1. The probability of being fully functional after two years for a single television is:

$$\begin{pmatrix}0.82 & 0.10 & 0.08\end{pmatrix}\begin{pmatrix}0.82\\0.60\\0.00\end{pmatrix} = 0.82*0.82+0.10*0.60+0.08*0.00 = 0.7324$$

The number of the five televisions being fully functional has a binomial distribution with parameters of $n=5$ and $p=0.7324$. The probability that there will be exactly two televisions that are fully functioning is therefore:

$$\binom{5}{2}0.7324^2(1-0.7324)^3 = 10*0.53641*0.019163 = 0.10279$$

2.

$$\mu_x = -\frac{d}{d_x}lnS_0(x) = -\frac{1}{3}\frac{d}{d_x}ln\left(1-\frac{x}{60}\right)$$

$$= \frac{1}{180}\left(1-\frac{x}{60}\right)^{-1} = \frac{1}{3(60-x)}$$

Therefore, $\mu_{35} = \frac{1}{3(25)} = \frac{1}{75} = 0.0133.$

3. Out of 400 lives initially, we expect $400*{}_{40}p_{25} = 400*\frac{l_{65}}{l_{40}} = 400*\frac{7,533,964}{9,565,017} = 315.0633$

Survivors with standard deviation of $\sqrt{400*{}_{40}p_{25}(1-{}_{40}p_{25})} = 8.1793$

To ensure 86% funding, using the normal distribution table, we plan for $315.0633+1.08(8.1793) = 323.8969.$

The initial fund must therefore be $F = 324*15200*\left(\frac{1}{1.06}\right)^{40} = 478,799.80.$

4. Probability

$$= \int_0^5 {}_tp^{\overline{00}} \mu^{01}\, {}_{5-t}p^{\overline{11}} dt$$
$$= \int_0^5 e^{-0.06t} 0.05 e^{-0.08(5-t)} dt$$
$$= e^{-0.40}(0.05)\int_0^5 e^{+0.02t} dt$$
$$= e^{-0.40}\left(\frac{5}{2}\right)\left(e^{0.10}-1\right) = 0.1762$$

5.

$$\ddot{a}_{[x]:\overline{n}|} = 1 + vp_{[x]}\ddot{a}_{x+1:\overline{n-1}|} = 1 + (1+k)vp_x\ddot{a}_{x+1:\overline{n-1}|} = 1 + (1+k)\left(\ddot{a}_{x:\overline{n}|} - 1\right)$$

Therefore, we have

$$k = \frac{\ddot{a}_{x:[n]} - 1}{\ddot{a}_{x:\overline{n}|} - 1} - 1 = \frac{21.167}{20.854} - 1 = 0.015$$

6.

$$100{,}000 A_{\overline{\overset{1}{50:60}:\overline{10}|}} = 100{,}000\left[A^1_{50:\overline{10}|} + A^1_{60:\overline{10}|} - A_{\overset{1}{50:60}:\overline{10}|}\right]$$
$$= 100{,}000\left[0.060495 + 0.136785 - 0.186751\right] = 1{,}052.89$$

where

$$A^1_{50:\overline{10}|} = A_{50} - {}_{10}E_{50}A_{60} = 0.24905 - (0.51081)(0.36913) = 0.060495$$
$$A^1_{60:\overline{10}|} = A_{60} - {}_{10}E_{60}A_{70} = 0.36913 - (0.45120)(0.51495) = 0.136785$$
$$A_{\overset{1}{50:60}:\overline{10}|} = A_{50\,60} - (1.06)^{10}\, {}_{10}E_{50}\; {}_{10}E_{60}A_{60:70}$$
$$= 0.42296 - 1.79085(0.51081)(0.45120)(0.57228) = 0.186751$$

7. Let G be the annual gross premium. By the equivalence principle, we have
$G\ddot{a}_{35} = 100,000A_{35} + 0.15G + 0.04G\ddot{a}_{35}$

so that

$$G = \frac{100,000A_{35}}{0.96\ddot{a}_{35} - 0.15} = \frac{100,000(0.12872)}{0.96(15.3926) - 0.15} = 880.023$$

8. By the equivalence principle,

$$4500\bar{a}_{x:\overline{20|}} = 100,000\bar{A}^{1}_{x:\overline{20|}} + R\bar{a}_{x:\overline{20|}}$$

where

$$\bar{A}^{1}_{x:\overline{20|}} = \frac{\mu}{\mu+\delta}\left(1 - c^{-20(\mu+\delta)}\right) = \frac{0.04}{0.12}\left(1 - e^{-20(0.12)}\right) = 0.3031$$

$$\bar{a}_{x:\overline{20|}} = \frac{1 - e^{-20(\mu+\delta)}}{\mu+\delta} = \frac{1 - e^{-20(0.12)}}{0.12} = 7.5774$$

Solving for R, we have

$$R = 4500 - 100,000\left(\frac{0.3031}{7.5774}\right) = 500$$

9. By the equivalence principle, we have

$$G\ddot{a}_{35:\overline{10|}} = 50,000A_{35} + 100a_{35} + 100A_{35}$$

so that

$$G = \frac{50,100A_{35} + 100(\ddot{a}_{35} - 1)}{\ddot{a}_{35} - {}_{10}E_{35}\ddot{a}_{45}}$$
$$= \frac{50,100(0.12872) + 100(14.3926)}{15.3926 - 0.54318(14.1121)}$$
$$= 1020.828$$

10. Let P be the annual net premium

$$P = \frac{1000\bar{A}_{x:\overline{n|}}}{\ddot{a}_{x:\overline{n|}}} = \frac{1000(0.192)}{\ddot{a}_{x:\overline{n|}}}$$

where

$$\ddot{a}_{x:\overline{n|}} = \frac{1-A_{x:\overline{n|}}}{d} = \frac{(1.05)}{(0.05)}\left(1 - A^{1}_{x:\overline{n|}} - A_{x:\overline{n|}}^{\;\;1}\right)$$

$$\bar{A}_{x:\overline{n|}} = \frac{i}{\delta}\left(A^{1}_{x:\overline{n|}}\right) + {}_nE_x$$

$$\Rightarrow 0.192 = \frac{0.05}{0.0488}\left(A^{1}_{x:\overline{n|}}\right) + 0.172$$

$$\Rightarrow A^{1}_{x:\overline{n|}} = 0.01952$$

$$\Rightarrow \ddot{a}_{x:\overline{n|}} = \frac{1.05}{0.05}(1 - 0.01952 - 0.172) = 16.97808$$

Therefore, we have

$$P = \frac{1000(0.192)}{16.97808} = 11.31$$

11. Premium at issue for (20): $65.28/16.5133 = 3.9531$
Premium at issue for (50): $249.05/13.2668 = 18.7724$

Lives in force after ten years:

Issued at age 20: $10,000\,{}_{10}p_{20} = 10,000 \times \dfrac{9,501,381}{9,617,802} = 10,000 \times 0.9878953 = 9878.953$

Issued at age 50: $10,000\,{}_{10}p_{50} = 10,000 \times \dfrac{8,188,074}{8,950,901} = 10,000 \times 0.9147765 = 9147.765$

The total number of lives after ten years is therefore: $9878.953 + 9147.765 = 19,026.718$

The average premium after ten years is therefore:

$$\frac{(3.9531 \times 9878.953) + (18.7724 \times 9147.765)}{19,026.718} = 11.078$$

12.

$$V\left[L_0\#1\right]=\left(B_1+\frac{P_1}{d}\right)^2\underbrace{\left({}^2A_x-A_x^2\right)}_{w}=20.55$$

$$=\left(8+\frac{1.25(1.06)}{(0.06)}\right)^2\times w=20.55$$

$$V\left[L_0\#2\right]=\left(12+\frac{1.875}{0.06}(1.06)\right)^2\times w$$

$$\frac{V\left[L\#2\right]}{V\left[L\#1\right]}=\frac{\left[12+\frac{1.875}{0.06}(1.06)\right]^2}{\left[8+\frac{1.25}{0.06}(1.06)\right]^2}=2.25$$

$$\Rightarrow V\left[L\#2\right]=2.25\times 20.55=46.24$$

Or:

$W=\left(\frac{12}{8}\right)^2\times V\left[L_0\#1\right]=(1.5)^2\times\left(20.55\right)=46.24$ (because both premium and benefit are scaled by 1.5)

13.

Calculating the reserve, ${}_{15}V=A_{50:\overline{15}|}-\frac{A_{35:\overline{30}|}}{\ddot{a}_{35:\overline{30}|}}\ddot{a}_{50:\overline{15}|}$

Where $\ddot{a}_{35:\overline{30}|}=\frac{1-A_{35:\overline{30}|}}{d}=\frac{1-0.255}{\frac{0.05}{1.05}}=15.645$

And $\ddot{a}_{50:\overline{15}|}=\frac{1-A_{50:\overline{15}|}}{d}=\frac{1-0.506}{\frac{0.05}{1.05}}=10.374$

So that ${}_{15}V=0.506-\frac{0.255}{15.645}10.374=0.3369128$

SC = surrender charge

$${}_{15}V-SC=0.40A_{50:\overline{15}|}\Rightarrow SC={}_{15}V-0.40A_{50:\overline{15}|}$$

$$=0.3369128-0.40(0.506)$$

$$=0.1345128$$

For insurance of 2000, SC = 269.0256

14.

$AV_0 = 0$

$P_1 = 4,450$

$EC_1 = 56 + 2\% * 4,450 = 145.00$

COI rate $= q_{36^*} = 1.2 * 0.00214 = 0.002568$

$\text{COI}_1 = 200,000 * 0.002568 * (1/1.06) = 484.53$

Credited Interest: $6\% * (\$4,450 - 145 - 484.53) = 229.23$

$AV_1 = 4,450 - 145 - 484.53 + 229.23 = 4,049.70$

15. We have

$$vq_x + \beta\left(\ddot{a}_{25:\overline{20}|} - 1\right) + P\ {}_{20}E_{25}\ddot{a}_{45:\overline{20}|} = P\ddot{a}_{25:\overline{40}|}$$

$$\Rightarrow \beta = \frac{P\left(\ddot{a}_{25:\overline{20}|}\right) + P\ {}_{20}E_{25}\ddot{a}_{45:\overline{20}|} - vq_x}{\ddot{a}_{25:\overline{20}|} - 1} = \frac{P\ \ddot{a}_{25:\overline{20}|} - vq_x}{\ddot{a}_{25:\overline{20}|} - 1}$$

Where $P = \dfrac{A_{25:\overline{40}|}}{\ddot{a}_{25:40}} = \dfrac{1}{\ddot{a}_{25:\overline{40}|}} - d = 0.02161656$

$$\Rightarrow \beta = \frac{0.02161656(11.087) - \frac{1}{1.04}(0.005)}{11.087 - 1} = 0.02328295$$

For insurance of 10,000, $\beta = 233$.

16.

$q_{50} = 0.00592, \quad q_{51} = 0.00642$

$\Rightarrow AV_1 = 1369.895$

$$AV_2 = \left(AV_1 + 5000(1 - 0.035) - 75 - (500,000 - AV_2)\left(\frac{1.20q_{51}}{1.03}\right)\right)(1.045)$$

$\Rightarrow AV_2 = 2506.787$

17. Let P be the annual net premium at $x+1$.

$$P\ddot{a}_{x+1} = 1000\sum_{k=0}^{\infty}(1.03)^{k+1} v^{k+1} {}_{k|}q_{x+1} = 1000A^{*}_{x+1}$$

We are given

$$110\ddot{a}_{x+1} = 1000\sum_{k=0}^{\infty}(1.03)^{k+1} v^{k+1} {}_{k|}q_{x+1} = 1000A^{*}_{x}$$

Which implies that

$$110\left(1 + vp_x\ddot{a}_{x+1}\right) = 1000\left(1.03vq_x + 1.03vp_xA^{*}_{x+1}\right)$$

Solving for A^{*}_{x+1}, we get

$$A^{*}_{x+1} = \frac{\frac{110}{1000}\left[1 + v(0.95)(7)\right] - 1.03v(0.05)}{1.03v(0.95)} = 0.8141032$$

Thus, we have

$$P = \frac{1000(0.8141032)}{7} = 116.3005$$

18. Under PUC:

${}_tV =$ accrual rate × years of past service × survival to retirement × discount to retirement × retirement benefit

$${}_{36}V = \frac{\text{years of service} + 1}{\text{years of service}}\,{}_{35}V$$

$${}_{35}V + C = {}_{36}V = \frac{36}{35}\,{}_{35}V$$

$$C = \frac{36}{35}\,{}_{35}V - {}_{35}V \Rightarrow C = \frac{{}_{35}V}{35}$$

19. By age 65, member would have served total of 35 years in which case, benefit would be $35 \times 0.02 = 70\%$. Thus set it at 60%.

$$\text{EPV(benefits)} = 0.60 \times 50{,}000 \times (1.03)^{19} \times \frac{1}{1.05^{20}} \frac{l_{65}^{(\tau)}}{l_{45}^{(\tau)}} \ddot{a}_{65}^{(12)}$$

$$= 0.60 \times 50{,}000 \times \left(\frac{1}{1.05}\right)^{20} \left(\frac{3}{5}\right)(7.8)(1.03)^{19}$$

$$= 92{,}787.29$$

20. Replacement ratios

Plan 1: $R = \dfrac{1250 * 25}{S_0 (1.04)^{24}}$

Plan 2: $R = \dfrac{S_0 * 0.02 * 25 * \dfrac{1.04^{25} - 1}{0.04} * \dfrac{1}{25}}{S_0 (1.04)^{24}}$

The two are equal, so that

$$S_0 = \frac{1250 \times 25}{0.02 \left(\dfrac{1.04^{25} - 1}{0.04} \right)} = 37{,}518.69$$