SOCIETY OF ACTUARIES
Advanced Topics in General Insurance

Exam GIADV

Date: Tuesday, October 31, 2017
Time: 2:00 p.m. – 4:15 p.m.

INSTRUCTIONS TO CANDIDATES

General Instructions

1. This examination has a total of 40 points.

   This exam consists of 8 questions, numbered 1 through 8.

   The points for each question are indicated at the beginning of the question.
2. Failure to stop writing after time is called will result in the disqualification of your answers or further disciplinary action.
3. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions on the exam booklet.

Written-Answer Instructions

1. Write your candidate number at the top of each sheet. Your name must not appear.
2. Write on only one side of a sheet. Start each question on a fresh sheet. On each sheet, write the number of the question that you are answering. Do not answer more than one question on a single sheet.
3. The answer should be confined to the question as set.
4. When you are asked to calculate, show all your work including any applicable formulas.
5. When you finish, insert all your written-answer sheets into the Essay Answer Envelope. Be sure to hand in all your answer sheets because they cannot be accepted later. Seal the envelope and write your candidate number in the space provided on the outside of the envelope. Check the appropriate box to indicate Exam GIADV.
6. Be sure your written-answer envelope is signed because if it is not, your examination will not be graded.

Tournez le cahier d'examen pour la version française.

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Exam GIADV-Front Cover

****BEGINNING OF EXAMINATION****

1. (*5 points*) Casualty R Us Reinsurance Company has been presented with an opportunity to write a casualty per occurrence excess treaty covering the layer 500,000 excess of 500,000 on a portfolio of umbrella policies. The following limits profile has been provided:

Subject Premium	Underlying Limit	Policy Limit
1,200,000	1,000,000	1,000,000
785,000	1,000,000	2,000,000

An analysis of historical data indicates that the following increased limits factors apply to this portfolio:

Policy Limit	Increased Limits Factor
500,000	1.00
1,000,000	1.50
1,500,000	1.80
2,000,000	2.00
2,500,000	2.10
3,000,000	2.15

The aggregate excess factor ϕ on the underlying policies is 0.10.

(a) (*0.5 points*) Explain how the aggregate excess factor is analogous to a Table M charge factor.

(b) (*3 points*) Calculate the approximate expected losses in the layer using an exposure rating approach with an expected loss ratio of 60%.

Casualty R Us has also been presented with an opportunity to write a casualty per occurrence excess treaty covering the layer 3,000,000 excess of 2,000,000 on the same portfolio of umbrella policies.

(c) (*0.5 points*) Identify the treaty category under which this treaty would fall.

(d) (*1 point*) Describe two ways that a loss on this treaty could occur.

2. (*4 points*) You are calculating a risk margin for claim liabilities using the methodology set out in "A Framework for Assessing Risk Margins."

The risk margin is to be calculated at the 75% adequacy level and is to be based on the following sources of uncertainty, which are assumed to be mutually independent:

		Coefficients of Variation		
Line of Business	**Claim Liabilities**	**Independent Risk**	**Internal Systemic Risk**	**External Systemic Risk**
Motor	7,500	7.0%	6.0%	4.5%
Property	4,000	10.0%	9.0%	4.0%
Total	**11,500**			**3.2%**

- The correlation between lines for internal systemic risk was assessed at 25%.
- Claims are assumed to be normally distributed.
- The *z*-value of the 75th percentile of the normal distribution is 0.674.

(a) (*1 point*) Describe each of the following sources of risk:

(i) Parameter selection error for internal systemic risk

(ii) Random component of parameter risk

(b) (*2 points*) Calculate each of the following:

(i) The independent risk coefficient of variation for both lines combined

(ii) The internal systemic risk coefficient of variation for both lines combined

(iii) The aggregate coefficient of variation for both lines combined

(iv) The amount of the risk margin

Your manager also asked you to provide an estimate of the risk margin at the 99.5% adequacy level on the same portfolio.

(c) (*1 point*) Explain why using the approach of part (b) may not be appropriate.

3. (*5 points*) You are using the following assumptions to set the premium for a one-year policy:

- The premium will be collected at policy inception.
- Expenses of 26 will be paid at policy inception.
- Losses are expected to be 70 and will be paid at policy expiration.
- The target total rate of return is 15%.
- Investable assets are equal to premium minus expenses plus owners' equity.
- The ratio of premium to owners' equity is 2 to 1.
- The investment return is the risk-free rate of 1.75%.
- The tax rate on all income is 35% and taxes will be paid at policy expiration.

(a) (*1.5 points*) Calculate the premium using the Target Total Rate of Return Model.

The Risk Adjusted Discount Technique produces the same premium as determined in part (a).

(b) (*1.5 points*) Determine the implied risk-adjusted rate for losses.

Your company is considering quota share reinsurance. Two reinsurers have offered the following terms:

- Reinsurer A: 60% of premium ceded with commission rate the same as the current expense ratio, payable at policy inception.
- Reinsurer B: 40% of premium ceded with commission rate of 30%, payable at policy inception.

Your company's equity is reduced to maintain the ratio of premium to owners' equity at 2 to 1. The two proposals are to be evaluated based on the resulting Total Rate of Return.

(c) (*2 points*) Calculate the Total Rate of Return under each reinsurance offer.

4. (*6 points*) You are interested in determining the variability of unpaid claim estimates. The triangle of paid claims data you are working with, by accident year (AY) and development year, is presented below. The shaded cells have been completed using the standard chain ladder method. It is assumed that all claims are fully developed after seven years.

Mack's method of estimating reserve variability has been applied to this triangle. The key results are provided in the table.

	Development Year							
AY	1	2	3	4	5	6	7	Standard error
1	9,146	12,176	17,670	18,546	18,128	18,517	18,888	0
2	10,834	15,902	20,884	23,304	22,887	23,371	23,839	0.04
3	11,946	15,697	20,478	22,854	20,718	21,159	21,583	5.64
4	12,414	19,333	38,991	42,905	40,935	41,806	42,644	1,761
5	14,284	20,888	25,210	27,675	26,405	26,967	27,507	1,514
6	15,648	17,240	25,293	27,767	26,492	27,056	27,598	7,217
7	17,221	23,473	34,438	37,806	36,070	36,838	37,576	9,765
f_k	1.36304	1.46713	1.09779	0.95408	1.02128	1.02004		
α_k^2	366.962	2012.50	18.3273	40.0504	0.00098	2.4×10^{-8}		

(a) (*1.5 points*) Demonstrate that the standard error for accident year 4 was correctly calculated.

The formula for the square of the standard error of the overall reserve estimator is a sum taken over accident years 2 through 7. For each accident year, the term is the sum of two components.

(b) (*2 points*) Calculate the value of the term for accident year 2.

The second component in each term must be positive because the reserve estimators for pairs of accident years are positively correlated. In discussing this formula with your actuarial student, he questions this statement by noting that under Mack's assumptions, future development depends only on current development for that accident year and hence reserve estimators for different accident years are independent.

(c) (*1 point*) Explain why the estimators are dependent.

(d) (*0.5 points*) Calculate the weighted residual as defined by Mack for the observation at accident year 4 and development year 3.

4. Continued

Venter suggests a method for testing the Mack model against other models. The method begins by using the model to predict the values for those cells that have observations and then sums the squared errors over all such cells. The total sum of squared errors is 184,086,659. Venter then suggests that this value be adjusted using a formula that involves n, the number of terms in the sum of squared errors, and p, the number of estimated parameters.

(e) (*0.5 points*) Determine the values of n and p.

(f) (*0.5 points*) Calculate the adjusted sum of squared errors using one of Venter's three recommended formulas.

5. (*7 points*) You are given the following triangle of cumulative reported losses:

	Months of Development		
Accident Year	12	24	36
2014	8,000	9,500	10,000
2015	4,000	6,000	
2016	6,000		

You apply Clark's stochastic reserving model using the LDF method and a loglogistic distribution with cumulative distribution function $G(x) = \frac{x^{\omega}}{(x^{\omega} + \theta^{\omega})}$ where x is in months. The maximum likelihood estimates of ω and θ are 1.1736 and 3.0544, respectively.

(a) (*1 point*) Explain why no truncation adjustment is necessary.

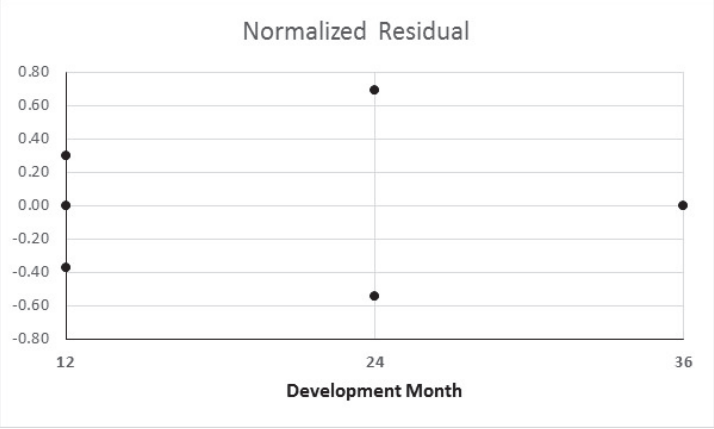
(b) (*1.5 points*) Calculate the aggregate IBNR reserve for accident years 2014 to 2016.

The estimate of σ^2 is 647.

(c) (*2.5 points*) Verify the calculation of the estimate of σ^2.

(d) (*0.5 points*) Calculate the process standard deviation of the aggregate IBNR reserve.

You have plotted the following normalized residuals:



(e) (*0.5 points*) Verify that the normalized residual for accident year 2015 at 24 months is 0.69.

(f) (*1 point*) Assess whether the residuals support the use of the chosen model.

6. (*4 points*) Unknown Reinsurance Company (URC) is renewing two accounts, X and Y, each of which is exposed to two possible independent claim events, 1 and 2. You are given the following information:

Event (*i*)		**Loss for Account**	
i	***p(i)***	**X**	**Y**
1	1%	20,000	4,000
2	2%	10,000	6,000

- *p(i)* represents the probability of Event *i*.
- The risk load multiplier, λ, is 0.000024.

(a) (*2 points*) Calculate the renewal risk load for each account using the Marginal Variance method.

(b) (*0.5 points*) Demonstrate that the Marginal Variance method is not renewal additive.

URC is considering using the Covariance Share method to calculate risk loads. The shared covariance of each event will be allocated to each account in proportion to its loss for that event.

(c) (*1.5 points*) Calculate the risk load for each account using the Covariance Share method.

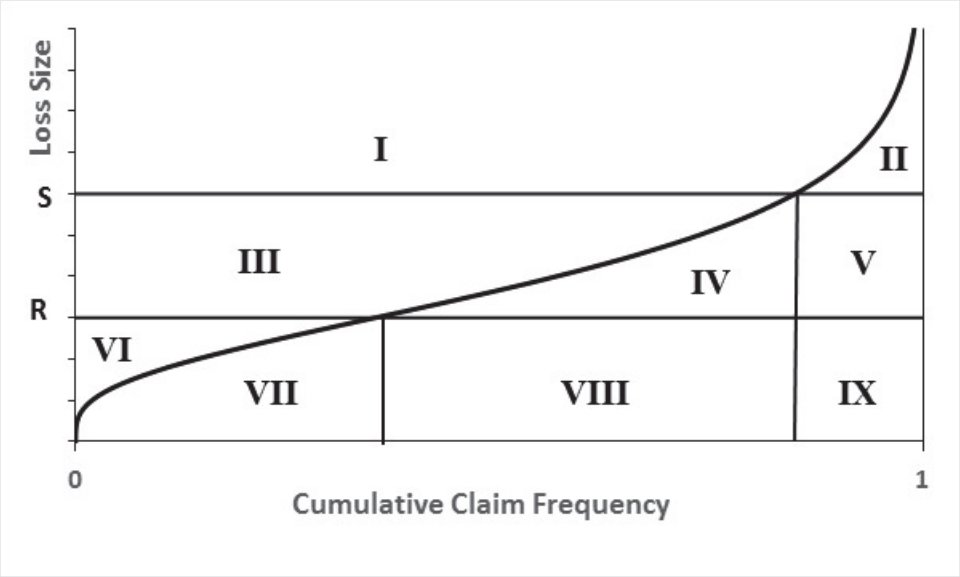
7. (*4 points*) The equivalence between the size method and the layer method of expressing the expected payment per ground-up claim on an excess of loss contract can be represented by the equation

$$\int_R^S G(x)dx = \int_R^S xdF(x) + S \cdot G(S) - R \cdot G(R),$$

where R is the retention, S is the retention plus the limit, F is the cumulative distribution function, and $G = 1 - F$.

(a) (*0.5 points*) Identify which side of the equation represents the layer method.

(b) (*2.5 points*) Identify the areas on the following graph that correspond to each of the terms in the equation.



Now suppose that R and S are entry ratios in a retrospective rating plan and F is the cumulative distribution function of the actual loss in units of expected loss.

(c) (*1 point*) Identify the areas on the graph that correspond to $\varphi(S)$, the Table M charge at entry ratio S, and $\psi(R)$, the Table M savings at entry ratio R.

8. (*5 points*) You project that the number of catastrophe losses next year for your company will follow a distribution with mean 1, variance 2 and probability function

$$p(n) = (0.5)^{n+1},\ n = 0,1,2,\ldots\ .$$

You also project that each loss size will have the following probability function:

Loss Size	Probability
1 billion	0.4
2 billion	0.3
3 billion	0.2
4 billion	0.1

Loss sizes are assumed to be independent of one another and independent of the number of losses.

The aggregate distribution of catastrophe losses has the following probability function for aggregate losses below 10 billion:

Aggregate Losses	Probability
0 billion	0.5000
1 billion	0.1000
2 billion	0.0950
3 billion	0.0840
4 billion	0.0661
5 billion	0.0403
6 billion	0.0311
7 billion	0.0231
8 billion	0.0166
9 billion	0.0119

(a) (*3 points*) Calculate the probability that aggregate catastrophe losses will be 10 billion.

(b) (*1.5 points*) Calculate the mean and coefficient of variation of aggregate catastrophe losses.

(c) (*0.5 points*) Identify one disadvantage of using the recursive formula to calculate aggregate distribution probabilities.

****END OF EXAMINATION****

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