
SOCIETY OF ACTUARIES
Quantitative Finance and Investment Advanced Exam

Exam QFIADV

MORNING SESSION

Date: Thursday, May 5, 2016

Time: 8:30 a.m. – 11:45 a.m.

INSTRUCTIONS TO CANDIDATES

General Instructions

1. This examination has a total of 100 points. It consists of a morning session (worth 60 points) and an afternoon session (worth 40 points).
 - a) The morning session consists of 9 questions numbered 1 through 9.
 - b) The afternoon session consists of 7 questions numbered 10 through 16.

The points for each question are indicated at the beginning of the question.
2. Failure to stop writing after time is called will result in the disqualification of your answers or further disciplinary action.
3. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions on the exam booklet.

Written-Answer Instructions

1. Write your candidate number at the top of each sheet. Your name must not appear.
2. Write on only one side of a sheet. Start each question on a fresh sheet. On each sheet, write the number of the question that you are answering. Do not answer more than one question on a single sheet.
3. The answer should be confined to the question as set.
4. When you are asked to calculate, show all your work including any applicable formulas. When you are asked to recommend, provide proper justification supporting your recommendation.
5. When you finish, insert all your written-answer sheets into the Essay Answer Envelope. Be sure to hand in all your answer sheets because they cannot be accepted later. Seal the envelope and write your candidate number in the space provided on the outside of the envelope. Check the appropriate box to indicate morning or afternoon session for Exam QFIADV.
6. Be sure your written-answer envelope is signed because if it is not, your examination will not be graded.

Tournez le cahier d'examen pour la version française.

****BEGINNING OF EXAMINATION***
Morning Session

- 1.** (6 points) ABC Bank, a new bank that has just been chartered, is building a portfolio of loans. ABC Bank has extended a 20,000 credit line to Company D, a technology company that specializes in personal gadgets.

Company D is allowed to draw 8,000 as cash and use the remaining 12,000 of the credit line for so-called contingent liabilities.

So far Company D has drawn 5,000.

Assume the following:

- For the cash portion of their credit line, creditors on average draw on their remaining credit 80% of the time and on average use 2/3 of remaining funds.
 - For the contingent liabilities portion of their credit line, creditors on average draw on their remaining credit 50% of the time and on average use 80% of remaining funds.
 - ABC Bank has determined that it takes 50,000 per 1,000,000 of loan extended to administer the loan. Also, the cost of originating a loan is 15,000 per 1,000,000 of loan extended.
 - The hurdle rate for economic capital is set at 8%.
 - Company D's Probability of Default (PD) is 25%.
 - Company D's Loss Given Default (LGD) is 65%.
- (a) (1 point) Calculate Company D's unexpected loss, treating EAD as if it were deterministic and equal to its expected value.

After conducting an extensive analysis on theoretical loss patterns, ABC Bank has determined that the loss for the loan portfolio should follow a Normal distribution. Also, ABC Bank decided to set the level of confidence for Economic Capital EC at 97.5%. Note this is the first loan, so EC contributory = EC. Loan Charge = admin charge + origination fee + expected loss + EC charge

- (b) (2 points) Calculate the annual percentage fee ABC Bank should charge for the loan to Company D.

1. Continued

After a successful installation of the first loan, ABC Bank has decided to extend a loan to Company E with the same terms as the loan to Company D for 20,000. Company E is a competitor of Company D in the same industry.

You are given the following:

- Company E's PD is 10%.
 - Company E's LGD is 70%.
 - The covariance of default indicators between Company D and Company E is 5%.
- (c) (2 points) Calculate the unexpected loss of the new portfolio again treating EAD as if it were deterministic and equal to its expected value.
- (d) (1 point) Discuss the impact of default correlation on economic capital for loan portfolios.

2. (7 points) As an investment actuary at ABC Life, you are asked to help senior management understand the use of the Target Volatility Fund (TVF) for your company's new Variable Annuity (VA) product with embedded guarantees. You plan to demonstrate TVF's periodic portfolio rebalancing to maintain its target volatility level using the following assumptions and data:

- The expected volatility between today and next rebalancing date is driven by an Exponentially Weighted Moving Average (EWMA) estimator. The weight assigned to the prior estimated volatility, is denoted by λ .
- Target annual volatility is 10%
- The rebalancing frequency in consideration is one of the following:
Daily (every 1 business day)
Monthly (every 22 business days)
Quarterly (every 66 business days)
- As of 3/31/2015, the equity index and estimated volatility are as follows:

Date	Equity Index	Prior Estimated Volatility	Estimated Volatility	Equity Weight
3/31/2015	2015.6			
1 day ago	1975.3	15.20%	15.46%	Y
1 month ago	1564.1	4.50%	W	Z
1 quarter ago	1560.0	11.20%	X	90%

- (a) (1 point) Explain the impact of the assumption λ on the results.
- (b) (2 points) Calculate W and X.

Senior management believes that the most frequent rebalancing (daily) would need the highest proportion of equity weight and suggests allocating 100% of equity for the daily rebalancing strategy.

- (c) (1 point) Calculate the un-leveraged equity weight as of 3/31/2015 that corresponds to each of two rebalancing frequencies: daily and monthly (Y and Z).
- (d) (1 point) Evaluate senior management's suggestion.

2. Continued

Now, senior management is concerned about potential market jumps and asks you to estimate results using a model including jumps. Management believes that it is very important to keep transaction costs down.

Using two new models: one without jumps and one with 'equity jumps', the following volatilities have been projected:

Frequency	without jump	with jump
Daily	10.24%	11.57%
Monthly	10.48%	11.73%
Quarterly	10.60%	11.85%

- (e) (2 points) Justify which model and rebalancing frequency should be used for TVF implementation.

- 3.** (6 points) You are comparing different approaches for fitting a volatility surface.
- (a) (1 point) List the pros and cons of using directly observable market prices versus preprocessed prices for purposes of this fitting.
 - (b) (3 points) Describe four fitting approaches, identifying their merits and any possible drawbacks.

You are implementing the fitting procedure of mixture of normal. All the normal densities in the mixture are ‘centered’ in log space around the forward value and weights are left unconstrained.

- (c) (1 point) Discuss two potential problems of this implementation process.
- (d) (1 point) Recommend how the problems in (c) might be overcome.

4. (6 points) Country A is experiencing a similar crisis as the U.S. did during the 2008 global credit crisis. The following events have occurred over the past couple of years:

- A flu epidemic has occurred, severely damaging the tourism business, the most important industry in Country A;
- In order to stimulate the economy and increase the GDP, the central bank of Country A has cut the interest rates to the lowest level in its history;
- With the low interest rates and easy borrowing, more and more people are buying a second house as an investment; they plan to rent it to tourists;
- Both the mortgage-per-person and total mortgage outstanding balances have soared;
- Most loans are issued as floating rate loans;
- To reduce capital, some large banks have bundled their mortgage loans and sold them to domestic and foreign investors ; In order to increase profit, these banks have encouraged their agents to generate more mortgages, which has triggered low quality mortgages;
- One neighboring country, which has better beaches and weather, surprised the financial markets and announced that it will open its border to tourists.

(a) (2 points) Explain how each of the events above contributed to the Country A Mortgage Market Credit Crisis.

(b) (2 points) Identify and describe four additional events that could push Country A into the second and third phase as defined by Saunders and Allen in “Credit Risk Measurement In and Out of the Financial Crisis”.

Country A created a volatility index (A-VIX) similar to the CBOE’s VIX and a stress index (A-SI) similar to KCFSI.

(c) (1 point) Describe Country A’s indices.

(d) (1 point) Explain how the events above would impact these indices.

5. (7 points) ABC Life purchased a zero-coupon bond that was privately issued by DeF, a publicly traded stock company. On a given valuation date, the Chief Investment Officer of ABC Life asked three of his analysts to independently estimate the fair market value of this zero-coupon bond. Below are the data and formula the three analysts used to estimate the fair market value of the zero-coupon bond on the same valuation date.

Data and formula that is common to all analysts:

This zero-coupon bond is the only liability of DeF. The maturity value of this zero-coupon bond is 100 million and the remaining time-to-maturity is 12 years.

- DeF has 1 million outstanding shares and is operating business as usual.
- DeF's current stock price is 22 per share.
- The continuously compounded risk-free interest rate is 2.0% for all maturities.

Assumptions and the estimates that vary by analysts:

Assume DeF's Asset Volatility	Estimated Market Value of the Zero-Coupon Bond
Analyst One: 7.1%	82 million
Analyst Two: 5.4%	78 million
Analyst Three: 8.6%	75 million

Taking DeF's total asset value to be the sum of the market value of its equity and the analysts' estimated market value of the zero-coupon bond, and using the analysts' assumed asset volatilities:

- (a) (2 points) Calculate, for each analyst, the implied estimate of DeF's stock price volatility using the Merton Asset Value Model.
- (b) (2 points) Calculate, for each analyst, the implied estimate of DeF's equity value using the Merton Asset Value Model.

You believe that DeF's stock price volatility will be 30%.

- (c) (1 point) Determine which analyst provided estimates that are most consistent with your view using the Merton Asset Value Model, based on your calculations in (a) and (b).

5. Continued

Your manager wants to use the following simplified equation for bond value.

$$\begin{aligned} \text{Market Value estimate of Zero-Coupon Bond} \\ = \exp(-rT)(PD * (1 - LGD) * EAD + (1 - PD) * EAD) \end{aligned}$$

- (d) (1 point) Calculate the PD under the Merton model for Analyst Two.
- (e) (1 point) Calculate the implied LGD (as percent of EAD) for Analyst Two's estimated market value of the zero-coupon bond, using the model proposed by your manager, and the PD from part (d).

- 6.** (7 points) You are the Chief Risk Officer of ABC Life. Recently, you sat down with your derivative traders to discuss volatility smiles and more specifically, pricing options in the presence of smiles.

One of your derivative traders purchased an option that pays \$1 if the underlying asset price ends above the strike of \$100 at expiry, and zero otherwise. In a Black-and-Scholes world with risk free rate equal to zero and in the absence of market frictions, the cost of this option is $N(d_2)$.

- (a) (1 point) Describe how the existence of volatility smiles in the real world changes the cost of this option.
- (b) (3 points) Explain how the delta of an asset in the real world can be estimated using the Black formula and market implied volatility from Black-Scholes.

Your discussion shifts to floating and sticky volatility smiles.

- (c) (1 point) Compare a floating volatility smile to a sticky volatility smile.

One of the traders is working on a strategy that buys and sells plain-vanilla options on forward rates spanning different portions of the same yield curve.

- (d) (2 points) Explain how this strategy would be impacted by a sticky volatility smile environment.

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7. (8 points) As the CFO of XYZ Annuity, you have been approached and asked to consider if you would like to invest 100 Million into a private equity fund. This fund invests in residential real estate by purchasing homes and renting them out. It is a unique private equity fund initiated to take advantage of the recent decline in home values. The current allocation to any real estate by XYZ Annuity is 2% and consists entirely of their office building which has a market value of 50 Million.

The investment committee of XYZ Annuity has agreed to increase the real estate exposure by investing 125 Million into real estate. This will bring the total investment in Non Fixed Income assets to 275 Million, as they already have 100 Million invested in Farmland.

XYZ Annuity was traditionally invested heavily in REITs although currently are not. You argue that an investment in that private equity fund is not different than investing in REITs. Zack, from the investment committee, disagrees. He claims that it is very different and thus doesn't have the same risk and opportunity.

- (a) (3 points) Compare and contrast an investment into the new fund above with an investment into a diversified REIT portfolio.
- (b) (2 points) Discuss diversification issues in the allocation of the 125 Million to be invested in private equity real estate for XYZ Annuity considering their existing portfolio

In the following year the total real estate holdings (with a beginning of year market value of 175 Million) earned an effective rate of 8% which is exactly in line with XYZ Annuity's expectation.

XYZ Annuity's Farmland holdings consist of the following with a total start of year market value of 100 Million:

Farm	% of XYZ Holdings	Prior Year Yield
Sunny Acres	25%	175
Green Valley	25%	265
Sandy Dunes	25%	325
Prairie Fields	25%	125

The holding percentage and the yield are not expected to change from year to year. Each unit of Yield translates to 10,000 of income.

7. Continued

The subsequent year experienced weather significantly different from any other year. This impacted the above Farms as follows:

- Sunny Acres shifted its production from corn with a photosynthetic efficiency of 1.5% to Sugar cane with a photosynthetic efficiency of 8% for the second half of the year (this did not impact the harvest of corn for the first half of the year).
 - Sandy Dunes suffered a severe drought which had its harvest index drop by 80%.
 - Both Prairie Fields and Green Valley observed an increase in total solar radiation by 10%.
- (c) (2 points) Calculate the actual income from Farmland that took place in the subsequent year.
- (d) (1 point) Calculate the difference between the expected return on XYZ Annuity Non Fixed Income Portfolio vs. the Actual Return in percentage terms.

8. (5 points) The Chief Investment Officer (CIO) of ABC Life is concerned about the credit risk and risk-return trade-off of ABC Life's bond portfolios and has asked you to apply Altman's optimization approach to help improve returns.

The first portfolio to be reviewed includes two bonds with the following characteristics:

The correlation between the returns of Bond A and Bond B is 3%.

	Bond A	Bond B
Book Value	\$1,500,000	\$2,500,000
Original Rating	A	BB
Maturity(year)	10	10
Expected Annualized Loss	0.03%	1.15%
Yield to Maturity	6.50%	8.25%
Standard Deviation of Return	4%	8%

- (a) (2 points) Calculate the annual expected rate of return and variance of loss for the portfolio.

The second portfolio to consider is a 10-bond portfolio with equally weighted bonds:

Asset ID	Asset Class	Country
Bond 1	Government	United States
Bond 2	Government	Canada
Bond 3	Government	Germany
Bond 4	Corporate	United States
Bond 5	Corporate	United States
Bond 6	Corporate	United States
Bond 7	Corporate	United States
Bond 8	Corporate	Canada
Bond 9	Corporate	Canada
Bond 10	Corporate	Germany

8. Continued

The portfolio's objective is to attain at least a 6% expected annual return. However, the corporate investment policy prohibits concentration of risk and has the following restrictions:

- No short selling.
 - No bond may make up more than 30% of the portfolio.
 - At least 50% of the portfolio must be invested in the United States.
 - At least 10% of the portfolio must be invested in Government securities.
 - The portfolio must have at least 5% of the portfolio from every country.
- (b) (2 points) Determine and explain the objective function and all constraints that would be used to find an optimum weighting for the portfolio provided above using Altman's optimization approach.
- (c) (1 point) Describe the reasons why Altman introduced the Z-Score model to determine unexpected losses.

9. (8 points) You are given the following information based on the daily log-returns of 5 stocks:

Covariance Matrix					
Stock	1	2	3	4	5
1	0.0282%	0.0089%	0.0049%	0.0127%	0.0037%
2	0.0089%	0.0204%	0.0060%	0.0182%	0.0052%
3	0.0049%	0.0060%	0.0084%	0.0079%	0.0034%
4	0.0127%	0.0182%	0.0079%	0.0372%	0.0070%
5	0.0037%	0.0052%	0.0034%	0.0070%	0.0088%

Principal Component					
Stock \ PC	PC1	PC2	PC3	PC4	PC5
1	-0.4480	0.8894	0.0749	-0.0307	0.0418
2	-0.4663	-0.2036	-0.6526	-0.5529	0.0970
3	-0.2122	-0.0271	-0.3151	0.4160	-0.8258
4	-0.7090	-0.4035	0.5738	0.0715	0.0125
5	-0.1848	-0.0629	-0.3741	0.7178	0.5539

Proportion of Total Variance Explained					
	59.51%	19.43%	9.03%	7.09%	4.93%

You are also given that:

- The sum of the eigenvalues for the covariance matrix is 0.00103; none of the stocks exhibit significant serial correlation.

Your manager makes the following statement: “If we use Principal Component Analysis, we need to be certain that we are covering at least 99% of the variation.”

- (1 point) Critique your manager’s statement.
- (1 point) Determine the eigenvalues of the covariance matrix.
- (2 points) Describe the process to calculate each principal component.
- (3 points) Propose a statistical factor model using two orthogonal factors from the Principal Component Analysis.
- (1 point) Estimate an upper bound for the sum of the squared elements of the error matrix for the model in (d).

****END OF EXAMINATION****

Morning Session

USE THIS PAGE FOR YOUR SCRATCH WORK

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