} C_{j,k}}\right) = C_{4,7}^2\left[\frac{\alpha_4^2}{f_4^2}\left(\frac{1}{C_{4,4}}+\frac{1}{\sum_{j=1}^{7-4=3} C_{j,4}}\right)+\frac{\alpha_5^2}{f_5^2}\left(\frac{1}{C_{4,5}}+\frac{1}{\sum_{j=1}^{7-5=2} C_{j,5}}\right)+\frac{\alpha_6^2}{f_6^2}\left(\frac{1}{C_{4,6}}+\frac{1}{\sum_{j=1}^{7-6=1} C_{j,6}}\right)\right]$$

$$= 42,644^2\left[\frac{40.0504}{0.95408^2}\left(\frac{1}{42,905}+\frac{1}{18,546+23,304+22,854}\right)\right.$$
$$\left.+\frac{0.00098}{1.02128^2}\left(\frac{1}{40,935}+\frac{1}{18,128+22,887}\right)+\frac{2.4\times10^{-8}}{1.02004^2}\left(\frac{1}{41,806}+\frac{1}{18,517}\right)\right]$$
$$= 3,101,519$$

The standard error is the square root, 1,761.

(b) Calculate the value of the term for accident year 2.

Commentary on Question:

Candidates performed poorly on this question. This particular formula was not tested on previous exams.

4. Continued

The term for AY 2 is the square of the standard error plus $C_{2,7}\left(\sum_{j=3}^{7} C_{j,7}\right)\sum_{k=6}^{6}\frac{2\alpha_k^2 / f_k^2}{\sum_{n=1}^{7-k} C_{n,k}}$. It is

$$0.04^2 + 23,839\left(21,583 + 42,644 + 27,507 + 27,598 + 37,576\right)\frac{2(2.4\times10^{-8}/1.02004^2)}{18,517}$$

$= 0.0109.$

(c) Explain why the estimators are dependent.

Each reserve estimate depends on the sequence of development factors. Some factors are used in more than one reserve estimate. For example, f_6 is used for both AY 4 and AY 5 reserve estimates. Any error in this factor will appear in both calculations.

(d) Calculate the weighted residual as defined by Mack for the observation at accident year 4 and development year 3.

The expected value is 19,333(1.46713) = 28,364. The weighted residual is (38,991 – 28,364)/sqrt(19,333) = 76.43.

(e) Determine the values of *n* and *p*.

n = 21, the number of estimated values. *p* = 6, the number of *f* values.

(f) Calculate the adjusted sum of squared errors using one of Venter's three recommended formulas.

The three formulas are:

- $\frac{184,086,659}{(21-6)^2} = 818,163$;
- $184,086,659e^{2(6)/21} = 325,979,727$; and
- $184,086,659(21^{6/21}) = 439,338,494$.

5. Learning Objectives:

1. The candidate will understand how to use basic loss development models to estimate the standard deviation of an estimator of unpaid claims.

Learning Outcomes:

(1e) Apply a parametric model of loss development.

(1f) Estimate the standard deviation of a parametric estimator of unpaid claims.

Source:

LDF Curve Fitting and Stochastic Reserving: A Maximum Likelihood Approach, Clark

Solution:

(a) Explain why no truncation adjustment is necessary.

The oldest age in the table is 30 and $G(30) = 0.93591$. With only 6.4% in the tail, a truncation adjustment will make little difference.

(b) Calculate the aggregate IBNR reserve for accident years 2014 to 2016.

$G(30) = 0.93591$, $G(18) = 0.88911$, $G(6) = 0.68834$
$ULT_{14} = 10{,}000/0.93591 = 10{,}685$
$ULT_{15} = 6{,}000/0.88911 = 6{,}748$
$ULT_{16} = 6{,}000/0.68834 = 8{,}717$
$IBNR = 10{,}685 + 6{,}748 + 8{,}717 - 22{,}000 = 4{,}150$

(c) Verify the calculation of the estimate of σ^2.

The estimated increments are:
AY14: 10,685(0.68834) = 7,355, 10,685(0.20077) = 2,145, 10,685(0.04680) = 500
AY15: 6,748(0.68834) = 4,645, 6,748(0.20077) = 1,355
AY16: 8,717(0.68834) = 6,000
Numerator:

$$\frac{(8,000-7,355)^2}{7,355}+\frac{(1,500-2,145)^2}{2,145}+\frac{(500-500)^2}{500}$$
$$+\frac{(4,000-4,645)^2}{4,645}+\frac{(2,000-1,355)^2}{1,355}=647$$

Estimate is 647/(6 – 5) = 647 where 6 is the number of data points and 5 is the number of parameters.

(d) Calculate the process standard deviation of the aggregate IBNR reserve.

It is sqrt[4,150(647)] = 1,639.

5. **Continued**

(e) Verify that the normalized residual for accident year 2015 at 24 months is 0.69.

It is $\dfrac{2,000-1,355}{\sqrt{1,355(647)}}=0.69.$

(f) Assess whether the residuals support the use of the chosen model.

Commentary on Question:
The most important point is that the two considerations be identified. It is reasonable to answer the second consideration "Yes" in that with the small number of points a trend is hard to establish. Due to the nature of the model and the formulas, the point at 36 months must be zero. Candidates were penalized for including that point in their analysis.

There are two considerations:

1. Are the residuals randomly scattered about zero? Yes, the points are about the same amount above and below.
2. Is the variability roughly constant? No, it seems to be increasing.

The residuals do not support the model.

6. **Learning Objectives:**

5. The candidate will understand methodologies for determining an underwriting profit margin.

Learning Outcomes:

(5d) Allocate an underwriting profit margin (risk load) among different accounts.

Source:

An Application of Game Theory: Property Catastrophe Risk Load, Mango

Solution:

(a) Calculate the renewal risk load for each account using the Marginal Variance method.

All calculations are done in thousands, with the multiplier adjusted accordingly.
$\mathrm{Var}(X) = 20^2(0.01)(0.99) + 10^2(0.02)(0.98) = 5.92$
$\mathrm{Var}(Y) = 4^2(0.01)(0.99) + 6^2(0.02)(0.98) = 0.864$
$\mathrm{Var}(X + Y) = 24^2(0.01)(0.99) + 16^2(0.02)(0.98) = 10.72$
Renewal risk load for X: $(10.72 – 0.864)(24) = 236.544$
Renewal risk load for Y: $(10.72 – 5.92)(24) = 115.2$

(b) Demonstrate that the Marginal Variance method is not renewal additive.

The total risk load using the Marginal Variance method is 236.544 + 115.2 = 351.744. The risk load for the two accounts combined is 10.72(24) = 257.28. The two do not match.

(c) Calculate the risk load for each account using the Covariance Share method.

Event 1:
Covariance to share is $\mathrm{Var}(X+Y) – \mathrm{Var}(X) – \mathrm{Var}(Y) = 24^2(0.01)(0.99) – 20^2(0.01)(0.99) – 4^2(0.01)(0.99) = 5.7024 – 3.96 – 0.1584 = 1.584$.
X’s share is $(20/24)(1.584) = 1.320$. Y’s share is $(4/24)(1.584) = 0.264$.

Event 2:
Covariance to share is $\mathrm{Var}(X+Y) – \mathrm{Var}(X) – \mathrm{Var}(Y) = 16^2(0.02)(0.98) – 10^2(0.02)(0.98) – 6^2(0.02)(0.98) = 5.0176 – 1.96 – 0.7056 = 2.352$.
X’s share is $(10/16)(2.352) = 1.470$. Y’s share is $(6/16)(2.352) = 0.882$.

X’s total: $1.320 + 1.470 = 2.790$. Y’s total: $0.264 + 0.882 = 1.146$.

X’s risk load: $(5.920 + 2.790)(24) = 209.04$.
Y’s risk load: $(0.864 + 1.146)(24) = 48.24$.

7. **Learning Objectives:**

3. The candidate will understand excess of loss coverages and retrospective rating.

Learning Outcomes:

(3a) Explain the mathematics of excess of loss coverages in graphical terms.

(3b) Calculate the expected value premium for increased limits coverage and excess of loss coverage.

(3e) Explain Table M and Table L construction in graphical terms.

Source:

The Mathematics of Excess of Loss Coverages and Retrospective Rating – A Graphical Approach, Lee

Solution:

(a) Identify which side of the equation represents the layer method.

The left side of the equation represents the layer method.

(b) Identify the areas on the following graph that correspond to each of the terms in the equation.

Commentary on Question:

This graph matches Figure 8 in Lee and the breakdown is presented in Equation (2.15).

Left-hand side: Areas IV and V
First term on right-hand side: Areas IV and VIII
Second term on right-hand side: Areas V and IX
Third term on right-hand side: Areas VIII and IX

(c) Identify the areas on the graph that correspond to $\varphi(S)$, the Table M charge at entry ratio *S*, and $\psi(R)$, the Table M savings at entry ratio *R*.

Commentary on Question:

See Figure 15 in Lee for the solution.

The Table M charge at entry ratio *S* is represented by Area II.
The Table M savings at entry ratio *R* is represented by Area VI.

8. Learning Objectives:

4. The candidate will understand how to apply the fundamental techniques of reinsurance pricing.

Learning Outcomes:

(4d) Apply an aggregate distribution model to a reinsurance pricing scenario.

Source:

Basics of Reinsurance Pricing, Clark

Solution:

(a) Calculate the probability that aggregate catastrophe losses will be 10 billion.

Commentary on Question:

While many candidates attempted to use the recursive formula, few were able to complete it correctly for the geometric frequency distribution.

With this geometric distribution, $a = 0.5$ and $b = 0$. The recursive formula is
$P(10) = 0.5[0.4(0.0119) + 0.3(0.0166) + 0.2(0.0231) + 0.1(0.0311)] = 0.0087$.

(b) Calculate the mean and coefficient of variation of aggregate catastrophe losses.

$E(S) = 0.4(1) + 0.3(2) + 0.2(3) + 0.1(4) = 2$
$Var(S) = 0.4(1) + 0.3(4) + 0.2(9) + 0.1(16) – 4 = 1$
$E(N) = 1$ (given)
$Var(N) = 2$ (given)
$E(A) = 1(2) = 2$
$Var(A) = 1(1) + 2(4) = 9$
$CoV = 3/2 = 1.5$.

(c) Identify one disadvantage of using the recursive formula to calculate aggregate distribution probabilities.

Commentary on Question:

Any one of the following was sufficient for full credit.

- The calculation is inconvenient when $E(N)$ is large.
- Only one severity distribution can be used.
- It requires an evenly spaced severity distribution.