

QFI ADV Model Solutions

Spring 2016

1. Learning Objectives:

2. The candidate will understand and be able to apply a variety of credit risk theories and models.

Learning Outcomes:

- (2b) Demonstrate an understanding of the basic concepts of credit risk modeling such as probability of default, loss given default, exposure at default, and expected loss.

Sources:

Sec.2 - Bluhm, An Introduction To Credit Risk Modeling, 2nd Ed, Ch 1

Commentary on Question:

This question tested a candidate's understanding of credit risk models and credit risk management

Solution:

- (a) Calculate Company D's unexpected loss treating EAD as if it were deterministic and equal to its expected value.

Commentary on Question:

The candidates performed above average on this section.

Most candidates used the correct formula to calculate both EAD and unexpected loss.

$$\begin{aligned} \text{EAD} &= \text{outstanding amount} + \text{undrawn (Cash DDF)} + \text{contingent liability (contingent DDF)} \\ &= 5,000 + (8,000 - 5,000) * 80\% * 66.67\% + 12,000 * 50\% * 80\% \\ &= 5,000 + 1,600 + 4,800 = 11,400 \end{aligned}$$

$$\begin{aligned} \text{UL}_D &= \sqrt{V[L]} = \text{EAD} \times \text{LGD} \times \sqrt{PD(1 - PD)} \\ &= 11,400 \times 0.65 \times \sqrt{0.25(0.75)} = 3,208.62 \end{aligned}$$

- (b) Calculate the annual percentage fee ABC Bank should charge for the loan to Company D.

1. Continued

Commentary on Question:

The candidates performed as expected on this section.

Most candidates correctly calculated the Expected loss. However, some candidates did not multiply the EC by the given hurdle rate and thus calculated an incorrect EC Charge.

$$EC_{97.5\%} = 1.96 * UL_D = 6,288.89$$

$$EC \text{ charge} = 6,288.89 * 0.08 = 503.11$$

$$\text{Expected loss} = 11,400 * 0.65 * 0.25 = 1,852.50$$

$$\text{Loan Charge} = 50K/1M + 15K/1M + 1,852.50 / 20,000 + 503.11 / 20,000$$

$$= 5\% + 1.5\% + 9.26\% + 2.51\%$$

$$= 18.27\%$$

- (c) Calculate the unexpected loss of the new portfolio again treating EAD as if it were deterministic and equal to its expected value.

Commentary on Question:

The candidates performed below average on this section.

The key to the question here was getting the unexpected loss equation right when there are two loss variables. Only a few candidates were able to arrive at the correct final answer. Note that since the co-variance was given, candidates were not necessarily required to derive the correlation term. Those that did however, were still awarded credit as appropriate.

$$\begin{aligned} UL_{D,E}^2 &= V[L_D] + V[L_E] + 2 * EAD_D * EAD_E * LGD_D * LGD_E * COV_{D,E} \\ &= 11,400^2 * 0.65^2 * 0.25 * 0.75 + 11,400^2 * 0.70^2 * 0.10 * 0.90 + 2 * 11,400 * \\ &11,400 * 0.65 * 0.70 * 0.05 \\ &= 10,295,269 + 5,731,236 + 5,913,180 = 21,939,448 \\ UL_{D,E} &= 4,683.96 \end{aligned}$$

- (d) Discuss the impact of default correlation on economic capital for loan portfolios.

Commentary on Question:

The candidates performed as expected on this section.

Most candidates identified that a higher default correlation would increase EC (and vice versa) but not discussed it well. Candidates were generally given full credit if they mentioned at least 3 of the bullets below.

- When $\rho=0$, the loans in the portfolio are completely unrelated that leads to lower risk (optimal diversification).
- Lower default correlation reduces the overall portfolio risk (EC).
- Positive correlation means default of one company increases likelihood of another counterparty defaulting.

1. Continued

- As ρ increases to 1, there will be more concentration risk so EC will increase.
- As ρ tends to -1, it could be a better hedge.

2. Learning Objectives:

7. The candidate will understand various investment related considerations with regard to liability manufacturing and management.

Learning Outcomes:

- (7d) Demonstrate understanding of Target Volatility funds and their impact on option costs.

Sources:

Guarantees and Target Volatility Funds, Steven Morrison and Laura Tadrowski, Moody's Analytics, 2013

Commentary on Question:

This question tested the candidate's understanding of the Target Volatility Funds, dynamic rebalancing, and guarantee costs associated with different equity models, including volatility and jumps.

Solution:

- (a) Explain the impact of the assumption λ on the results.

Commentary on Question:

The candidates performed as expected on this section. Most candidates explained that a higher λ leads to a higher weight assigned to past data. Few candidates referred λ to be rate of decay.

λ is rate of decay. Higher λ refers to more weight on the old data deriving the volatility at time $t-\Delta t$.

- (b) Calculate W and X .

Commentary on Question:

The candidates performed as expected on this section. Most candidates listed and applied the formula correctly. Regarding $\Delta t = 1/252$: Alternate answers assuming 365/264/250 days per year also received credit since the question did not provide 252 business days per year assumption.

$$\text{Formula: } (\sigma_{t^{\text{equity}}})^2 = \lambda (\sigma_{t-\Delta t}^{\text{equity}})^2 + (1-\lambda)(1/\Delta t)(\ln(St/St-\Delta t))^2$$

Assume 252 business days in one year,

Calculate $\lambda=.99$, by using the daily calculation

$$\text{Daily} = (.99*(15.2\%)^2+(1-.99)(1/(1/252))(\ln(2015.6/1975.3))^2)^.5=15.46\%$$

$$W = \text{Monthly} = (.99*(4.5\%)^2+(1-.99)(1/(22/252))(\ln(2015.6/1564.1))^2)^.5=9.68\%$$

2. Continued

$$X = \text{Quarterly} = (.99*(11.2\%)^2 + (1-.99)(1/(66/252))(\ln(2015.6/1560.0))^2)^{.5} = 12.22\%$$

Alternate answer for X:

Formula: $W_t^{\text{equity}} = \min(\sigma_{\text{target}} / \sigma_t^{\text{eq}}, 100\%)$;

$W_t^{\text{equity}} = 90\%$, $\sigma_{\text{target}} = 10\%$

$\Rightarrow X = \sigma_t^{\text{equity}} = 10\% / 90\% = 11.11\%$

Although the EWMA estimator was the intended method, because sufficient information was provided for the simpler approach, alternate approach for X was acceptable as well.

- (c) Calculate the un-leveraged equity weight as of 3/31/2015 that corresponds to each of two rebalancing frequencies: daily and monthly (Y and Z).

Commentary on Question:

The candidates performed above average on this section. Candidates needed to ensure equity weight was capped at 100% in order to receive full credits.

$$Y = \min(10\% / 15.46\%, 100\%) = 65\%$$

$$Z = \min(10\% / 9.68\%, 100\%) = 100\%$$

- (d) Evaluate senior management's suggestion.

Commentary on Question:

The candidates performed below average on this section. Candidates needed to provide both a justification and a final conclusion (management comment incorrect) in order to receive full credit.

Management comment is incorrect. The monthly weight is highest. The lower the equity volatility, the higher proportion of the equity needed in the target fund to meet the target volatility, capped at 100%.

- (e) Justify which model and rebalancing frequency should be used for TVF implementation.

Commentary on Question:

The candidates performed as expected on this section. Note both a justification and a final conclusion (quarterly with jumps) needed to be provided in order to receive full credits.

2. Continued

Since jumps in equity returns can be observed in reality, we should include jumps in the modelling of target volatility funds. There is no material difference of the volatilities among rebalancing frequencies. Hence we should pick quarterly rebalancing since it is least costly.

3. Learning Objectives:

1. The candidate will understand the standard yield curve models, including:
 - One and two-factor short rate models
 - LIBOR market modelsThe candidate will understand approaches to volatility modeling.

Learning Outcomes:

- (1r) Describe and explain various issues and approaches for fitting a volatility surface.

Sources:

Volatility Correlation - The Perfect Hedger and the Fox, Rebonato, R., 2nd Edition, Section 9.1, p. 250-251

Volatility Correlation - The Perfect Hedger and the Fox, Rebonato, R., 2nd Edition, Section 9.5-9.8, p. 252-259

Volatility Correlation - The Perfect Hedger and the Fox, Rebonato, R., 2nd Edition, Section 9.9, p. 259-265

Commentary on Question:

This question tested the candidates' ability to understand the various issues and approaches for fitting a volatility surface. It required the candidate to recall the pros and cons of various fitting approaches, and also asked the candidate to provide solutions to overcome some drawbacks of different fitting approaches.

Solution:

- (a) List the pros and cons of using directly observable market prices versus preprocessed prices for purposes of this fitting.

Commentary on Question:

Candidates performed as expected on this section. Almost all candidates mentioned that this was the cleaner approach, and made some mention of the drawback of noisy prices. Most candidates failed to mention how to decide between raw market prices and smoothed prices by testing if the fitting procedure was stable under small perturbations of input quantities.

Pros:

- This is a 'cleaner' approach because it is the least data polluting.
- Plain vanilla traders may find a model's inability to price certain options exactly conveys important relative-value information.

3. Continued

Cons:

- There is no possibility of a trade-off between the quality of the global fit and a small variation in the input prices. The market prices are invariably noisy, so it is safer to work with a globally smooth surface, and to impose the recovery of many 'synthetic' prices, rather than using the observable prices directly.
- Impossible for some approaches in practice as the fitting methodology become a noise amplifier. Whether one should use directly raw market prices or smoothed "synthetic" prices depends on whether the chosen fitting procedure is stable under small perturbations of the input quantities.

- (b) Describe four fitting approaches, identifying their merits and any possible drawbacks.

Commentary on Question:

Candidates performed above average on this section. Most candidates correctly listed the four fitting approaches, and correctly described their merits. A few candidates missed points for not describing all the drawbacks of fitting to prices and implied volatilities. For the merits of fitting to price densities, almost all candidates missed the merit point on the link between the unconditional and conditional price densities.

(i) Fitting to prices:

Create a smooth function of the observed market prices as the strike varies for each available expiry.

Merits: This approach is direct and simple.

Drawbacks:

- The inputs are price; hence we mix and use on the same footing quantities of very different magnitudes (out-of-the-money and in-the-money options).
- The fitting procedure assumes that certain quantities are known with infinite precision. It does not address the question of whether a much more 'desirable' solution could be obtained if a few of the reference prices were modified even by a very small amount. Therefore, there is no possibility of a trade-off between the quality of the global fit and a small variation in the input prices.
- Any direct fitting to prices has the drawback that out-of-the-money options have smaller prices than at-the-money ones.

(ii) Fitting to transformed prices:

Rescale the price by using their logarithms and then fit to the rescaled price.

3. Continued

Merits: Fitting to log-prices brings the magnitude of the data being fitted to a more uniform scale and improves the fit for low delta strikes. It ensures that the prices remain positive while allowing for arbitrarily large errors.

(iii) **Fitting to the implied volatilities:**

Merits: It is direct and simple.

Drawbacks: The approach has great sensitivity of the associated price density function to the details of the fitting to implied volatilities.

(iv) **Fitting to the associated price densities**

Merits:

- This approach is most useful and robust because option prices are obtained by integrating over a price density which is a smooth operation. Therefore, a smooth density function ensures that the associated prices and implied volatilities will be smooth.
- A smooth price density is important because there is a link between the unconditional (marginal) price densities obtained from the quoted prices today, and the conditional densities that will prevail in the future.

- (c) Discuss two potential problems of this implementation process.

Commentary on Question:

Candidates performed as expected on this section. They were successful in recognizing that unconstrained weights were a problem, and that normal densities ‘centered’ in log space wouldn’t create fat tails for the price density. However, many candidates failed to recognize the main problem of unconstrained weights: it does not guarantee that the price density is positive everywhere.

- If the normal densities in the mixture are ‘centered’ in log space around the forward value, the resulting pricing density will display no skewness. This is at odds with empirical findings.
- If the weights are left unconstrained, there is no guarantee that the resulting density will be everywhere positive. Obviously, a density cannot be negative.

- (d) Recommend how the problems in (c) might be overcome.

Commentary on Question:

Candidates performed as expected on this section. Candidates were successful in recognizing that the weights should be positive. However, many candidates did not describe a particular method for making them positive. In addition, most candidates did not recognize that the Gaussian densities needed to be centered around different coefficients.

3. Continued

For the first problem, one can overcome it by allowing different constituent Gaussian densities to be centered around different location coefficients. When doing this, one must exercise extra care to ensure the first moment of the density is recovered exactly.

For the second problem, to ensure that the resulting density is everywhere positive, we require that all the weights should be positive. One way to do this is to impose $w_k^i = (\alpha_k^i)^2$.

4. Learning Objectives:

2. The candidate will understand and be able to apply a variety of credit risk theories and models.

Learning Outcomes:

- (2a) Demonstrate an understanding of events and causes of the recent global credit crisis.

Sources:

Saunders, A., Allen, L., Credit Risk Measurement In and Out of the Financial Crisis, 3rd Edition, 2008, Ch. 2

Commentary on Question:

This question tested the candidates' understanding of events surrounding the 2008 financial crisis. The candidates were given a similar situation in an imaginary country and asked to explain the events and causes of a possible credit crisis and related indices.

Solution:

- (a) Explain how each of the events above contributed to the Country A Mortgage Market Credit Crisis.

Commentary on Question:

The candidates performed above average on this section. Most of the candidates were able to tie the events in the Country A to the 2008 financial crisis and explained how they contributed to the Mortgage Market Credit Crisis. However some candidates described events from the US crisis which was not relevant to the Country A crisis.

- *A flu epidemic has occurred, severely damaging the tourism business, the most important industry in Country A* Similar to 911 effects on the US economy
- *...central bank of Country A has cut the interest rates to the lowest level in its history* This promotes lending money to anyone which, in turn, will start to deteriorate the overall quality of mortgages generated.
- *... more and more people are buying a second house as an investment; they plan to rent it to tourists* The increase in home purchases could inflate the price of houses creating a 'housing bubble'.
- *Both the mortgage-per-person and total mortgage outstanding balances have soared; Most loans are issued as floating rate loans* With the increase in home purchases using floating rate loans in a historical low interest rate environment means that if/when rates increase there is the possibility that home owners will not be able to afford the mortgage payments.

4. Continued

- *To reduce capital some Country A banks have bundled their mortgage loans and sold them to investors; In order to increase profit, these banks have encouraged their agents to generate more mortgages, which has triggered low quality mortgages* By removing the poor credit risk from the bank's books, the credit crisis is now impacting the investors and not just the banks who originated the loans. Investments made by foreign countries will also be impacted. This expands the impact of the credit crisis beyond just the originating banks and beyond just Country A.
 - *One neighboring country, which has better beaches and weather, surprised the financial markets and announced that it will open its boarder to tourists; With tourism declining, there will not be enough tourists to support the increase in rental properties. Increased competition will just make it that much more difficult for the new rental properties to make enough money to cover their mortgage payments. It could be the last factor that bursts the bubble.*
- (b) Identify and describe four additional events that could push Country A into the second and third phase as defined by Saunders and Allen in "Credit Risk Measurement In and Out of the Financial Crisis".

Commentary on Question:

The candidates performed as expected on this section. Most candidates identified "Too Big To Fail" banks that could default as an event. However, many candidates provided events which were included in phase 1 which was not asked for.

- The second phase of 2008 credit crisis: The Crisis Spreads – Liquidity Risk.
- Investors who purchased a second home could draw on lines of credit.
- Country A central bank could raise interest rates for the fear of inflation this would increase the cost of mortgagors interest payments.
- Country A banks may not be able to securitize poor quality assets still on their books and have to take losses.
- Investors in Country A may move to higher quality assets driving the cost of lower quality assets down and widening spreads.

- The third phase of 2008 credit crisis: The Lehman Failure – Underwriting and Political Intervention Risk.
- Large, seemingly Too Big To Fail, banks could default without help from government.
- Country A bank agents were encouraged to "sell, sell, sell" because of large profit margins so lower quality underwriting was done.
- If there are no 'bargain hunters' or workout specialist that can help with the recovery process the crisis may be prolonged.

4. Continued

- (c) Describe Country A's indices.

Commentary on Question:

The candidates performed poorly on this section. Many candidates simply stated the index represents volatility or stress, but did not describe anything further. Since this was already provided in the question, no additional credit was given for these answers.

CBOE VIX/A-VIX

- Viewed as the “investor fear gauge”.
- The higher the index, the more fear and uncertainty in the market.

KCFSI/A –SI

- These indices include 11 component variables (spreads, correlation, etc.).
- The higher the index, the more fear and uncertainty in the market.
- The higher the stress index, the more severe the financial crisis is.

- (d) Explain how the events above would impact these indices.

Commentary on Question:

The candidates performed as expected on this section. Most candidates said the indices would increase, but did not explain why they made that comment.

- The events provided will push the index higher.
- Will see these indexes start to increase as the mortgage market collapses.
- With the highest level being seen during the third phase.
- Could see unprecedented increases and unprecedented levels.

5. Learning Objectives:

2. The candidate will understand and be able to apply a variety of credit risk theories and models.

Learning Outcomes:

- (2b) Demonstrate an understanding of the basic concepts of credit risk modeling such as probability of default, loss given default, exposure at default, and expected loss.
- (2d) Demonstrate an understanding of Merton asset value models in the context of credit risk.

Sources:

Bluhm 2nd Ed, Ch 3 parts a-c, Ch1 part d

Commentary on Question:

The question primarily tests a candidate's understanding of the Merton Asset Value Model.

Note to candidates: This question was an exact copy of Question #4 from the Spring 2014 exam with the values changed. Also note that the formula in part (a) of the Spring 2014 exam solution showed $\sigma_E = \sigma_A (E_t / A_t) (\partial C_t / \partial A_t)$ which is incorrect because E and A are flipped, however the calculations in that solution were all correct. The correct formula is shown below

Solution:

- (a) Calculate, for each analyst, the implied estimate of DeF's stock price volatility using the Merton Asset Value Model.

Commentary on Question:

Candidates performed as expected on this section. However the candidates specifically either scored well, or poorly in this section, depending on whether they were able to successfully apply the valuation formula. Points were awarded based on the correct application of values into the formula.

Under the Merton Asset Value Model, the implied estimate of DeF's stock price volatility is calculated using the following equation.

$$\sigma_E = \sigma_A (A_t / E_t) (\partial C_t / \partial A_t)$$

$$\text{where } \partial C_t / \partial A_t = N(dI_t)$$

Calculate asset value

$$A_t = B_t + E_t$$

5. Continued

$A_0 = 82 + 22 = 104$ million for analyst one

$A_0 = 78 + 22 = 100$ million for analyst two

$A_0 = 75 + 22 = 97$ million for analyst three

Calculate $d1$ and $N(d1)$ for Merton Asset Value Model

$$d1 = [\ln(A_t/F) + (r + \sigma_A^2/2)T] / \sigma_A \sqrt{T}$$

Where $r = 2\%$
 $T = 12$
 $F = 100$

For analyst one, $\sigma_A = 0.071$, $A_0 = 104$.

$$d1 = [\ln(104/100) + (0.02 + 0.071^2/2)(12)] / (0.071 * \sqrt{12}) = 1.26$$
$$N(d1) = 0.896$$

For analyst two, $\sigma_A = 0.054$, $A_0 = 100$. $d1 = 1.38$. $N(d1) = 0.915$

For analyst three, $\sigma_A = 0.086$, $A_0 = 97$. $d1 = 1.38$. $N(d1) = 0.803$

Calculate estimated stock price volatility for each analyst

$$\sigma_E = \sigma_A (A_t / E_t) (\partial C_t / \partial A_t)$$

where $\partial C_t / \partial A_t = N(d1_t)$

For analyst one,

$$\sigma_E = \sigma_A (A_0 / E_0) * N(d1) = (0.071)(104/22)(0.896) = 30.0\%$$

For analyst two, $\sigma_E = 22.5\%$

For analyst three, $\sigma_E = 30.4\%$

- (b) Calculate, for each analyst, the implied estimate of DeF's equity value using the Merton Asset Value Model.

Commentary on Question:

Candidates performed as expected on this section. . Most candidates recognized that the implied equity value is an implicit call option, although some of the candidates copied the wrong formula for $d2$. Errors from (a) were carried forward and candidates were not penalized for using the $d1$ values calculated in (a).

5. Continued

The implied estimate of DeF's equity value is the value of a call option under a Merton Asset Value Model.

$$C_t = A_t N(d1_t) - e^{-rT} F N(d2_t)$$

Note that all variables are defined/calculated in part (a), with the exception of $d2$.

$$d2_t = d1_t - \sigma_A \sqrt{T}$$

For analyst one, $d2 = 1.26 - 0.071\sqrt{12} = 1.01$. $N(d2) = 0.844$

For analyst two, $d2 = 1.38 - 0.054\sqrt{12} = 1.19$. $N(d2) = 0.883$

For analyst three, $d2 = 0.85 - 0.086\sqrt{12} = 0.55$. $N(d2) = 0.710$

Substituting values into formula for call option and hence the implied equity value,

For analyst one, $C = 104(0.896) - 100e^{-0.02*12}(0.844) = 26.8$ million

For analyst two, $C = 100(0.916) - 100e^{-0.02*12}(0.883) = 22.1$ million

For analyst three, $C = 97(0.803) - 100e^{-0.02*12}(0.710) = 22.0$ million

- (c) Determine which analyst provided estimates that are most consistent with your view using the Merton Asset Value Model, based on your calculations in (a) and (b).

Commentary on Question:

Candidates performed above average on this section. Although most candidates were able to conclude that analyst three provided the best estimate, some failed to highlight both the fit in volatility and value, as well as why other analysts' estimates were lacking.

Analyst Three provided the best estimate as it nearly fulfills consistency requirement in Merton's Asset Value Model by having both its implied equity volatility of 30.4% and equity value of \$22.0 million closely match actual equity volatility of 30% and equity value of \$22 million, respectively.

Analyst One's implied equity value of \$26.8 million is higher than the market value of \$22 million; Analyst Two's implied equity volatility of 22.5% is lower than the market volatility of 30%.

5. Continued

- (d) Calculate the PD under the Merton model for Analyst Two.

Commentary on Question:

Candidates performed as expected on this section. The formula is fairly straightforward, although a few candidates mistakenly stated that $PD=N(-d1)$.

The risk neutral probability of default, PD , is given by

$$PD = N(-d2) = 1 - N(d2) = 1 - 0.883 = 0.117$$

- (e) Calculate the implied LGD (as percent of EAD) for Analyst Two's estimated market value of the zero-coupon bond, using the model proposed by your manager, and the PD from part (d).

Commentary on Question:

Candidates performed as expected on this section. Some candidates did not recognize that EAD is the face value of the bond. Errors from (d) were carried forward and candidates were not penalized for using the PD values calculated in (d).

Using the equation proposed by the manager,

$$MV = e^{-rt}[(PD)(1 - LGD)(EAD) + (1 - PD)(EAD)]$$

Substituting values from earlier parts, and noting that $EAD = F = 100$,

$$78 = e^{-0.02 \cdot 12}[(0.117)(1 - LGD)(100) + (1 - 0.117)(100)]$$

$$0.78e^{0.24} = 1 - 0.117LGD$$

$$LGD = \frac{1 - 0.78e^{0.24}}{0.117} = 7.2\%$$

6. Learning Objectives:

1. The candidate will understand the standard yield curve models, including:
 - One and two-factor short rate models
 - LIBOR market modelsThe candidate will understand approaches to volatility modeling.

Learning Outcomes:

- (1l) Define and explain the concept of volatility smile and some arguments for its existence.
- (1m) Calculate the hedge ratio for a call option given the dependency of the Black-Scholes volatility on the underlying.
- (1n) Compare and contrast “floating” and “sticky smiles.

Sources:

Volatility Correlation – The Perfect Hedger and the Fox, Rebonato, R., 2nd Edition, p. 172-182

Commentary on Question:

This question tests the concept of a volatility smile, hedge ratio and variations of volatility smiles.

Solution:

- (a) Describe how the existence of volatility smiles in the real world changes the cost of this option.

Commentary on Question:

Candidates performed below average on this section. Candidates were generally able to identify that cost of option does not converge to the value given by $N(d_2)$. Only a limited number of candidates identified issues regarding using implied volatility outside a domain of applicability.

With a volatility smile, the cost of the option does not converge to the value given by the $N(d_2)$ formula.

Using implied volatility used outside the areas where it is applicable does not provide information about equivalent processes.

- (b) Explain how the delta of an asset in the real world can be estimated using the Black formula and market implied volatility from Black-Scholes.

6. Continued

Commentary on Question:

Candidates performed below average on this part. Candidates were generally able to identify a formula for delta incorporating a BlackVega. Most candidates were not able to provide a complete walk-through of the process.

With volatility smile, we know that BS implied volatility, σ_{impl} , is not linked in a simple way to the volatility, σ , of the true asset price process.

$$\text{Call price} = BS(S, T - 0, K, \sigma_{\text{impl}}(S, K))$$

$$\text{delta} = \partial \text{Call Price} / \partial S$$

$$\text{delta} = \partial \text{Black}(S, T, K, \sigma_{\text{impl}}(S, K)) / \partial S$$

$$= N(h1) + \text{BlackVega}(S, T, K, \sigma_{\text{impl}}(S, K)) * [\partial \sigma_{\text{impl}}(S, K) / \partial S]$$

The only part of this equation for delta that is not known is $\partial \sigma_{\text{impl}}(S, K) / \partial S$

The implied volatility on the underlying asset price can be estimated from today's market prices:

- The relationship of implied volatility to the underlying asset price and the strike price is equal to the relationship of implied volatility to the difference between the underlying asset price and the strike price.

Switch from $\partial / \partial S$ to $\partial / \partial K$ and make use of the readily available data

- (c) Compare a floating volatility smile to a sticky volatility smile.

Commentary on Question:

Candidates performed brilliantly on this part. Most candidates were able to correctly compare a floating volatility smile to a sticky volatility smile

Floating volatility smile – the implied volatility remains the same for the same degree of (log-) in-the-moneyness.

Sticky smile – a K-strike option would always have the same implied volatility irrespective of where the underlying moved.

- (d) Explain how this strategy would be impacted by a sticky volatility smile environment.

6. Continued

Commentary on Question:

Candidates performed poorly on this part. A significant number of candidates were able to identify in a limited way how the assumption of a sticky smile provided to information to the trader. However, the majority of candidates did not provide a sufficient level of detail on the how the trader would use the information.

Sticky smile means the implied volatility versus strike curve does not move with the level of interest rates.

In terms of the forward rate strategy:

The trader could graph implied volatilities for non-at-the-money options with the same expiry but different strikes to obtain a smile curve.

If rates were to move up, the trader would still use the same graph to read the implied volatility of a same-strike option.

The trader's belief about the relative movement of the long position with respect to the short position will not be different according to whether rates have moved up or down.

The implied volatility versus strike curve would not move with the level of rates; a property known in the market as a sticky smile.

7. Learning Objectives:

6. The candidate will understand and be able to describe the variety and assess the role of alternative assets in investment portfolios. The candidate will demonstrate an understanding of the distinguishing investment characteristics and potential contributions to investment portfolios of the following major alternative asset groups:
- Real Estate
 - Private Equity
 - Commodities
 - Hedge Funds
 - Managed Futures
 - Distressed Securities
 - Farmland and Timber

Learning Outcomes:

- (6a) Demonstrate an understanding of the types of investments available in each market, and their most important differences for an investor.
- (6d) Demonstrate an understanding of the due diligence process for alternative investments.

Sources:

QFIA-111-13: Maginn & Tuttle, *Managing Investment Portfolios*, 3rd Ed. 2007, Ch. 8

QFIA-114-13: CAIA Level II: *Advanced Core Topics in Alternative Investment*, 2nd Ed., Ch. 21

Commentary on Question:

This question tested the candidate's understanding alternative investments specifically for Real Estate, Private Equity funds and direct ownership of land (Farmland).

Solution:

- (a) Compare and contrast an investment into the new fund above with an investment into a diversified REIT portfolio.

Commentary on Question:

The candidate performed as expected on this section. The candidates were successful in describing the concentration and diversification benefits, liquidity concerns and due diligence required with a PE investment. Some candidates missed the indirect nature of REIT and PE investments. Some candidates also missed describing each of the points in detail since this part was 3 points so a simple list of major points was not enough without outlining each of them. In the list below if the candidate has mentioned 3 or more points for each sub heading they would get full credit.

7. Continued

Due Diligence, Management and Monitoring:

- PE has private transaction so informational transparency is low
- High Due diligence costs is higher for PE than a REIT investment
- Hard to perform attribution analysis in a PE fund
- PE fund is loosely regulated than the REITs.
- PE fund will have a higher due diligence costs

Liquidity:

- PE requires long term commitment of funds.
- Some PE funds have horizons of 10 year or more.
- PE funds can be very illiquid and will have liquidity restrictions depending on the deal.
- REITs liquidity is high since shares can be sold in the market.
- In REITs the funds are not locked in as in the case of PE.

Diversification/Correlation:

- REITs are more geographically diverse than PE.
- REITs and PE both have low correlation with traditional investments (stocks/bonds).
- REITs have higher correlation with stocks and bonds than the PE.
- REITs investment is diversified geographically which PE investment will have geographic concentration.
- REITs can also be diversified in investing in different type of properties like residential, commercial, and even in mortgagees.

Indirect Investment:

- Both REITs and PE are indirect investment in real estate.
- However, PE is investing directly in real estate.
- REITs represent investing in a trust that can own real estate or debt on real estate.
- REITs are bought using shares traded on the public stock markets.

- (b) Discuss diversification issues in the allocation of the 125 Million to be invested in private equity real estate for XYZ Annuity considering their existing portfolio

Commentary on Question:

The candidate performed as expected on this section. There is no correct opinion in whether PE fund is good for diversification. What we were looking for is a valid opinion supported by correct reasoning.

7. Continued

- Current allocation of \$50 million (our own building) consists of 2% of the whole asset portfolio of XYZ annuity. This is an advantage since our company uses it.
 - Total current assets: = $\$50\text{M}/2\% = \2500M
 - The investment in PE fund will increase the allocation to real estate to $\$275\text{M}/\$2500 = 11\%$.
 - PE fund is a good diversification from the traditional investment (bond/equity).
 - PE fund allocation will be $\$125\text{M}/\$2500\text{M} = 5\%$.
 - 5% allocation in PE might be considered good investment as it is not very high in total percentage of assets.
 - However a better strategy might be to go back to investing in REITs as diversification benefits in REIT can be higher because the investments will not be geographically concentrated.
 - Also REIT investment can be more diversified than just residential real estate.
- (c) Calculate the actual income from Farmland that took place in the subsequent year.

Commentary on Question:

The candidate performed above average on this section. Many candidates incorrectly calculated the income from Sunny Acres farm. Most candidates correctly calculated the income for the other farms.

$$Y = S \times I \times E \times H$$

Y = Yield

S = Total Solar radiation over the area

I = Fraction of solar radiation captures by the crop company

E = Photo synthetic efficiency of the crop

H = Harvest Index

$$\text{Sunny Acres} = 175 (0.5 + 0.5 * 8\%/1.5\%) = 554.17$$

$$\text{Green Valley} = 265 * (1 + 10\%) = 291.5$$

$$\text{Sandy Dunes} = 325 * (1 - 80\%) = 65$$

$$\text{Prairie Fields} = 125 * (1 + 10\%) = 137.5$$

$$\text{Current year Yield} = 554.17 + 291.5 + 65 + 137.5 = 1048.17$$

$$\text{Actual income from Farmland} = 1048 * 10,000 = 10.48\text{M}$$

7. Continued

- (d) Calculate the difference between the expected return on XYZ Annuity Non Fixed Income Portfolio vs. the Actual Return in percentage terms.

Commentary on Question:

The candidate performed as expected on this section. Most candidates calculated the actual return correctly. However, many candidates incorrectly calculated the expected return where many mistakenly assumed that the expected return was 8%. Note that the exact calculation of the difference in return was not required to get full credit.

Initial value of non-fixed income portfolio = 100M+ 175M = 275M

Expected return on non-fixed income portfolio = $(890 * 10,000 + 8\% * 175M) / 275M = 8.33\%$

Actual return on non-fixed income portfolio = $(10.48M \text{ from part (c)} + 8\% * 175M) / 275M$

Difference = 8.90% - 8.33% = 0.57%

8. Learning Objectives:

2. The candidate will understand and be able to apply a variety of credit risk theories and models.

Learning Outcomes:

- (2c) Demonstrate an understanding of credit valuation models.
- (2l) Understand and apply various approaches for managing credit risk in a portfolio setting.

Sources:

QFIA-101-13: Managing Credit Risk: The Great Challenge for Global Financial Markets, Caouette, John B., et. al., 2nd Edition, 2008, Ch. 20

“Altman’s (1997) Optimization Approach” on pages 390 to 397.

“Portfolio Risk and Efficient Frontiers...” pages 397 to 399.

Commentary on Question:

This question tests apply Altman’s approaches for managing credit risk in a portfolio setting.

Solution:

- (a) Calculate the annual expected rate of return and variance of loss for the portfolio.

Commentary on Question:

The candidates performed above average on this section. However, the candidates typically got most or few of the points.

The general formula for portfolio variance is:

$$V_p = \sum_{i=1}^N \sum_{j=1}^N X_i X_j \sigma_i \sigma_j \rho_{ij}$$

Applying the above formula to our two bond portfolio

$$\begin{aligned} V_p &= X_A X_A \sigma_A \sigma_A \rho_{AA} + 2 \cdot X_A X_B \sigma_A \sigma_B \rho_{AB} + X_B X_B \sigma_B \sigma_B \rho_{BB} \\ V_p &= (37.5\%)^2 (4.0\%)^2 + 2 \cdot (37.5\%) (62.5\%) (4.0\%) (8.0\%) (3.0\%) + \\ & \quad (62.5\%)^2 (8.0\%)^2 \\ &= 0.27700\% \end{aligned}$$

Where

$$X_{\text{Bond A}} = 37.5\% = \frac{1,500,000}{1,500,000 + 2,500,000}$$

$$X_{\text{Bond B}} = 62.5\% = \frac{2,500,000}{1,500,000 + 2,500,000}$$

$$\sigma_{\text{Bond A}} = 4.0\% ; \sigma_{\text{Bond B}} = 8.0\% ; \rho_{A,B} = 3.0\%$$

8. Continued

Expected annual return (EAR) of each bond is calculated as

$$EAR_i = YTM_i - EAL_i$$

Thus the expected annual returns are:

$$EAR_A = 6.47\%; EAR_B = 7.10\%$$

Expected annual return (EAR) for the portfolio is calculated as

$$EAR_p = X_A EAR_A + X_B EAR_B = (37.5\%)(6.47\%) + (62.5\%)(7.10\%) = 6.864\%$$

- (b) Determine and explain the objective function and all constraints that would be used to find an optimum weighting for the portfolio provided above using Altman's optimization approach.

Commentary on Question:

The candidates performed as expected on this section. Most candidates expressed the constraints analytically, but some candidates just copied the question constraint wording which did not indicate whether the candidates could tie in the constraints with the objective function. The candidates also did not explain the objective function or the optimization goal well.

$$\text{Objective Function: } \eta = \frac{R_p}{\sqrt{V_P}}; \text{ Where } R_p = \sum_{i=1}^{10} X_i EAR_i; V_P = \sum_{i=1}^{10} \sum_{j=1}^{10} X_i X_j \sigma_i \sigma_j \rho_{ij}$$

$$\text{Subject to (constraints): } \sum_{i=1}^{10} X_i = 1; X_i \geq 0\%, i = 1 \dots 10$$

The distribution variables must sum to 100% and there should be no short selling.

Where X_i is the percentage invested in bond i .

$$\text{Expected return of portfolio should be at least 6\%: } R_p \geq 6\%$$

$$\text{No bond may make up more than 30\% of the portfolio: } X_i \leq 30\%, i = 1 \dots 10$$

50% of the portfolio must be invested in the United States:

$$X_1 + X_4 + X_5 + X_6 + X_7 \geq 50\%$$

$$\text{At least 10\% of the portfolio must be invested in Government securities: } X_1 + X_2 + X_3 \geq 10\%$$

The portfolio must have at least 1 bond from every country;

$$\text{U.S.: } X_1 + X_4 + X_5 + X_6 + X_7 \geq 5\%$$

$$\text{Canada: } X_2 + X_8 + X_9 \geq 5\%$$

$$\text{Germany: } X_3 + X_{10} \geq 5\%$$

8. Continued

Explaining the objective function and constraints

The purpose for the optimization is to find the optimal weights (X's) of the bonds such that $\sigma_p = \sqrt{V_p}$ (risk) as small as possible under the given constraints.

- (c) Describe the reasons why Altman introduced the Z"-Score model to determine unexpected losses.

Commentary on Question:

The candidates performed below average on this section. Most of them just stated the Z"-Score model is simple but few stated why.

- It is too complicated to use of the classic mean-variance framework to analyze a reasonable large number of bond and loan assets
- There is no sufficient historical high-yield bond and loan returns data to compute correlations

9. Learning Objectives:

4. The candidate will understand important quantitative techniques for analyzing financial time series Performance Measurement and Performance Attribution.

Learning Outcomes:

- (4a) Demonstrate an understanding of the mathematical considerations for analyzing financial time series.
- (4b) Apply various techniques for analyzing factor models including Principal Component Analysis (PCA) and Statistical Factor Analysis.

Sources:

Tsay, Chapter 9

Commentary on Question:

This question tested the construction of PCA and statistical factor models. It progressed from describing the process to a practical application of performing part of a PCA analysis.

Solution:

- (a) Critique your manager's statement.

Commentary on Question:

The candidates performed above average on this section. Most identified the shortfalls of the manager's statement but many candidates did not explain their critique.

Disagree with the constraint of 99%. To adhere to the manager's constraint, all 5 factors would be needed, which would defeat the purpose of using PCA, which aims to reduce the number of factors to only the most important ones.

- (b) Determine the eigenvalues of the covariance matrix.

Commentary on Question:

The candidates performed above average on this section. Most candidates correctly calculated the eigenvalues using the appropriate sum as the denominator in the calculation. Areas where candidates did not earn full credit were incorrect calculations, applying the correct calculation but on incorrect values, and merely restating the question.

The proportion of variance explained is the ratio of the respective eigenvalue and the sum of the eigenvalues.

9. Continued

Multiplying each percentage by 0.00103:

0.0006

0.0002

0.0001

0.0001

0.0001

- (c) Describe the process to calculate each principal component.

Commentary on Question:

The candidates performed below average on this section. Most candidates demonstrated a good understanding of the process. The candidates who lost credit in did not specify the constraints when identifying subsequent factors.

The idea behind applying PCA analysis is to find linear combinations w_i such that $y_i = w_i' r$ and $y_j = w_j' r$ are uncorrelated for $i \neq j$ and the variances of y_i are maximized.

The first PC is $y_1 = w_1' r$ that maximizes the variance of y_1 , subject to the constraint $w_1' w_1 = 1$.

The second PC is $y_2 = w_2' r$ that maximizes the variance of y_2 , subject to the constraint $w_2' w_2 = 1$ AND the covariance between y_1 and y_2 is 0.

Continue as for the second PC for the third and subsequent factors.

- (d) Propose a statistical factor model using two orthogonal factors from the Principal Component Analysis.

Commentary on Question:

The candidates performed poorly on this section. Most candidates correctly recommended the most significant factors to use but few described the model.

The most significant factors are PC1 and PC2, which explain ~79% of the total variance

9. Continued

The model is

$$r_t - \mu = \beta f_t + \epsilon_t$$

The resulting matrix of factor loadings is

$$\hat{\beta} = \begin{bmatrix} \sqrt{\hat{\lambda}_1} \hat{e}_1 & \sqrt{\hat{\lambda}_2} \hat{e}_2 \end{bmatrix}$$

-0.0111% 0.0126%

-0.0115% -0.0029%

-0.0053% -0.0004%

-0.0176% -0.0057%

-0.0046% -0.0009%

Finally, the covariance of the residuals is:

$$\hat{D} = Cov[\epsilon_t] = diag\{\hat{\sigma}_1^2, \hat{\sigma}_2^2\}$$

$$\hat{\sigma}_1^2 = \hat{\sigma}_{11,r}^2 - \hat{\beta}_{11}^2 - \hat{\beta}_{12}^2 = 0.0282\% - .0111\%^2 - .0115\%^2 = .0279\%$$

$$\hat{\sigma}_2^2 = \hat{\sigma}_{22,r}^2 - \hat{\beta}_{21}^2 - \hat{\beta}_{22}^2 = 0.0204\% - .0178\%^2 - .0041\%^2 = .0202\%$$

- (e) Estimate an upper bound for the sum of the squared elements of the error matrix for the model in (d).

Commentary on Question:

The candidates performed poorly on this section. Most candidates omitted this section. The few that attempted this section often failed to calculate the correct equation in determining the approximation error.

The approximation error is bounded by the factors not used in the analysis from the recommendation, i.e., factors 3, 4, and 5. The error is bounded by

$$\hat{\lambda}_3^2 + \hat{\lambda}_4^2 + \hat{\lambda}_5^2 = 3 \times 10^{-8}$$

10. Learning Objectives:

6. The candidate will understand and be able to describe the variety and assess the role of alternative assets in investment portfolios. The candidate will demonstrate an understanding of the distinguishing investment characteristics and potential contributions to investment portfolios of the following major alternative asset groups:
- Real Estate
 - Private Equity
 - Commodities
 - Hedge Funds
 - Managed Futures
 - Distressed Securities
 - Farmland and Timber

Learning Outcomes:

- (6a) Demonstrate an understanding of the types of investments available in each market, and their most important differences for an investor.
- (6d) Demonstrate an understanding of the due diligence process for alternative investments.

Sources:

QFIA-111-13 Maginn & Tuttle, Managing Investment Portfolios, 3rd Ed. 2007, Ch. 8

Commentary on Question:

This question tested the candidates' understanding of types of investments and their ability to identify and explain the characteristics of a private equity transaction.

Solution:

- (a) Identify the financial transaction taking place, the participants, and their respective roles.

Commentary on Question:

The candidates performed as expected on this section. Successful candidates were able to identify the transaction as private equity and identify the participants and their respective roles. Unsuccessful candidates misidentified the financial transaction.

This is a private equity transaction as can be seen by the fact that Adam is the sole owner of the business in need of capital. Donald and Eric are investors in the business.

10. Continued

- (b) Describe the main investment characteristics of this transaction.

Commentary on Question:

The candidates performed as expected on this section. Successful candidates were able to identify the relevant characteristics and describe them as they specifically relate to the transaction. Unsuccessful candidates did not identify or describe the characteristics.

The following are the main investment characteristics of this transaction:

- **Illiquidity:** There is no secondary market for that investment. In fact there is barely a primary market.
- **Long-term commitment required:** Donald and Eric have no idea when they'll get their investment back.
- **Higher-risk:** Donald and Eric may never get their investment back.
- **Limited information:** It is very difficult to perform due diligence on this transaction and there is limited information available about Adam's business ability.
- **High expected IRR required:** Donald and Eric are hoping for a higher expected return on this transaction given the items listed above.

- (c) Critique the fairness of the revenue split between Donald and Eric.

Commentary on Question:

The candidates performed poorly on this section. Successful candidates recognized that the main distinction determining the fairness of the split is the difference in majority ownership and control between Donald and Eric. Unsuccessful candidates did not recognize this and critiqued the fairness solely on the prices Donald and Eric paid per unit of revenue.

The revenue split between Donald and Eric is fair. Although Eric paid less per unit of revenue than Donald, this is justified because Donald has a majority and controlling position and control while Eric doesn't.

- (d) Develop a range of market values for Adam's share of the business based on the information provided.

Commentary on Question:

The candidates performed poorly on this section. Successful candidates recognized that based on Adam's percent stake and control, his per unit of revenue market value should be somewhere in between that of Donald and Eric. Unsuccessful candidates either did provide a market value or determined Adam's market value off the cost to build a glove.

10. Continued

Adam has a minority position but more control than Eric. Therefore his per unit of market value should be somewhere between that of Donald and Eric, which leads to a market value between 300 and 450.

- (e) Evaluate the prospect of market success for this transaction from each participant's perspective.

Commentary on Question:

The candidates performed below average on this section. Unsuccessful candidates did not identify any or only very few relevant prospects for success.

The key prospects for success are the following.

- **Markets, competition and sales prospects:** The success of the transaction is dependent on whether there are competitors to this product and whether there is interest. This item is key for Adam, Donald and Eric.
- **Management experience:** The success of the transaction is dependent on how capable Adam is managing the business. This is relevant for all three participants, but especially key for Adam because if he is not capable, Donald and Eric can try to replace him.
- **Management commitment:** The success of the transaction is dependent on how committed Adam is. In particular, he is doing most of the work but only has 25% ownership of the revenue. This is relevant for all three participants, but especially key for Adam because if he is not committed, Donald and Eric can try to replace him.
- **Capital (marbles) invested:** The amount of capital invested determines the success of the transaction for each participant with it being more relevant the more capital invested. Having invested only 50 marbles, this is not very relevant to Adam, but it is more relevant to Donald and Eric.
- **Opinion of customers:** The opinion of customers also helps determine the success of the transaction. Even if there is a market, if customers don't like the product, the business will fail. This affects all three participants.
- **Identity of current investors:** The current investors in the business also help determine its success, for example because they can contribute to the business, lend more capital, and lend their reputation. This affects all three participants.

11. Learning Objectives:

4. The candidate will understand important quantitative techniques for analyzing financial time series Performance Measurement and Performance Attribution.

Learning Outcomes:

- (4f) Calculate and interpret performance attribution metrics for a given asset, portfolio.

Sources:

QFIA-107-13: Ch 69&70: Handbook of Fixed Income Securities, Fabozzi, F.J., 8th Edition, 2012

Commentary on Question:

This question tests candidates' knowledge of performance attribution matrix for a given asset portfolio.

Solution:

- (a) List and define the important principles of a successful performance attribution algorithm.

Commentary on Question:

The candidates performed above average on this section. While nearly all candidates were able to list and define the first three principles (additivity, completeness and fairness), few were able to come up with the complete list. Full credit was given if the candidate was able to list and define four of the five principles; otherwise, partial credit was given based on the number of principles candidates listed and defined.

The important principles of a successful performance attribution algorithm are:
Additivity: The contribution of two or more agents combined is equal to the sum of the contributions from those agents.

Completeness: The sum of all outperformance contributions is equal to the total portfolio outperformance.

Fairness: The allocation of outperformance to different interacting agents is performed in a way that is perceived to be fair by all agents.

Multi-Period Attribution and Compounding: The attribution splits into two (or more) sub-periods and combines the results of the sub-periods to explain the total portfolio performance over the entire period.

Flexibility: The attribution reflects the decisions that a portfolio manager makes during the normal course of investment process (e.g. able to measure risk factors corresponding to manager decisions, able to measure contributions of factors, able to follow published returns of the fund and its benchmark, be available in timely manner, able to adapt and explain both short and long time periods)

11. Continued

- (b) Evaluate whether your performance attribution model can be a successful model.

Commentary on Question:

The candidates performed brilliantly on this section. While most candidates were able to conclude that the attribution model was not successful, some did not provide a numerical support for the conclusion. In this case, partial credit was given to the candidate.

A successful performance attribution model has to satisfy at least the additivity principle. However, this is not the case for the model because:

- (a) $0.3 + 1.1$ does not equal to 1 as asset allocation for bond A and bond B does not equal to total asset allocation and
(b) $1.1 + 1$ does not equal to 3 as the sum of asset allocation and security selection does not equal to total return for bond B

In light of the above, this model cannot be a successful model.

- (c) Calculate the portfolio outperformance due to reshaping of the yield curve.

Commentary on Question:

The candidates performed as expected on this section. About half the candidates were able to arrive at the correct answer; others made mistakes in terms of using an incorrect formula and/or incorrect numbers. Partial credit was given for using the correct formula and correct input numbers but with incorrect final answer due to arithmetic errors.

Outperformance is calculated using the formula

$$- \sum (KRD_j^P - KRD_j^B) * (\Delta y_j - \Delta y_{avg})$$

Plugging the values:

$$\begin{aligned} & - [(0.1 - 0.2) * (-0.15 + 0.2) + (0.4 - 0.5) * (-0.2 + 0.2) \\ & \quad + (0.7 - 0.8) * (-0.4 + 0.2) + (2.7 - 1.2) * (-0.6 + 0.2) \\ & \quad + (1.2 - 1.3) * (-0.45 + 0.2)] \\ & = 0.56 \end{aligned}$$

12. Learning Objectives:

1. The candidate will understand the standard yield curve models, including:
 - One and two-factor short rate models
 - LIBOR market modelsThe candidate will understand approaches to volatility modeling.

Learning Outcomes:

- (1h) Understand and explain the features of the G2++ model, including: The motivation for more than one factor, calibration approaches, the pricing of bonds and options, and the model's relationship to the two-factor Hull-White model.

Sources:

Interest Rate Models – Theory and Practice, 2nd Edition, Brigo, D and Mercurio F, Sections 4.1-4.2, page number: 138 – 139, 145, 148-149, 166-169

Commentary on Question:

This question tested the candidate's understanding of the motivation for using a two-factor model in interest rate modeling and the features of the G2++ model, including calibration approaches in the pricing of bonds.

Solution:

- (a) Describe the motivation of using a two-factor interest rate model vs. one-factor model in interest rate modeling.

Commentary on Question: *The candidates performed above average on this question. Many of the candidates were able to describe the pros and cons of the two-factor model and the one-factor model in interest rate modeling and pricing complex instruments.*

For one-factor model, all the points of the yield curve are perfectly correlated. In reality, yield curve exhibits de-correlation.

Some instruments have payoffs that depend on more than one interest rate. For these instruments, correlation structure is important in calculating the payoffs. The two-factor model is a better process of capturing this correlation.

- (b) Show that $f(t, T)$, the instantaneous forward rate for this model, is

$$f(t, T) = \varphi(T) + e^{-a(T-t)} x(t) + e^{-b(T-t)} y(t) - \frac{\partial V(t, T)}{\partial T} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

12. Continued

Commentary on Question:

Candidates performed brilliantly on this section. Most of the candidates were able to show the final result of $f(t,T)$ while showing full development of differentiation of each of the component of the distribution. Candidates who failed to get full credit had computation errors such as an incorrect sign and/or an incorrect differentiation of one or more of the exponential components above.

$$f(t,T) = -\frac{\partial \ln P(t,T)}{\partial T}$$

$$P(t,T) = \exp\left\{-\int_t^T \varphi(u)du - (1 - e^{-a(T-t)})\frac{x(t)}{a} - (1 - e^{-b(T-t)})\frac{y(t)}{b} + \frac{V(t,T)}{2}\right\}$$

$$-\frac{\partial}{\partial T}(\ln \varphi(T))$$

$$-\frac{\partial}{\partial T}(\ln \left(\exp\left\{(1 - e^{-a(T-t)})\frac{x(t)}{a}\right\}\right)) = e^{-a(T-t)}x(t)$$

$$-\frac{\partial}{\partial T}(\ln \left(\exp\left\{(1 - e^{-b(T-t)})\frac{y(t)}{b}\right\}\right)) = e^{-b(T-t)}y(t)$$

$$-\frac{\partial}{\partial T}(\ln \left(\exp\left\{\frac{V(t,T)}{2}\right\}\right)) = -\frac{1}{2}\frac{\partial V(t,T)}{\partial T}$$

Combining the above 4 lines gives you the answer of

$$f(t,T) = \varphi(T) + e^{-a(T-t)}x(t) + e^{-b(T-t)}y(t) - \frac{\partial V(t,T)}{\partial T}\left(\frac{1}{2}\right)$$

- (c) Determine which set of the parameters most likely came from the cap market volatility data and explain why.

Commentary on Question:

The candidates performed above average on this section. Most of the candidates were able to provide a clear explanation of how the value of ρ being close to -1 is consistent with the expected results for cap market volatility. Some candidates were also able to provide clear reasons disqualifying the other parameter sets. Candidates who did not get full credit often demonstrated they did not understand the calibration of the the G2++ model using cap market data,.

Caps prices do not depend on the correlation of forward rates. So even a one-factor model that implies perfectly correlated rates can fit cap data well in many situations. The set of parameters that has a ρ value close to -1 will work as this implies the G2++ model degenerates into a single factor process. Set II is the answer.

13. Learning Objectives:

2. The candidate will understand and be able to apply a variety of credit risk theories and models.

Learning Outcomes:

- (2h) Demonstrate an understanding of credit default swaps (CDS) and the bond-CDS basis, including the use of CDS in portfolio and trading contexts.
- (2i) Demonstrate an understanding an understanding of CDS valuations

Sources:

V-C183-10, J.P. Morgan, Bond CDS Basis Handbook, February 2009

Commentary on Question:

This question was tested candidates understanding of Bond CDS basis trades and their associated calculations.

Solution:

- (a) Describe two main motivations relating to trading and structuring basis trades.

Commentary on Question:

The candidates performed above average on this section. The most common mistake was listing the motivations rather than describing them.

Lock-In “Risk-free” Spread - If bond and CDS share the same credit risk but they are pricing it differently, it might be possible to construct something akin to an “arbitrage-free” trade to profit from it.

Trade the Basis - A negative basis trade (buy bond and buy CDS protection) can be used to bet that an already negative basis will disappear, or to bet that the basis will become positive. For example, CDS spreads might react faster to negative news regarding corporate events. In those cases, the basis can become positive until bond spreads catch up. A negative basis trade established prior to the negative news should profit from it.

- (b) Describe how the “cheapest-to-deliver option” on a CDS contract impacts the CDS spread.

Commentary on Question:

The candidates performed above average on this section. The most common mistake was an incorrect directional impact on the CDS spread.

13. Continued

CDS protection buyers have the so-called “cheapest-to-deliver” option: in case of default, they are contractually allowed to choose which bond to deliver in exchange for the notional amount. Thus, investors will generally deliver the cheapest bond in the market.

When there is a credit event, bonds at the same level of the capital structure generally trade at or near the same price (except for potential differences in accrued interest) as they will be treated similarly in a restructuring. Still, there is the potential for price disparity.

- (c) Calculate the minimum and maximum Bond-CDS basis of the entire group of bonds.

Commentary on Question:

The candidates did brilliantly on this section. The most common mistakes were reversing the bond-CDS basis formula and not stating which basis was the maximum and which was the minimum which was asked for in the question.

$$\text{CDS spread} = (\text{Probability of default}) (1 - \text{Expected recovery rate})$$

$$\text{Bond-CDS Basis} = \text{CDS spread} - \text{Bond spread}$$

Maturity (years)	1	2	3	4
Bond spread (bps)	225	257	305	340
Probability of default	6%	8%	8%	10%
Expected recovery rate	1- (0.0180/0.06) = 70%	1- (0.0240/0.08) = 70%	1- (0.0320/0.08) = 60%	1- (0.0400/0.10) = 60%
CDS spread (bps)	6%(1-70%)= 180	8%(1-70%)= 240	8%(1-60%)= 320	10%(1-60%)= 400
Bond-CDS basis (bps)	180-225 = -45	240 - 257 = -17	320 - 305 = 15	400 - 340 = 60

Minimum Bond-CDS basis = -45 bps (Maturity 1-year)

Maximum Bond-CDS basis = 60 bps (Maturity 4-year)

13. Continued

- (d) Identify the bond that offers the best negative basis trade arbitrage opportunity and describe the strategy.

Commentary on Question:

The candidates performed brilliantly on this section. The most common mistake was not describing the strategy.

Negative basis trade - If the basis is negative, the CDS spread is lower (tighter) than the bond spread. To capture the pricing discrepancy when a negative basis arises, an investor could buy the bond (long risk) and buy CDS protection (short risk) with the same maturity as the bond.

The 1-year maturity bond offers the greatest negative basis trade opportunity. The underlying positions for the strategy include buying both the 1-year maturity bond and the 1-year CDS.

14. Learning Objectives:

3. Candidate will understand the nature, measurement and management of liquidity risk in financial institutions.

Learning Outcomes:

- (3b) Measure and monitor liquidity risk, using various liquidity measurement tools and ratios.
- (3d) Understand liability termination provisions such as book-value surrender and the impact on a company's overall liquidity risk.

Sources:

QFIA-105-13: Report of the Life Liquidity Work Group of the American Academy of Actuaries to the Life Liquidity Risk Working Group of the NAIC (7 points)

Commentary on Question:

This question tests the candidate's knowledge of a practical approach to deal with liquidity risk for an insurance company relative to the embedded option into insurance contract.

Solution:

- (a) Explain the potential liquidity exposures embedded in XYZ's liabilities.

Commentary on Question:

The candidates performed as expected on this section. Some candidates did not identify the risk that policyholders will borrow as much as they can in a stress scenarios. Also, some candidates identified that liquidity risk increases with reduced surrender charges, but failed to specify the potential impact on the 8th year and after for WL.

Non-participating whole life:

- Older policies are more likely to surrender because of higher cash surrender values especially at the end of the 8th policy year and after. When there will be no more surrender charges, there could be large surrenders. Then XYZ could have a large amount of cash outflow at the end of the 8th year.
- We need to consider the ability of the policyholders to borrow against their policies cash value and cause liquidity concerns. In a stress scenario, we assume many knowledgeable people will borrow as much as they can against their policies if interest on policy loan can provide a gain in the context of current interest rate on the market at time of loan.

GIC:

- In a downgrade scenario, XYZ may not be able to gather enough cash within the 7 days' notice to pay back the investors. The 7-day period may be too short for XYZ in a stress scenario.

14. Continued

- (b) Describe ways to manage the liquidity risks embedded in XYZ's liabilities.

Commentary on Question:

The candidates performed below average on this section. Many candidates forgot to explain the specific investment tool required to protect the cash need in event of specific risk as downgrade, stress scenario, or mismatch of asset and liability. Many candidates alternatively suggested policy modifications to reduce the liquidity risk which was not asked for in the question. Additionally, policy provisions for inforce policies cannot be changed and there were many suggestions that were not feasible.

XYZ could consider some of the following:

For both WL and GIC products:

- Use repurchase agreements (repos) to mitigate short-term cash needs.
- Purchase liquidity options from an investment dealer to protect against the liquidity risks.
- Have back-stop liquidity lines for both products to draw cash at a short-term notice, with a guaranteed price.
- Purchase equity puts that will pay off during an industry stress scenario reflecting the sector the company is in (i.e., *not just any industry stress scenario*).

For GIC product:

- Ladder the asset and the liability maturities to offset the risk of needing to sell new GICs in order to fund the maturity of older ones.
- Purchase credit derivatives that will pay in the event of a downgrade to protect against the risk of a downgrade of XYZ.

- (c) Propose ways to improve this calculation to make it more effective.

Commentary on Question:

The candidate performed below average on this section. Most candidates identified stressed scenarios, however they often did not specify how the stresses apply to both asset and liability cashflows, both expected and unexpected.

We suggest the calculation be performed to consider many scenarios especially using stressed scenarios to value the stress liquidity risk impact on both assets and liabilities of the company.

14. Continued

For each one we need to consider those items:

- For the liabilities liquidity risk:
 - The projection of liability cash flows shouldn't be limited to scheduled payments, unexpected payments caused by stress liquidity scenarios should be considered as well.
 - Consider any ability of policyholders to surrender the policy without penalty – in this example, as surrender charges drop over time, likelihood of surrenders increase, thus increasing liquidity risk.
 - In a stressed situation, knowledgeable institutional investors will exercise the puts, thus creating dramatic drains on a company's liquidity in a stressed scenario.
 - Policyholders who borrow policy loans may borrow as much as they can in a stress scenario.

For those items different factors should be assigned under various timeframes (base case vs. stress scenarios) to account for policyholders' likelihood to surrender and the timeframe they have to react.

- For the assets liquidity risk:
 - Project out future expected and unexpected asset principal and interest payments.
 - Calculate the potential asset haircuts to determine how much discount the assets will be sold at in a stress scenario.
 - Determine potential asset sales limits as the maximum amount/volume of a type of asset that the market can absorb in a stress scenario.

15. Learning Objectives:

1. The candidate will understand the standard yield curve models, including:
 - One and two-factor short rate models
 - LIBOR market modelsThe candidate will understand approaches to volatility modeling.

Learning Outcomes:

- (1f) Explain how deterministic shifts can be used to fit any given interest rate term structure.
- (1g) Demonstrate an understanding of the CIR++ model.

Sources:

Interest Rate Models – Theory and Practice: With Smile, Inflation and Credit, Brigo, D. & Mercurio F., 2nd Edition, p. 95-96, 102-110

Commentary on Question:

This question tests how deterministic shifts can be used to fit any given interest rate term structure and an understanding of the CIR++ model.

Solution:

- (a) List the properties of a deterministic shift extension to the CIR short-rate model.

Commentary on Question:

The candidates performed above average on this section. Many candidates were able to identify at least two of the properties of a deterministic shift extension.

Exact fit of any observed term structure.

Analytical formulas for bond prices, bond-option prices, swaptions and cap prices.

The distribution of the instantaneous spot rate has tails that are fatter than in the Gaussian case and, through restriction on the parameters, it is always possible to guarantee positive rates without worsening the volatility calibration in most situations.

- (b) Describe any fitting quality issues when using the CIR++ model.

Commentary on Question:

The candidates performed below average on this section. The common omission was failing to identify imposing positive rates on calibration to market prices.

The important issue for the CIR++ model is whether calibration to market prices is feasible while imposing positive rates.

Calibration to market prices breaks down in the case of an inverted yield curve. However, inverted curves are not common in liquid markets so the problem can generally be avoided.

15. Continued

- (c) Explain how you would use the CIR++ model to price caps and floors.

Commentary on Question:

The candidates performed poorly on this section. Nearly all candidates failed to document the key point that is CIR++ model is the extension of the CIR model.

Since the CIR++ model is the extension of the Cox-Ingersoll-Ross model, we calculate the analytical formulas implied by such extension.

Assuming exact fitting of the initial term structure of discount factors, we have $\varphi(t) = \varphi^{\text{CIR}}(t; \alpha)$.

Solve differential equations to obtain the price at time t of a zero-coupon bond maturing at time T is $P(t, T) = \bar{A}(t, T)e^{-B(t, T)r(t)}$

Caps and floors can be priced since they can be viewed as portfolios of zero bond options.

- (d) Identify and explain any problems in your manager's proposal.

Commentary on Question:

The candidates performed above average on this section. The majority of candidates were able to identify the two problems.

$\sigma\sqrt{r_t}$ should be in front of dW_t and not in front of J_t .

$\alpha < 0$ is inaccurate because the arrival rate is non-negative by definition.

16. Learning Objectives:

3. Candidate will understand the nature, measurement and management of liquidity risk in financial institutions.

Learning Outcomes:

- (3c) Understand the levels of liquidity available with various asset types, and the impact on a company's overall liquidity risk.
- (3e) Apply liquidity risk models, including modeling cashflow of various types of assets (e.g. indeterminate maturity assets) and liabilities.
- (3f) Apply liquidity scenario analysis with various time horizons.

Sources:

Liquidity Risk: Measurement and Management - A Practitioner's Guide to Global Best Practices, Matz, Leonard & Neu, Peter Chapter 3 (6 points)

Commentary on Question:

This question tested the candidate's knowledge of scenario analysis and liquidity risk and an understanding of the nature, measurement and management of liquidity risk in financial institution.

Solution:

- (a) Identify four considerations when creating and evaluating deterministic scenarios.

Commentary on Question:

The candidates performed below average on this section. Many candidates identified that considerations should be bank specific, should address short and long term liquidity concerns, and be internally consistent.

Evaluate both long time and short term scenarios. Banks and regulators may look at seven and 30-day hypothetical liquidity events.

Stress levels need to be set properly. A mild insurance-company specific funding problem and a worst case problem are simply one scenario described at two different stress levels.

Synchronize ordinary course of business liquidity scenario with budget

Funding problems can build up over months.

The scenarios need to be relevant to the strategic nature of the banks

The scenarios need to be relevant to the stability or stickiness of the liabilities.

Include seasonal fluctuations in the firm's liquidity

16. Continued

- (b) Explain the impact of interest rates on liquidity needs.

Commentary on Question:

The candidates performed as expected on this section. Many candidates were able to describe 1-3 impacts on bank's liquidity needs from an interest rate change. However, some candidates explained how the MV and/or duration of the asset and liability would change which is not liquidity management

Term structure of bank's assets or liabilities may be affected by changes in prevailing interest rates

Holdings of bank assets and liabilities with embedded options can be materially affected by changes in prevailing interest rates

The volume of new bank assets can be directly affected by changes in interest rates

The volume of the bank's liabilities can be directly affected by changes in interest rates

The volume of the bank's assets and liabilities may be indirectly affected as changes in prevailing interest rates track (or cause) changes in business conditions

- (c) Calculate the cash flow cushion as of June 30 for each of the two scenarios.

Commentary on Question:

The candidates performed above average on this section. Many candidates calculated the bond and MBS cashflow under stress scenario 2 using a MV instead of the par value.

cash flow cushion = forecast cash inflows for the period / forecast cash outflows for the period

Scenario 1:

Cash inflows = coupons + maturities = $(600 \cdot 4\% / 2 + 200 \cdot 5\% / 2 + 200 \cdot 3\% / 2) + (600 \cdot 25\% + 200 \cdot 20\% + 200 \cdot 20\%) = (12 + 5 + 3) + (150 + 40 + 40) = 250$

Cash outflows = 200 (only the CDs)

Cash flow cushion = $250 / 200 = 125\%$

Scenario 2:

Cash inflows = coupons + maturities = $(600 \cdot 4\% / 2 + 200 \cdot 5\% / 2 + 200 \cdot 3\% / 2) + (600 \cdot 25\% + 200 \cdot 20\% + 200 \cdot 15\%) = (12 + 5 + 3) + (150 + 40 + 30) = 240$

Cash outflows = $200 + 400 \cdot 10\% = 240$

Cash flow cushion = $240 / 240 = 100\%$

16. Continued

- (d) Recommend actions which could be taken to meet the requirement.

Commentary on Question:

The candidates performed as expected on this section. Almost all candidates identified that scenario 2 does not meet the minimum level of cash flow cushion. Candidates who calculated the shortage of cash flow correctly in scenario 2 based on their answer in part c were given credit. Some other common answers which did not receive credit are: 1) reduce holding on commercial mortgage since it is not liquid, 2) increase allocation to bond since it is more liquid, which implies to reduce holdings on commercial mortgage and MBS.

The cash flow cushion for Scenario 1 (125%) exceeds the minimum level, but the one for Scenario 2 (100%) is below the level. [1] So, would need to recommend actions to increase the amount of inflows.

To have a cash flow cushion of 110%, the amount of inflows would need to be $240 \times 1.1 = 264$

Selling either the corporate bonds or the MBS would increase the amount of inflows.

Recommend selling the corporate bonds to raise 24 of cash. The capital loss from selling the corporate bonds is less than the capital loss from selling the MBS.

Selling bonds reduces coupons – required for full credit (see note below)

Some common other answers which received partial credit if the candidate showed a good understanding of their approaches:

1) replace MBS and/or bond with cash and/or short term treasury which is safe and liquid.

2) obtain a funding or credit line,

3) encourage more deposit,

4) entering into interest rate derivatives to hedge against rising interest rate.