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**SOCIETY OF ACTUARIES**  
**Quantitative Finance and Investment Advanced Exam**

# Exam QFIADV

## AFTERNOON SESSION

**Date:** Thursday, May 5, 2016

**Time:** 1:30 p.m. – 3:45 p.m.

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### INSTRUCTIONS TO CANDIDATES

#### General Instructions

1. This afternoon session consists of 7 questions numbered 10 through 16 for a total of 40 points. The points for each question are indicated at the beginning of the question.
2. Failure to stop writing after time is called will result in the disqualification of your answers or further disciplinary action.
3. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions on the exam booklet.

#### Written-Answer Instructions

1. Write your candidate number at the top of each sheet. Your name must not appear.
2. Write on only one side of a sheet. Start each question on a fresh sheet. On each sheet, write the number of the question that you are answering. Do not answer more than one question on a single sheet.
3. The answer should be confined to the question as set.
4. When you are asked to calculate, show all your work including any applicable formulas. When you are asked to recommend, provide proper justification supporting your recommendation.
5. When you finish, insert all your written-answer sheets into the Essay Answer Envelope. Be sure to hand in all your answer sheets because they cannot be accepted later. Seal the envelope and write your candidate number in the space provided on the outside of the envelope. Check the appropriate box to indicate morning or afternoon session for Exam QFIADV.
6. Be sure your written-answer envelope is signed because if it is not, your examination will not be graded.

Tournez le cahier d'examen pour la version française.





**\*\*BEGINNING OF EXAMINATION\*\***

**Afternoon Session**  
***Beginning with Question 10***

- 10.** (8 points) Adam, an awesome marble player at his elementary school, developed a special glove that greatly improves performance. Other players at his school, and other schools around, have noticed the glove and are willing to buy gloves from him if he can make some for them. Adam figures that he can market his idea and thus finance his entry into regional marbles tournaments this way.

The cost for him to build the gloves requires 1,250 marbles. Adam has 50 marbles.

Fortunately, Donald and Eric are willing to provide some marbles, but would want to be involved in his business and have a share of the revenue.

The following has been agreed to.

Donald will provide 900 marbles and will obtain 50% of the revenue. He will also be involved in the marketing and decide the price for the gloves. Eric will provide 300 marbles and will obtain 25% of the revenue in exchange.

- (a) (1 point) Identify the financial transaction taking place, the participants, and their respective roles.
- (b) (2 points) Describe the main investment characteristics of this transaction.
- (c) (1 point) Critique the fairness of the revenue split between Donald and Eric.
- (d) (1 point) Develop a range of market values for Adam's share of the business based on the information provided.
- (e) (3 points) Evaluate the prospect of market success for this transaction from each participant's perspective.

11. (5 points) You analyze results from a performance attribution model for a fixed income portfolio manager.

You are given the following results:

Performance Attribution Results (in %):

	Asset Allocation	Security Selection	Total
Bond A	0.3		
Bond B	1.1	1	3
Total	1	1	2

- (a) (2 points) List and define the important principles of a successful performance attribution algorithm.
- (b) (1 point) Evaluate whether your performance attribution model can be a successful model.

You are given the following information from a different performance attribution:

		Yield		
		Change (bps)	Duration x weight (years)	
			Portfolio	Benchmark
<b>Parallel Shift</b>				
Average	-20			
<b>Key Rates</b>				
2Y	-15	0.1	0.2	
5Y	-20	0.4	0.5	
10Y	-40	0.7	0.8	
20Y	-60	2.7	1.2	
30Y	-45	1.2	1.3	

- (c) (2 points) Calculate the portfolio outperformance due to reshaping of the yield curve.

**12.** (6 points) You are proposing to use a two-factor interest rate model to replace the one-factor model currently used to price long-term financial guarantees.

- (a) (1 point) Describe the motivation of using a two-factor interest rate model vs. one-factor model in interest rate modeling.

You are considering a G2++ model as following:

$$\begin{aligned} r(t) &= x(t) + y(t) + \phi(t) \\ dx(t) &= -ax(t)dt + \sigma dW_1(t) \\ dy(t) &= -by(t)dt + \eta dW_2(t) \\ dW_1(t)dW_2(t) &= \rho dt \end{aligned}$$

$$P(t, T) = \exp \left\{ -\int_t^T \varphi(u) du - \frac{(1 - e^{-a(T-t)})}{a} x(t) - (1 - e^{-b(T-t)}) \frac{y(t)}{b} + V(t, T) / 2 \right\}$$

$P(t, T)$  is the price of the bond, with  $W_1(t)$  and  $W_2(t)$  are two Brownian motions with instantaneous correlation  $\rho$ ,  $a$ ,  $b$  are constants,  $V(t, T)$  is the variance of  $I(t, T)$  where

$$I(t, T) = \int_t^T [x(u) + y(u)] du$$

- (b) (3 points) Show that  $f(t, T)$  the instantaneous forward rate for this model is

$$f(t, T) = \varphi(T) + e^{-a(T-t)} x(t) + e^{-b(T-t)} y(t) - \frac{\partial V(t, T)}{\partial T} \left( \frac{1}{2} \right)$$

Your colleague used this model to calibrate to real cap market volatility data and handed over the following three sets of parameters.

Estimated Parameters	I	II	III
a	0.969	0.900	0.63
b	0.240	0.116	0.08
$\sigma$	0.47%	0.91%	1.00%
$\eta$	1.18%	0.90%	1.00%
$\rho$	-0.79	-0.99	0.56

- (c) (2 points) Determine which set of the parameters most likely came from the cap market volatility data and explain why.

**13.** (5 points) You sit on the trading desk of XYZ Life Co. and your managing director has been asked to research trading strategies in the fixed income market. He asked you to answer the following questions in preparation for the upcoming investment committee meeting regarding investments into CDS.

- (a) (1 point) Describe two main motivations relating to trading and structuring basis trades.
- (b) (1 point) Describe how the “cheapest-to-deliver option” on a CDS contract impacts the CDS spread.

You are given the following information for ABC Inc. bonds of different maturities. Your analyst while not giving you the calculation informs you that the Expected recovery rate trends down as the maturity of each bond increases although it can remain the same for some consecutive periods.

<b>Maturity (years)</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>
<b>Bond spread (bps)</b>	225	257	305	340
<b>Default probability</b>	6%	8%	8%	10%
<b>Expected Recovery Rate</b>	70%	70%	60%	60%

You assume a simplified one-step time period approach.

- (c) (2 points) Calculate the minimum and maximum Bond-CDS basis of the entire group of bonds.
- (d) (1 point) Identify the bond that offers the best negative basis trade arbitrage opportunity and describe the strategy.

**14.** (5 points) You are a risk management actuary at XYZ, an insurance and financial services company. The product portfolio at XYZ consists of the following two products:

- Non-participating whole life (sold to individuals):
  - Guaranteed cash value that grows as the policy ages.
  - Surrender charges start at 8% and linearly decrease to 0% over the first 8 years.
  - Policyholders may take out loans against the cash value.
- GIC (sold to institutional investors):
  - No premature surrenders allowed.
  - However, if XYZ's credit rating drops below the investment-grade level, investors can choose to surrender at book value with 7-days' notice.

- (a) (1 point) Explain the potential liquidity exposures embedded in XYZ's liabilities.
- (b) (2 points) Describe ways to manage the liquidity risks embedded in XYZ's liabilities.

XYZ has traditionally estimated its liquidity risk by calculating a single liquidity ratio, on a quarterly basis, using current economic assumptions. The numerator is the sum of market values of assets at the end of a quarter, and the denominator is the sum of scheduled benefit payments of total liabilities for the next three months at the end of the same quarter.

Your CFO has declared that this liquidity ratio is not providing her with sufficient information on XYZ's liquidity profile.

- (c) (2 points) Propose ways to improve this calculation to make it more effective.



**15.** (5 points) You are working in the Corporate division of ABC Life and your manager has asked you to extend a time-homogeneous short-rate model. You have chosen to extend the Cox-Ingersoll-Ross (CIR) model.

- (a) (1 point) List the properties of a deterministic shift extension to the CIR short-rate model.
- (b) (1 point) Describe any fitting quality issues when using the CIR++ model.
- (c) (2 points) Explain how you would use the CIR++ model to price caps and floors.

Your manager proposes to add a jump component to your model as follows:

- (1)  $dr_t = k(\theta - r_t)dt + dW_t + \sigma\sqrt{r_t}dJ_t$
- (2)  $J$  is a pure jump process with jumps arrival rate  $\alpha < 0$ .
- (d) (1 point) Identify and explain any problems in your manager's proposal.

**16.** (6 points) You work in the risk management department at ABC Bank. It is January 1 and the CFO has heard that the Federal Reserve may raise rates soon and is concerned about the impact on ABC. She would like you to perform a scenario analysis.

- (a) (1 point) Identify four considerations when creating and evaluating deterministic scenarios.
- (b) (1 point) Explain the impact of interest rates on liquidity needs.

The balance sheet of the bank as of yesterday, December 31, is as follows:

Asset Class	Current Value	Par Value	Annual Coupon
Corporate Bonds	600	600	4%
Commercial Mortgages	200	200	5%
AAA-Rated MBS	200	200	3%

Liability Type	Current Value
CDs (maturing June 30)	200
Money Market Deposits	400
Long-term Debt	300
Equity	100

The asset portfolio has the following cash flow characteristics for the following year:

- The corporate bonds have semi-annual coupons and maturities are laddered, with 25% of them maturing on June 30. These bonds are all frequently traded.
- The mortgages have semi-annual coupons and 20% of them are scheduled to mature June 30. The mortgages cannot be sold.
- The MBS have semi-annual coupons and 20% are projected to pay off on June 30.
- No interest is expected to be paid on the bank's liabilities within the current year.

The two scenarios that the CFO is interested in are:

- Scenario 1:
  - No change in rates.
  - MBS pay off as projected.
  - No money market withdrawals.
- Scenario 2:
  - Interest rates increase.
  - 10% of money market liabilities are withdrawn.
  - The corporate bonds decline in value by 5%.
  - Only 15% of the MBS pay off as projected.
  - The MBS decline in value by 10%.

**16. Continued**

- (c) (2 points) Calculate the cash flow cushion as of June 30 for each of the two scenarios.

The CFO has stated that she would require a minimum cash flow cushion of 110% under each scenario.

- (d) (2 points) Recommend actions which could be taken to meet the requirement.

**\*\*END OF EXAMINATION\*\***  
**Afternoon Session**

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