MLC Spring 2016 Written Answer Questions

Model Solutions

MLC Spring 2016

Question 1 Model Solution

Learning Outcome: 1(b), 2(a), 3(a), 4(a)

Chapter References: AMLCR Chapter 8

General comments:

The question was done well, overall. Most candidates whose written answers were graded achieved full credit for parts (a) to (d); fewer than half for part (e). Parts (f) and (g) were less well done, with many candidates omitting these parts entirely, and many others achieving few or no points for their work.

(a)

$$\begin{aligned} A = {}_3p_{90}^{00} &= {}_2p_{90}^{00}\, p_{92}^{00} + {}_2p_{90}^{01}\, p_{92}^{10} \\ &= 0.5880 * 0.51 + 0.1245 * 0.2 = 0.32478 \\ B = {}_3p_{90}^{01} &= {}_2p_{90}^{00}\, p_{92}^{01} + {}_2p_{90}^{01}\, p_{92}^{11} \\ &= 0.5880 * 0.19 + 0.1245 * 0.25 = 0.142845 \\ C = {}_3p_{90}^{02} &= {}_2p_{90}^{00}\, p_{92}^{02} + {}_2p_{90}^{01}\, p_{92}^{12} + {}_2p_{92}^{02} \\ &= 0.5880 * 0.30 + 0.1245 * 0.55 + 0.2875 = 0.532375 \end{aligned}$$

OR

$$C = 1 - A - B = 0.532375$$

where

$${}_2p_{90}^{00} = 0.5880 \quad {}_2p_{90}^{01} = 0.1245 \quad {}_2p_{90}^{02} = 0.2875$$
$$p_{92}^{01} = 0.19 \quad p_{92}^{02} = 0.30$$
$$p_{92}^{00} = 1 - p_{92}^{01} - p_{92}^{02} = 1 - 0.19 - 0.30 = 0.51$$
$$p_{92}^{10} = 0.20 \quad p_{92}^{12} = 0.55$$
$$p_{92}^{11} = 1 - p_{92}^{10} - p_{92}^{12} = 1 - 0.20 - 0.55 = 0.25$$

(b)

$$\begin{aligned} \text{EPV Disability Benefit} &= 25\,000 \left(v\, p_{90}^{01} + v^2\, {}_2p_{90}^{01} + v^3\, {}_3p_{90}^{01} \right) \\ &= 25\,000 * 0.26643 = 6660.7 \end{aligned}$$

(c)

$$\begin{aligned}\text{EPV Death Benefit} &= 100\,000\left(v\,p_{90}^{02} + v^2\left({}_2p_{90}^{02} - p_{90}^{02}\right) + v^3\left({}_3p_{90}^{01} - {}_2p_{90}^{02}\right)\right)\\ &= 100\,000\left(v * 0.1 + v^2(0.2875 - 0.1) + v^3(0.5324 - 0.2875)\right)\\ &= 100\,000 * 0.44773 = 44\,773\end{aligned}$$

(d)

$$\begin{aligned}\text{EPV annual premium } P &= P\left(1 + vp_{90}^{00} + v^2\,{}_2p_{90}^{00}\right)\\ &= 2.29115P\end{aligned}$$

$$\Rightarrow P = \frac{6660.7 + 44\,773}{2.29115} = 22\,449$$

(e)

$$\begin{aligned}{}_2V^{(0)} &= 25000\,v\,p_{92}^{01} + 100\,000\,v\,p_{92}^{02} - P\\ &= 25\,000 * v * 0.19 + 100\,000 * v * 0.30 - 22449\\ &= 9726.95\\ {}_2V^{(1)} &= 25000\,v\,p_{92}^{11} + 100\,000\,v\,p_{92}^{12}\\ &= 25\,000 * v * 0.25 * v + 100\,000 * v * 0.55\\ &= 56712.96\end{aligned}$$

(f) The probability that no benefit is paid out over the three year policy term is

$$\begin{aligned}p_{90}^{00} * p_{91}^{00} * p_{92}^{00} &= 0.85 * 0.68 * 0.51\\ &= 0.29478\end{aligned}$$

Let P^* denote the revised premium. The EPV of the return of premium benefit is

$$3 * P^* * (0.29478) * v^3 = 0.70202P^*$$

so that

$$\begin{aligned}P^* &= \frac{6660.7 + 44\,773}{2.29115 - 0.70202}\\ &= 32\,366\end{aligned}$$

The difference in premium is therefore

$$32\,366 - 22\,449 = 9917$$

Comment: The refund of premiums without interest means that the amount paid is $3P$*. However, this is still a present value calculation, so the value must include the discount term* v^3*. Many candidates omitted this term.*

(e) The return of premium benefit would pay out $97,098 to eligible policyholders, considerably more than they would get from the disability benefit. It would be rational in some cases for policyholders to forgo the disability benefit in order to qualify for the return of premium benefit.

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Question 2 Model Solution

Learning Outcome: 2(a)

Chapter References: AMLCR Chapter 4

(a)

$$E[Z_1] = 10\bar{A}_{35:\overline{25|}}$$

$$E[Z_2] = 100\bar{A}^{\,1}_{35:\overline{25|}} + 200\,{}_{25}E_{35}\,\bar{A}^{\,1}_{60:\overline{20|}}$$

Or $\quad 100\bar{A}^{\,1}_{35:\overline{25|}} + 200\,{}_{25|}\bar{A}^{\,1}_{35:\overline{20|}}$

Or $\quad 200\bar{A}^{\,1}_{35:\overline{45|}} - 100\bar{A}^{\,1}_{35:\overline{25|}}$

Comments

1. *There are several ways of writing $E[Z_2]$. Full credit was given for any correct answer using A or E functions. No credit was given for expressions in integral or summation form. This applies also to the variance and covariance formulas in parts (b) and (c).*
2. *Candidates either did very well or very poorly on this part. The most common mistake was to work with benefits payable at the end of the year of death instead of benefits payable at time of death.*

(b)

$$Z_1^2 = \begin{cases} 100(v^2)^{T_{35}} & 0 < T_{35} \le 25 \\ 100(v^2)^{25} & 25 < T_{35} \end{cases}$$

$$\Rightarrow E\left[Z_1^2\right] = 100\ {}^{2}\bar{A}_{35:\overline{25|}}$$

$$\Rightarrow Var[Z_1] = 100\left(\ {}^{2}\bar{A}_{35:\overline{25|}} - \left(\bar{A}_{35:\overline{25|}}\right)^2\right)$$

$$Z_2^2 = \begin{cases} 10\,000(v^2)^{T_{35}} & 0 < T_{35} \le 25 \\ 40\,000(v^2)^{T_{35}} & 25 < T_{35} \le 45 \\ 0 & 45 < T_{35} \end{cases}$$

$$\Rightarrow E[Z_2^2] = 10\,000\ {}^{2}\bar{A}^{\,1}_{35:\overline{25|}} + 40\,000\,{}_{25}E_{35}\,v^{25}\ {}^{2}\bar{A}^{\,1}_{60:\overline{20|}}$$

Also

$$E[Z_2]^2 = 10\,000\left(\bar{A}^{\,1}_{35:\overline{25}|}\right)^2 + 40\,000\,\bar{A}^{\,1}_{35:\overline{25}|}\,{}_{25}E_{35}\,\bar{A}^{\,1}_{60:\overline{20}|} + 40\,000\,({}_{25}E_{35})^2\,\left(\bar{A}^{\,1}_{60:\overline{20}|}\right)^2$$

$$\begin{aligned}\Rightarrow Var[Z_2] = 10\,000 &\left({}^2\bar{A}^{\,1}_{35:\overline{25}|} - \left(\bar{A}^{\,1}_{35:\overline{25}|}\right)^2\right) \\ &+ 40\,000\left({}_{25}E_{35}\,v^{25}\ {}^2\bar{A}^{\,1}_{60:\overline{20}|} - \left({}_{25}E_{35}\,\bar{A}^{\,1}_{60:\overline{20}|}\right)^2\right) \\ &\quad - 40\,000\,\bar{A}^{\,1}_{35:\overline{25}|}\,{}_{25}E_{35}\,\bar{A}^{\,1}_{60:\overline{20}|}\end{aligned}$$

Comments

1. *The variance of Z_1 was answered correctly by most candidates but many lost some marks for not providing a derivation.*
2. *The variance of Z_2 proved to be more challenging. Many candidates treated this insurance policy as a combination of a 25-year temporary insurance and a 25-year deferred 20-year temporary insurance, say $Z_2 = Z_3 + Z_4$ and calculated $Var[Z_2]$ as $Var[Z_3] + Var[Z_4]$. which is incorrect as it ignores the covariance between Z_3 and Z_4. Few candidates achieved full credit.*

(c)

$$Z_1\,Z_2 = \begin{cases} 1\,000(v^2)^{T_{35}} & 0 < T_{35} \le 25 \\ 2\,000v^{25+T_{35}} & 25 < T_{35} \le 45 \\ 0 & 45 < T_{35} \end{cases}$$

$$E\left[Z_1Z_2\right] = 1000\ \ {}^2\bar{A}^{\,1}_{35:\overline{25}|} + 2\,000v^{25}\,{}_{25}E_{35}\bar{A}^{\,1}_{60:\overline{20}|}$$

$$E\left[Z_1\right]\ E\left[Z_2\right] = 1000\bar{A}_{35:\overline{25}|}\left(\bar{A}^{\,1}_{35:\overline{25}|} + 2\,{}_{25}E_{35}\bar{A}^{\,1}_{60:\overline{20}|}\right)$$

$$\begin{aligned}\Rightarrow Cov[Z_1, Z_2] = 1000 &\left({}^2\bar{A}^{\,1}_{35:\overline{25}|} + 2\,v^{25}\,{}_{25}E_{35}\bar{A}^{\,1}_{60:\overline{20}|}\right) \\ &- 1000\bar{A}_{35:\overline{25}|}\left(\bar{A}^{\,1}_{35:\overline{25}|} + 2\,{}_{25}E_{35}\bar{A}^{\,1}_{60:\overline{20}|}\right)\end{aligned}$$

Comments

1. *This part proved very challenging to most candidates.*
2. *The small number of candidates who knew how to apply the concept of covariance between the present values of benefits payable over different periods did very well, generally receiving full credit for this part.*

3. *The most common mistakes were 1) assuming no covariance between the present values of benefits payable over different periods; 2) missing the v^{25} factor in the second term.*
4. *Many candidates omitted this part.*

(d)

$$Var[Z_1] = 100\left({}^2\bar{A}_{35:\overline{25}|} - \left(\bar{A}_{35:\overline{25}|}\right)^2\right)$$

$$\bar{A}_{35:\overline{25}|} = \frac{i}{\delta}A_{35} - \frac{i}{\delta}\,{}_{25}E_{35}A_{60} + {}_{25}E_{35}$$

$${}_{25}E_{35} = \frac{8188074}{9420657}v^{25} = 0.202513$$

$$\bar{A}_{35:\overline{25}|} = 1.02971 * 0.12872 - 0.202513 * 1.02971 * 0.36913 + 0.202513$$

$$= 0.25808$$

$${}^2\bar{A}_{35:\overline{25}|} = \frac{i^*}{2\delta}\ {}^2A_{35} - \frac{i^*}{2\delta}\,{}_{25}^{2}E_{35}\,{}^2A_{60} + {}_{25}\,{}^2E_{35}$$

$$\text{where } \ 1+i^* = (1.06)^2 \Rightarrow \frac{i^*}{2\delta} = 1.0606$$

$${}_{25}\,{}^2E_{35} = v^{25}\ {}_{25}E_{35} = 0.047185$$

$$\Rightarrow\ {}^2\bar{A}_{35:\overline{25}|} = 1.0606 * 0.03488 - 0.047185 * 1.0606 * 0.17741 + 0.047185 = 0.07530$$

$$\Rightarrow Var[Z_1] = 100\left(0.07530 - 0.25808^2\right) = 0.86947$$

$$\Rightarrow SD[Z_1] = 0.93245$$

Comments

1. *The candidates who correctly found $Var[Z_1]$ in (b) did very well on this part.*
2. *Candidates who worked with benefits payable at the end of the year of death only received partial credit.*
3. *A common mistake was to calculate EPVs for temporary insurance policies instead of EPVs for the endowment policies.*

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Question 3 Model Solution

Learning Outcome: 3(a), 3(d)

Chapter References: AMLCR Chapter 6

(a)

$$ {}_0L = 50\,000\, v^{K_{40}^{(12)}+\frac{1}{12}} + 0.2(12P) - 0.95(12P)\ddot{a}^{(12)}_{\overline{K_{40}^{(12)}+\frac{1}{12}}|} $$

Comment: Relatively few candidates achieved full credit for this part. The most common mistakes included

1. *Using annual rather than monthly factors.*
2. *Omitting the* $\frac{1}{12}$ *in* $K_{40}^{(12)} + \frac{1}{12}$ *.*
3. *Confusing annual and monthly premium rates.*
4. *Spreading initial expenses over the first year.*
5. *Double counting expenses.*

(b)

$$ {}_0L = 50\,000\, v^{K_{40}^{(12)}+\frac{1}{12}} + 0.2(12P) - 0.95(12P)\left(\frac{1 - v^{K_{40}^{(12)}+\frac{1}{12}}}{d^{(12)}}\right) $$
$$ = \left(50\,000 + \frac{0.95(12P)}{d^{(12)}}\right) v^{K_{40}^{(12)}+\frac{1}{12}} + (12P)\left(0.2 - \frac{0.95}{d^{(12)}}\right) $$

$$ Var[{}_0L] = \left(50\,000 + \frac{0.95(12P)}{d^{(12)}}\right)^2 Var\left[v^{K_{40}^{(12)}+\frac{1}{12}}\right] $$
$$ = \left(50\,000 + \frac{0.95(12P)}{d^{(12)}}\right)^2 \left({}^2A_{40}^{(12)} - \left(A_{40}^{(12)}\right)^2\right) $$

Comment: This part was done well by many candidates. As the question did not ask for a derivation, candidates who wrote down the correct formula without derivation received full credit.

(c) Let Π denote the equivalence principle premium.

$$\Pi = \frac{50\,000\, A_{40}^{(12)}}{12\left(0.95\ddot{a}_{40}^{(12)} - 0.2\right)}$$

$$A_{40}^{(12)} = \frac{i}{i^{(12)}} A_{40} = 0.16571$$

$$\ddot{a}_{40}^{(12)} = \frac{1 - A_{40}^{(12)}}{d^{(12)}} = 14.352$$

$$\Rightarrow \Pi = \frac{8285.50}{161.21} = 51.40$$

Comment: This part was done well by many candidates.

(d)

$$ {}^{2}A_{40}^{(12)} = \frac{j}{j^{(12)}}\, {}^{2}A_{40} = 0.05132$$

where $j = (1+i)^2 - 1 = 0.1236 \qquad j^* = 12((1+j)^{\frac{1}{12}} - 1) = 0.11711$

$$Var[{}_0L] = \left(50\,000 + \frac{0.95(12P)}{d^{(12)}}\right)^2 \left({}^{2}A_{40}^{(12)} - \left(A_{40}^{(12)}\right)^2\right)$$

$$SD[{}_0L] = \left(50\,000 + \frac{0.95(12P)}{d^{(12)}}\right) \sqrt{\left({}^{2}A_{40}^{(12)} - \left(A_{40}^{(12)}\right)^2\right)}$$

$$= (50\,000 + 196.11P)\, 0.15447$$

$$= 7723.50 + 30.293P$$

Let L denote the aggregate loss at issue for 10,000 policies.

$$E[L] = 10,000\left(50\,000 A_{40}^{(12)} + 12P(0.2 - 0.95\, \ddot{a}_{40}^{(12)})\right)$$

$$= 10,000\,(8285.5 - 161.21P)$$

$$SD[L] = \sqrt{10\,000}\,(7723.50 + 30.293P)$$

$$\Pr[L \le 0] = 0.9 \Rightarrow \Phi\left(\frac{-E[L]}{SD[L]}\right) = 0.9$$

$$\Rightarrow \frac{-E[L]}{SD[L]} = 1.282$$

$$\Rightarrow \frac{10\,000\,(161.21P - 8285.5)}{\sqrt{10000}\,(7723.50 + 30.293P)} = 1.282$$

$$\Rightarrow 16121P - 828550 = 9901.53 + 38.8356P$$

$$\Rightarrow P = \frac{838\,451}{16082} = 52.14$$

Comment: This part proved quite challenging. Candidates who repeated mistakes from earlier parts were not generally penalised in this part if the calculations were otherwise correct. Many candidates omitted this part, or gave incomplete answers.

(d) As the number of policies increases, the premium P will tend to the equivalence principle premium, $\Pi = 51.40$.. This is because as the number of policies increases, the mortality risk is diversified away. Intuitively this means that the cash flows of the portfolio become increasingly certain; the cost of benefits will be (asymptotically) exactly 8285.5 per policy, and the value of the premiums of P per month, after expense deductions, will be (asymptotically) exactly $161.21P$. If the premium is less than Π, then a loss is (asymptotically) certain. If the premium is greater than Π then a gain is asymptotically certain. If the premium is equal to Π, then the income will exactly meet outgo.

Comment: For full credit, candidates were required to state that the equivalence principle premium is the limiting value of the portfolio percentile premium, and to give some explanation of this result. Many candidates omitted this part, or wrote answers that did not give sufficient detail, such as "The premium will decrease".

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Question 4 Model Solution

Learning Outcomes: 4(a), 4(c), 4(f)

Chapter References: AMLCR Chapters 7, 13

(a)

Modified premium is $P^* = \dfrac{100\,000 A_{41}}{\ddot{a}_{41}} = 1148.61$

${}_2V^{FPT} = 100\,000 A_{42} - P^* \ddot{a}_{42} = 922.52$

OR

$${}_2V^{FPT} = 100\,000 \left(1 - \frac{\ddot{a}_{42}}{\ddot{a}_{41}}\right) = 921.94$$

Comment: Most candidates who attempted this part did well, but a substantial number wrongly incorporated expenses into the FPT net premium reserve calculation.

(b) Note that ${}_1V^{FPT} = 0$. Profit is income (gross premiums, reserve brought forward, interest) minus outgo (expenses, claims, reserve carried forward. That is

$$\begin{aligned}\text{Profit}_2 &= (990(2000)(0.9) - 200000)\,(1.08) - 6 \times 100\,000 - 984\,({}_2V^{FPT}) \\ &= 201\,312\end{aligned}$$

Comment: This part was done well by the candidates who attempted it. The main error was not knowing that the reserve brought forward at the start of the year is ${}_1V^{FPT} = 0$.

(c)

${}_9V^{FPT} = 110\,000 A_{49} - P^* \ddot{a}_{49} = 10\,824$

Or

${}_9V^{FPT} = 8436 + 10\,000 A_{49} = 10\,824$

Comment: Most candidates received little or no credit for this part, but those who did make a serious attempt generally earned full credit.

(d)

$$\text{Profit}_{10} = (800(2000)(0.9) + 800\,(\,{}_9V^{FPT}) - 21500)\,(1.08) - 6 \times 110\,000 - 794\;{}_{10}V^{FPT}$$

$${}_{10}V^{FPT} = 9666 + 10\,000A_{50} = 12\,157$$

$$\Rightarrow\ \text{Profit}_{10} = 571,258$$

Comment: As for part (c), the best candidates scored very well, but many candidates did not achieve even partial credit for this part.

(e) Profit distribution, 60% of Year 10 Profit: 342 755

Cash equivalent, per policy in force at $t = 9$: 428.44

Reversionary Bonus Amount: $\dfrac{428.44}{A_{50}} = 1720.31$

Reversionary Bonus Rate: $\dfrac{1720.31}{110\,000} = 1.56\%$

Comment: This part was done better than (c) and (d), with many candidates earning full credit.

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Question 5 Model Solution

Learning Outcomes: 4(a), 4(c), 4(g)

Chapter References: AMLCR Chapter 7

(a)

$$
\begin{aligned}
{}_{11}V &= \frac{({}_{10}V + G - E)\,(1.05) - q_{65}(100300)}{p_{65}} \\
&= \frac{(29250 + 5500 - 100)(1.05) - (0.04)(100\,300)}{0.96} \\
&= 33\,719.27
\end{aligned}
$$

Comment: This part was done very well by almost all candidates.

(b)(i) Profit is income (gross premiums, reserve brought forward, interest) minus outgo (expenses, claims, reserve carried forward). That is

$$
\begin{aligned}
\text{Profit} &= (160\,({}_{10}V + G) - 17\,700)\,(1.06) \\
&\qquad - 9(100\,000) - 2500 - 7(0.9)({}_{11}V) - 144({}_{11}V) \\
&= (160\,(29\,250 + 5500) - 17\,700)(1.06) \\
&\qquad - 9(100\,000) - 2500 - 7(0.9)(33\,719.27) - 144(33\,719.27) \\
&= -95\,668
\end{aligned}
$$

That is **Loss of 95 668**

(b)(ii) **Interest Gain**: use (actual – expected) interest income, expected mortality, expenses, surrenders.

$(1.06 - 1.05)(160)\,({}_{10}V + G - 100) = 55\,440$

So the **interest profit is 55 440.**

Mortality gain: use actual interest, (expected – actual) cost of mortality (net amount at risk), expected expenses and surrenders.

Using the valuation basis mortality, we expect (0.04)(160)=6.4 deaths, compared with the 9 deaths experienced.

$(6.4 - 9)\,(100\,300 - {}_{11}V) = -173\,111$

So the **Mortality Profit is -173 111** (or loss of 173 111)

Expense gain: use actual interest and mortality, (expected– actual) expense outgo, expected surrenders

$160(100)(1.06) + 9(300) - (17700(1.06) + 2500) = -1602$

So the **expense profit is -1602** (or loss of 1602)

Surrender gain: use actual interest, mortality and expenses, and (expected-actual) outgo on surrenders (net amount at risk).

$(0-7)\,(0.9\,{}_{11}V - {}_{11}V) = 23\,603$

Hence the **surrender profit is 23 603**

Comment: Very few candidates scored full credit here, but many candidates earned substantial partial credit. Some candidates did not identify whether the difference represented a gain or a loss. A common error was omitting the reserve in the calculation of the surrender and mortality profit.

(c) The asset share at the year end is the rolled up asset share from the previous year, minus the net outgo on the portfolio, divided by the number of survivors. That is

$$AS_{11} = \frac{(160(40\,100 + 5500) - 17\,700)\,(1.06) - 9(100\,000) - 2500 - 7(0.9)({}_{11}V)}{144}$$
$$= 45\,834$$

Comment: Some candidates were not sure how to calculate the asset share, for example, how to incorporate the reserve. There was also some confusion around the appropriate denominator.

(d) The gross premium reserve represents the capital required to meet future net outgo, assuming experience follows the valuation basis, expressed per policy in force.

The asset share represents the total accumulated value of past income less outgo for the portfolio, expressed per policy in force.

If the past experience exactly matched the valuation basis assumptions, the values would be the same.

In this case, the AS is greater than the reserve, which means that the portfolio to date has been profitable, even though the most recent year showed a loss overall.

Comment: This part was not well answered in general. Only a few candidates demonstrated an understanding of the relationship between the gross premium reserve and the asset share.

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Question 6 Model Solution

Learning Outcomes: 5(a), 5(d), 5(f)

Chapter References: AMLCR Chapter 10

(a)(i)

$$V_0^{TUC} = 0.015 \times 5 \times S_{44} \times \left(\frac{1 + 1.03^{-1} + 1.03^{-2}}{3} \right) v_{5\%}^{20} \times {}_{20}p_{45} \times \ddot{a}_{65\,(6\%)}^{(12)}$$

$$\ddot{a}_{65\,(6\%)}^{(12)} = 1.00028(9.8969) - 1.46812 = 9.43155$$

$${}_{20}p_{45} = \frac{7\,533\,964}{9\,164\,051} = 0.822122$$

$$\Rightarrow V_0^{TUC} = 9578.5$$

(a)(ii)

$$V_0^{PUC} = 0.015 \times 5 \times S_{44} \times \left(\frac{1.03^{20} + 1.03^{19} + 1.03^{18}}{3} \right) v_{5\%}^{20} \times {}_{20}p_{45} \times \ddot{a}_{65\,(6\%)}^{(12)}$$

$$= 17299.8$$

(b)(i) The normal contribution is the difference between the expected present value of the TUC valuation in 1 year, denoted $EPV(V_1^{TUC})$, and the current TUC valuation V_0^{TUC}.

$$EPV(V_1^{TUC}) = vp_{45}(0.015)(6)S_{44} \left(\frac{1.03 + 1 + 1.03^{-1}}{3} \right) v_{5\%}^{19} \, {}_{19}p_{46} \, \ddot{a}_{65\,(6\%)}^{(12)}$$

$$= V_0^{TUC} \times (1.03) \times \frac{6}{5}$$

$$= 11839.03$$

$\Rightarrow$Normal Contribution $= 11839.03 - 9578.50 = 2260.53$

$\Rightarrow$Normal Contribution Rate $= \dfrac{2260.53}{45000(1.03)} = 4.88\%$

(b)(ii)

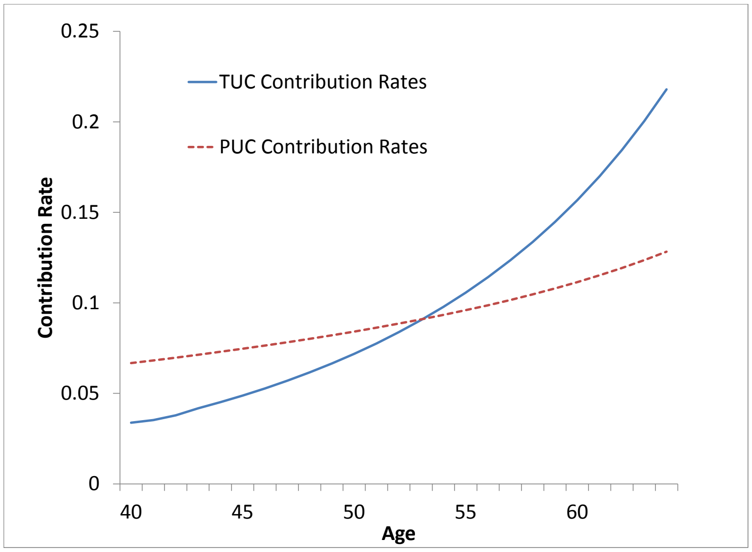
$$EPV(V_1^{PUC}) = 0.015 \times 6 \times S_{44} \times \left(\frac{1.03^{20} + 1.03^{19} + 1.03^{18}}{3} \right) v_{5\%}^{20} \times {}_{20}p_{45} \times \ddot{a}_{65\,(6\%)}^{(12)}$$

$$= \frac{6}{5} V_0^{PUC}$$

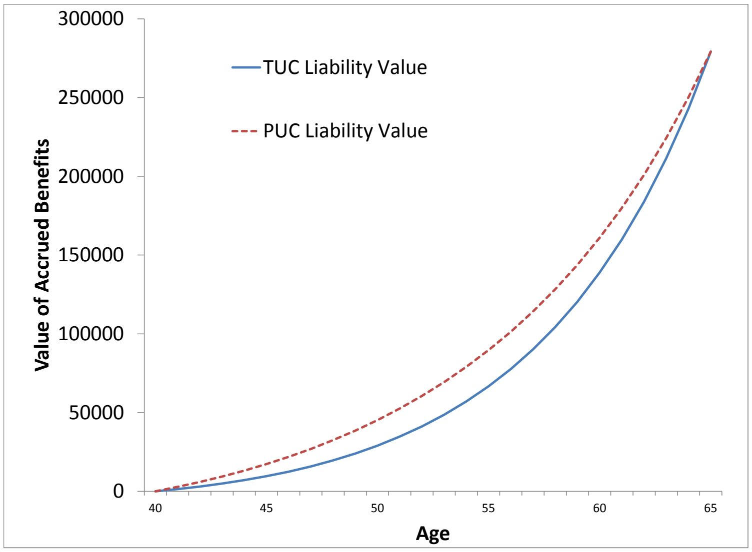
So the Normal Contribution is 17299.8/5 = 3460.0.

Which gives a NC rate, as % of 2016 salary, of 7.46%.

(c)(i) TUC and PUC Contribution Rates



(c)(ii) TUC and PUC Value of Accrued Liability



Comments:

1. *The last part of this question proved to be very challenging, requiring an understanding of the impact of lower (TUC) and faster (PUC) paced funding on the contribution rates and the accrued liability. The key features of the contribution graph is that the PUC contribution rate starts higher and finishes lower than TUC, and both are increasing over the working lifetime. The key features of the value of the accrued liability graph is that the start and end values are the same, but the faster funding pace (PUC) creates a higher liability inbetween the end points.*
2. *Most candidates who attempted this question achieved substantial credit for the PUC and TUC liability calculations, and for the PUC contribution. Fewer candidates correctly found the TUC contribution rate. Overall, this was one of the lower scoring questions on the exam.*