GI ADV Model Solutions
Spring 2016

1. **Learning Objectives:**
4. The candidate will understand how to apply the fundamental techniques of reinsurance pricing.

Learning Outcomes:
(4b) Calculate the price for a property per risk excess treaty.

(4e) Describe considerations involved in pricing property catastrophe covers.

Sources:
Basics of Reinsurance Pricing, Clark

Solution:
(a) Calculate the expected loss in the layer assuming an expected loss ratio of 60%.

The calculation is

$$\left[1,000,000\left(\left(1-\frac{1.0}{1.2}\right)^2-0\right)+1,000,000\left(\left(1-\frac{0.5}{1.2}\right)^2-0\right)+2\cdot1,000,000\left(\left(1-\frac{0.2}{1.2}\right)^2-\left(1-\frac{1.0}{1.2}\right)^2\right)\right]\cdot60\%$$

which equals 1,020,833.

(b) Explain why you might elect not to use an experience rating approach to price this treaty because of the issue of "free cover."

Commentary on Question:
A full credit answer needs to both explain the issue and indicate why the cover is "free." Most candidates did both.

There may be no trended losses in the top of the layer. In this case the experience rating approach would indicate zero premium for this part of the layer. Thus, the cover turns out to be free.

(c) State whether the property catastrophe cover will typically inure to the benefit of the surplus share reinsurance.

1. Continued

Commentary on Question:
For this question and part (d) it is sufficient to answer no because the question did not ask for justification.

The answer is no. The reverse is typically true. The surplus share reinsurance will inure to the benefit of the property catastrophe cover.

(d)   State whether the property catastrophe cover will typically inure to the benefit of the property per risk excess treaty.

The answer is no. The reverse is typically true. The property per risk excess treaty will inure to the benefit of the property catastrophe cover.

2. **Learning Objectives:**
5. The candidate will understand methodologies for determining an underwriting profit margin.

Learning Outcomes:
(5d) Allocate an underwriting profit margin (risk load) among different accounts.

Sources:
An Application of Game Theory: Property Catastrophe Risk Load, Mango

Solution:
(a) Calculate the variance risk load for the combined portfolio.

Commentary on Question:
There continues to be occasional confusion between the case where there are several risks, each with a single possible claim amount (as in this question) and the case where a single risk can have several possible claim amounts.

First the variances must be calculated:
ERU: $100^2(0.01)(0.99) + 200^2(0.02)(0.98) + 400^2(0.03)(0.97) = 5,539$
HRU: $500^2(0.02)(0.98) + 400^2(0.04)(0.96) + 200^2(0.06)(0.94)$
$\qquad + 100^2(0.07)(0.93) = 13,951$
Then the covariance is $0.2\sqrt{5,539(13,951)} = 1,758$.
The variance of the combined portfolio is $5,539 + 13,951 + 2(1,758) = 23,006$.
The risk load is $25(23,006) = 575,150$.

(b) Calculate the risk loads for each of ERU and HRU using the Shapley method.

Commentary on Question:
Candidates received full credit if incorrect numbers from part (a) were correctly used here.

For ERU the risk load is $25(5,539 + 1,758) = 182,425$.
For HRU the risk load is $25(13,951 + 1,758) = 392,725$.

(c) Identify which approach will ensure that the separate risk loads will add up to the combined portfolio risk load calculated in part (a).

Commentary on Question:
Because the question asked that the approach be identified, the name is sufficient for full credit.

The Build-up Risk Load approach.

2. Continued

(d)   Calculate the two possible sets of risk loads for ERU and HRU using the Marginal Variance method and the approach identified in part (c).

Commentary on Question:
Many candidates did not calculate both sets of risk loads.

The Build-up approach requires sequential evaluation. Assuming ERU is calculated first, the ERU risk load is 25(5,539) = 138,475. Then the risk load for HRU is 25(23,006 – 5,539) = 436,675. If HRU is calculated first, its risk load is 25(13,951) = 348,775 and then the risk load for ERU is 25(23,006 – 13,951) = 226,375.

3. **Learning Objectives:**
1.   The candidate will understand how to use basic loss development models to estimate the standard deviation of an estimator of unpaid claims.

Learning Outcomes:
(1e)   Apply a parametric model of loss development.

(1f)   Estimate the standard deviation of a parametric estimator of unpaid claims.

Sources:
LDF Curve Fitting and Stochastic Reserving: A Maximum Likelihood Approach, Clark

Solution:
(a)   State two advantages and one disadvantage of using a parametric cumulative distribution function to model loss development.

   Commentary on Question:
   Most candidates identified two advantages, but few were able to identify the disadvantage. Only two of the listed advantages were needed for full credit.

   Three advantages are (1) it provides smoothing, (2) there are a small number of parameters to estimate, and (3) it does not require equal spacing of data points. Disadvantage is that only increasing development patterns can be modeled.

(b)   Calculate the maximum likelihood estimate of accident year 2013 ultimate losses, ULT_{2013}.

   $$ULT_{2013} = 7,250 / F(30) = 7,250 / [1 - \exp(-30 / 7.804)] = 7,409$$

(c)   Calculate the values of *X* and *Y*.

   $$X = 7,409F(6) = 7,409[1 - \exp(-6 / 7.804)] = 3,975$$
   $$Y = 7,409F(18) = 7,409[1 - \exp(-18 / 7.804)] = 6,671$$

(d)   Identify the number of degrees of freedom associated with the estimate of σ^2.

   Degrees of freedom = 6 (data points) – 4 (parameters) = 2.

(e)   Estimate the process standard deviation of the accident year 2013 reserve.

   The reserve is 7,409 – 7,250 = 159.
   The standard deviation is sqrt[159(47)] = 86.

4. **Learning Objectives:**

1. The candidate will understand how to use basic loss development models to estimate the standard deviation of an estimator of unpaid claims.

Learning Outcomes:

(1a) Identify the assumptions underlying the chain ladder estimation method.

(1b) Test for the validity of these assumptions.

(1c) Identify alternative models that should be considered depending on the results of the tests.

(1d) Estimate the standard deviation of a chain ladder estimator of unpaid claims.

Sources:

Measuring the Variability of Chain Ladder Reserve Estimates, Mack. Testing the Assumptions of Age-to-Age Factors, Venter.

Solution:

(a) Demonstrate that the value of α_4^2 was correctly calculated. (Your calculation need not match to all three decimal places.)

$$
\begin{aligned}
\alpha_4^2 &= \frac{1}{7-4-1}\sum_{j=1}^{7-4} C_{j,4}\left(\frac{C_{j,5}}{C_{j,4}} - f_4\right)^2 \\
&= \frac{1}{2}\left[C_{1,4}\left(\frac{C_{1,5}}{C_{1,4}} - f_4\right)^2 + C_{2,4}\left(\frac{C_{2,5}}{C_{2,4}} - f_4\right)^2 + C_{3,4}\left(\frac{C_{3,5}}{C_{3,4}} - f_4\right)^2 \right] \\
&= \frac{1}{2}\left[14,212\left(\frac{14,486}{14,212} - 1.03608\right)^2 + 27,910\left(\frac{28,117}{27,910} - 1.03608\right)^2 \right. \\
&\quad \left. + 22,433\left(\frac{24,281}{22,433} - 1.03608\right)^2 \right] \\
&= 37.514.
\end{aligned}
$$

4. Continued

(b) Demonstrate that the standard error for accident year 3 was correctly calculated.

$$C_{3,7}^2 \sum_{k=7+1-3=5}^{7-1=6} \frac{\alpha_k^2}{f_k^2}\left(\frac{1}{C_{3,k}}+\frac{1}{\sum_{j=1}^{7-k}C_{j,k}}\right) = C_{3,7}^2\left[\frac{\alpha_5^2}{f_5^2}\left(\frac{1}{C_{3,5}}+\frac{1}{\sum_{j=1}^{7-5=2}C_{j,5}}\right)+\frac{\alpha_6^2}{f_6^2}\left(\frac{1}{C_{3,6}}+\frac{1}{\sum_{j=1}^{7-6=1}C_{j,6}}\right)\right]$$

$$= 25,310^2\left[\frac{0.308}{1.02256^2}\left(\frac{1}{24,281}+\frac{1}{14,486+28,117}\right)+\frac{0.00252}{1.01937^2}\left(\frac{1}{24,829}+\frac{1}{14,867}\right)\right]$$

$$= 12,367$$

The standard error is the square root, 111.

(c) For each of Mack's three assumptions:

   (i) State the assumption; and

   (ii) Explain why that assumption does or does not prevent the value from decreasing from one development year to the next.

Commentary on Question:
Candidates generally knew the three assumptions but did not always provide a solid argument for their relationship to possible decreasing development.

(1) The expected value is the previous value times a constant that depends only on the development year. This assumption does not prevent values from decreasing because it only relates to the expected, not the actual value. Alternatively, the constant can be less than one, which implies an expected decrease.
(2) The variance is the previous value times a factor that depends only on the development year. This assumption does not prevent values from decreasing because with variability in outcomes, that could extend to decreasing values.
(3) Values from different accident years are independent. This assumption makes no statement about the magnitude of the values and so allows for decreasing values.

(d) State the number of observations and the number of estimated parameters for this situation.

There is one parameter per development year after the first, so there are six. When there are *n* development years, there are *n*(*n* − 1)/2 or 7(6)/2 = 21 observations.

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4. Continued

(e) Calculate the adjusted SSE using one of the three methods suggested by Venter.

Commentary on Question:
Any one of the three listed answers earns full credit. If incorrect values were provided in part (d), there is full credit if they are used correctly here.

Using $n = 21$ and $p = 6$:
$$126,347,521/(21-6)^2 = 561,545$$
$$126,347,521e^{2(6)/21} = 223,735,552$$
$$126,347,521(21^{6/21}) = 301,539,122$$

(f) Describe one such alternative model, using words, not formulas.

Commentary on Question:
Venter lists three such models. Any one is sufficient.

(1) Add a constant after multiplying by the chain ladder development factor.
(2) Multiplication of factors representing the accident year and development year. This can also be described as a parameterized version of the Bornhuetter Ferguson method.
(3) As in number (2) but add a factor for calendar year.

5.

Learning Objectives:
5.   The candidate will understand methodologies for determining an underwriting profit margin.

Learning Outcomes:
(5a)   Calculate an underwriting profit margin using the target total rate of return model.

(5b)   Calculate an underwriting profit margin using the capital asset pricing model.

(5c)   Calculate an underwriting profit margin using the risk adjusted discount technique.

Sources:
Ratemaking: A Financial Economics Approach, D'Arcy and Dyer

Solution:
(a)   Calculate P.

Commentary on Question:
Candidates generally performed well. Each of the four components of the formula was evaluated separately.

$PV(L) = 35/0.94 + 35/0.94^2 = 76.84$
$PV(E) = 20$
$PV(TUW) = (P - 20)(0.4)/1.01 - 70(0.4)/0.94 = 0.3960P - 37.71$
$PV(TII) = (P + 100 - 20)(0.01)(0.4)/1.01 + (P + 50 - 20 - 35)(0.01)(0.4)/1.01^2 = 0.007882P + 0.2972$

The equation to solve is:
$P = 76.84 + 20 + 0.3960P - 37.71 + 0.007882P + 0.2972$.
The solution is $P = 99.69$.

(b)   Recommend which of the two should be used. Justify your recommendation.

Commentary on Question:
Full credit required three items from those listed below in support of the correct recommendation.

5. **Continued**

Actual equity should be used, because using statutory surplus has the following issues:
- It is usually lower than actual equity and thus will lead to a lower profit margin, which may lead to investment in risky assets.
- It ignores the time value of money.
- It excludes tangible assets and some reinsurance.
- It does not value bonds and real estate at market value.

(c)   Explain the adjustment to the Capital Asset Pricing Model (CAPM) needed to reflect increased risk.

The absolute value of beta should be increased.

6. **Learning Objectives:**
3.   The candidate will understand excess of loss coverages and retrospective rating.

Learning Outcomes:
(3e)   Explain Table M and Table L construction in graphical terms.

Sources:
The Mathematics of Excess of Loss Coverages and Retrospective Rating – A Graphical Approach, Lee

Solution:
(a)   Define the following terms associated with Table M in a retrospective rating plan:

   (i)   The entry ratio, r

   (ii)   The Table M charge, $\phi(r)$

   (iii) The Table M savings, $\psi(r)$

   The entry ratio: The multiple of a risk's expected loss or expected loss ratio
   The Table M charge: The average amount by which a risk's actual loss exceeds r times its expected loss, divided by its expected loss
   The Table M savings: The average amount by which a risk's actual loss falls short of r times its expected loss, divided by its expected loss

(b)   Complete the following Table M for this risk:

| r	$\phi(r)$	$\psi(r)$
0.00	1.00	0.00
0.20	0.80	0.00
0.40	0.60	0.00
0.60	0.40	0.00
0.80	0.24	0.04
1.00	0.12	0.12
1.20	0.04	0.24
1.40	0.00	0.40
1.60	0.00	0.60
1.80	0.00	0.80
2.00	0.00	1.00

6. **Continued**

Commentary on Question:
The table is completed above, with further explanation below.

One approach is to calculate the values directly from the definition. The average loss ratio is $(30 + 40 + 50 + 60 + 70)/5 = 50$. For $r = 0.80$ (for example), the charge is the expected excess over $0.80(50) = 40$, which is $(0 + 0 + 10 + 20 + 30)/5 = 12$, divided by the average of 50 to obtain 0.24. Similarly, the savings is $(10 + 0 + 0 + 0 + 0)/5 = 2$, divided by 50 to obtain 0.04.

A second approach is to construct a graph. The one below illustrates the calculation for $r = 0.80$. The charge is the area shaded with horizontal stripes, which goes from the line at $r = 0.80$ to the cumulative distribution function. The savings is the area shaded with vertical stripes, which goes from the cumulative distribution function to the line at $r = 0.80$.
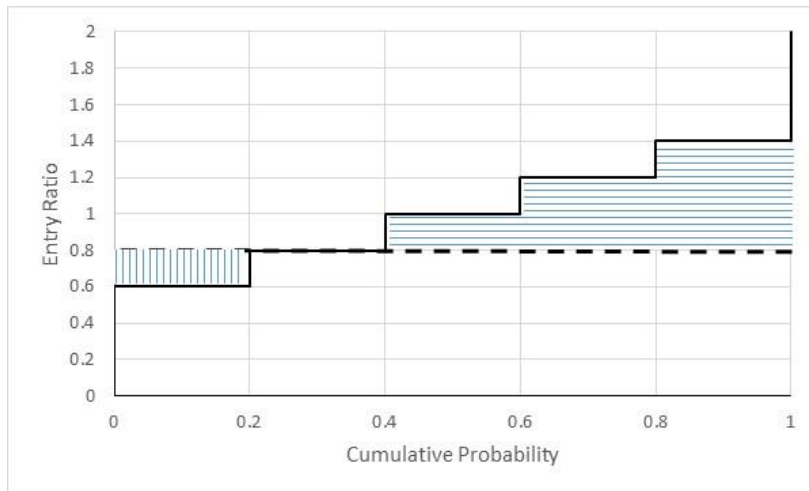
(c)   Explain how Table L differs from Table M.

Table L incorporates a per accident limit on losses.

7. **Learning Objectives:**

2. The candidate will understand the considerations in selecting a risk margin for unpaid claims.

Learning Outcomes:

(2a) Describe a risk margin analysis framework.

(2b) Identify the sources of uncertainty underlying an estimate of unpaid claims.

(2c) Describe methods to assess this uncertainty.

Sources:

A Framework for Assessing Risk Margins, Marshall, et al.

Solution:

(a) Calculate the combined coefficient of variation for all sources of uncertainty.

The combined coefficient of variation is $(0.06^2 + 0.08^2 + 0.16^2)^{0.5} = 0.1887$.

(b) Calculate the amount of the risk margin.

The risk margin is $0.1887(0.674)(216,000,000) = 27,468,734$.

(c) Describe two areas of additional analysis that you may conduct to provide further comfort regarding the outcomes from the deployment of this framework.

Commentary on Question:

Any two areas of additional analysis from the following five are sufficient to earn full credit.

- Sensitivity testing: Vary each assumption to see the effect on risk margins.
- Scenario testing: Tie outcomes to a set of valuation outcomes.
- Internal benchmarking: For each source of uncertainty compare the coefficients of variation between classes for various liabilities.
- External benchmarking: Review differences between benchmarks and the claim portfolio being analyzed.
- Hindsight analysis: Compare past estimates against the latest view of the equivalent liabilities.

7. **Continued**

(d)   Identify four approaches that can be used to analyze independent sources of risk.

  Commentary on Question:
  Any four of the following are sufficient to earn full credit. Providing the name (without a description) is sufficient.

  - Mack method
  - Bootstrapping
  - Stochastic chain ladder
  - Generalized linear models
  - Bayesian

(e)   Evaluate your student's proposal.

  Commentary on Question:
  Three of the following four statements are sufficient to earn full credit.

  - May not be possible to work at the granular level
  - Information may not be credible
  - May be costly to work at the granular level
  - Extra detail may not provide material improvement

8. **Learning Objectives:**

4.   The candidate will understand how to apply the fundamental techniques of reinsurance pricing.

Learning Outcomes:

(4a)   Calculate the price for a proportional treaty.

(4d)   Apply an aggregate distribution model to a reinsurance pricing scenario.

Sources:

Basics or Reinsurance Pricing, Clark

Solution:

(a)   Provide an example of each of the following adjustable features:

        (i)   Sliding scale commission

        (ii)   Profit commission

        (iii)   Loss corridor

Commentary on Question:

The solution provides examples as an illustration of what a full credit answer might look like. Any example that is consistent with the definition was acceptable.

- Sliding scale: 25% above a 65% loss ratio, slides 1:1 to be 35% at a 55% loss ratio, then slides 0.5:1 to be 45% at a 35% loss ratio, at which point it remains constant. At a 40% loss ratio, the commission is $35 + (15/20)(45 - 35) = 42.5\%$.
- Profit: Begin with 100%, then subtract the actual loss ratio (for example, 55%), then subtract the base commission (for example, 25%) and a margin (for example, 10%) to obtain a profit measure (10% in this example). The profit commission is a fixed percentage, for example 50%. In this case the profit commission is 5%.
- Loss corridor: The ceding company reassumes a portion of the liability if the loss ratio exceeds a certain amount. For example, the corridor may be 75% of the layer from 80% to 90%. If the actual loss ratio is 110%, the calculation is $100\%(80\%) + 25\%(90\% - 80\%) + 100\%(110\% - 90\%) = 102.5\%$.

8. Continued

(b)   Identify one disadvantage specific to each of the following approaches:

    (i)   Empirical distribution

    (ii)   Single distribution model

    (iii)   Recursive formula

Commentary on Question:
For each, only one disadvantage is required.

- Empirical distribution
  - Does not take into account all possible outcomes
  - If volume or mix changes, will not be able to reflect future volatility
  - If loss developed used Bornhuetter Ferguson or Cape Cod, then historical periods may provide an artificially smooth sequence of loss ratios, which will understate future volatility
- Single distribution model
  - There is no provision for a positive probability of zero claims
  - Difficult to reflect changes in per-occurrence limits
- Recursive formula
  - Calculation is difficult when the expected frequency is high
  - Only one severity distribution can be used

(c)   Describe one approach to pricing the impact of a carryforward provision and identify one problem with that approach.

Commentary on Question:
There are two possible approaches. Either is acceptable, but to receive full credit the identified problem must align with the stated approach. In the second case only one of the problems is needed.

- Include the carryforward from past years and estimate its effect on the current year only. This approach ignores the potential effect on later years.
- Look at the long run of the contract and extend the modification, incorporating a reduction in the variance. For this method there is no obvious way to reduce the variance and it is possible that the contract will not renew.