
SOCIETY OF ACTUARIES
Foundations of CFE Exam

Exam CFEFD

AFTERNOON SESSION

Date: Wednesday, November 2, 2016

Time: 1:30 p.m. – 3:45 p.m.

INSTRUCTIONS TO CANDIDATES

General Instructions

1. This afternoon session consists of 5 questions numbered 9 through 13 for a total of 40 points. The points for each question are indicated at the beginning of the question. No questions pertain to the Case Study.
2. Failure to stop writing after time is called will result in the disqualification of your answers or further disciplinary action.
3. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions on the exam booklet.

Written-Answer Instructions

1. Write your candidate number at the top of each sheet. Your name must not appear.
2. Write on only one side of a sheet. Start each question on a fresh sheet. On each sheet, write the number of the question that you are answering. Do not answer more than one question on a single sheet.
3. The answer should be confined to the question as set.
4. When you are asked to calculate, show all your work including any applicable formulas.
5. When you finish, insert all your written-answer sheets into the Essay Answer Envelope. Be sure to hand in all your answer sheets because they cannot be accepted later. Seal the envelope and write your candidate number in the space provided on the outside of the envelope. Check the appropriate box to indicate morning or afternoon session for Exam CFEFD.
6. Be sure your written-answer envelope is signed because if it is not, your examination will not be graded.

Tournez le cahier d'examen pour la version française.

****BEGINNING OF EXAMINATION****

Afternoon Session
Beginning with Question 9

9. (8 points) A company has hired you to price options and provide advice on modelling stock prices and interest rates.

(a) (2 points)

- (i) Identify one advantage and one disadvantage of using the Constant Elasticity of Variance (CEV) model rather than the Black-Scholes model.
- (ii) List the steps required to price a European put option using the CEV model and Monte Carlo simulation.

Your intern says, “If we want to double the accuracy of the Monte Carlo estimator, can’t we just double the number of runs?”

(b) (1 point) Critique your intern’s statement.

(c) (2 points) Describe three variance reduction methods that can be used to improve the speed of convergence of a simulation.

(d) (1 point) Identify two reasons why the CEV and Black-Scholes models are not suitable for modeling interest rates.

(e) (2 points)

- (i) Describe three approaches to interest rate modeling.
- (ii) Provide an example for each interest rate modeling approach.

- 10.** (10 points) A financial firm is setting up its economic capital (EC) framework using Least Squares Monte Carlo (LSMC). There are only two risk factors (RF): S&P 500 index (RF_1) and risk free rate (RF_2). The CRO has laid out the following steps:

- Step 1: Project many scenarios for one year.
 Step 2: Fill the fitting space (range of scenarios at year 1) with fitting points.
 Step 3: For each fitting point, value the liability by performing two simulations.
 Step 4: Calibrate the coefficients (a_0 to a_3) of a polynomial proxy function:

$$Y = a_0 \cdot RF_1 + a_1 \cdot RF_2 + a_2 \cdot RF_1 \cdot RF_2 + a_3$$

- Step 5: Develop a distribution of liabilities at year 1 to calculate the EC.

- (a) (1 point) Explain whether to use real world or risk neutral scenarios for each of Step 1 and Step 3 of the EC framework.

S&P 500 data is collected to model RF_1 . However, the CRO wants to build a Brownian Bridge from time 0 to T .

Time	S&P 500
0	2,000
$T = 1$	2,050

Assume

- Stock price process for S&P 500 from time 0 to T follows a Brownian motion with volatility, $\sigma = 40\%$.
- z_1 , z_2 , and z_3 are random numbers generated from the standard normal distribution:

z_1	z_2	z_3
0.24	-0.3	0.5

The Brownian Bridge from a to b is given by:

$$B_t^{a,b} = a \frac{T-t}{T} + b \frac{t}{T} + \left(W_t - \frac{t}{T} W_T \right), \quad t \in [0, T]$$

Where W_t is a one-dimensional Brownian motion.

- (b) (2 points) Calculate the S&P 500 index for $T = 1/4$, $1/2$ and $3/4$ using the Brownian Bridge. Show your work.

10. Continued

The Black-Karasinski (B-K) model is used to model the term structure of the risk free rate:

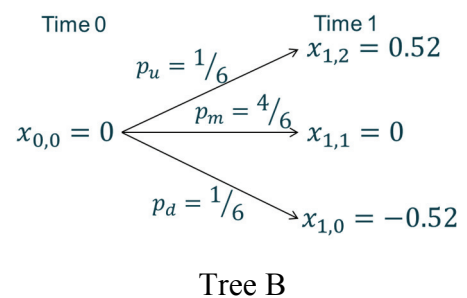
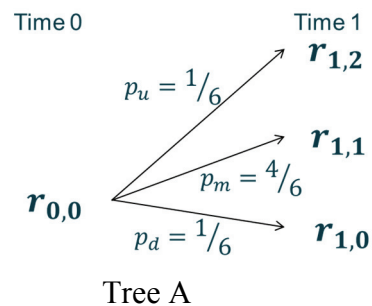
- The B-K model follows the stochastic differential equation (SDE):

$$d \ln(r) = [\theta(t) - 0.22 \ln(r)] dt + 0.3 dW_t$$

- The discrete trinomial tree corresponding to the B-K model is shown in Tree A.
- Tree B below corresponds to the following SDE:

$$dx = -0.22xdt + 0.3dW_t$$

Note that $x = \ln(r) - g(t)$, where $g(t)$ satisfies $dg = [\theta(t) - 0.22g(t)] dt$



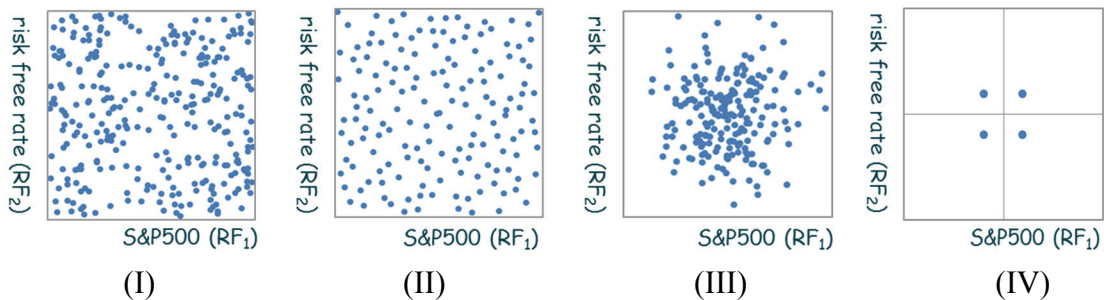
The market has two risk-free zero coupon bonds, which mature at times 1 and 2, and have prices \$0.96 and \$0.80 respectively. Both bonds pay \$1 at maturity.

- (c) (3 points) Calculate $r_{0,0}$, $r_{1,0}$, $r_{1,1}$ and $r_{1,2}$ in Tree A derived from the B-K model, by matching the two observed bond prices. Use $x = 0.84$ as the solution to the equation $x^{1.682} + 4x + x^{0.595} = 5$. Show your work.

Question 10 continued on next page.

10. Continued

The following graphs show the entire fitting space with fitting points at year 1. Points in graph (IV) are intersections of 2nd order Legendre polynomial's roots for each risk factor.



(d) (3 points)

- (i) Evaluate the appropriateness of each graph as the fitting space for Step 2 of the EC framework.
- (ii) Recommend the most appropriate fitting points for fitting the spaces in Step 2 of the EC framework.

You calibrated the coefficients (a_0 to a_3) of the proxy function in Step 4 of the EC framework. To validate its fit, you created an error surface of out-of-sample testing points. The CEO is unhappy with the fit in extreme scenarios.

- (e) (1 point) Recommend a change to the calibration method to improve the fit of the proxy model. Justify your recommendation.

11. (8 points)

- (a) (1 point) List the three reasons why information has value to a business.

The CFO of Bob's Birthdays, a multi-national birthday party supplies retailer, has asked you to review the work an external consultant produced. Information from the consultant's proposal is below:

- Bob's Birthdays needs to spend \$16.5 million on new technologies, in order to revive the balloon division that has been producing losses.
- To purchase the technology the company must issue bonds. Each bond has a face value of \$500 and can be sold at par. The administrative cost of issuing bonds is \$150,000.
- The maximum number of bonds sold is 1.5 times the minimum number sold, with 90% certainty.
- The Expected Value of Perfect Information (EVPI) is \$125,000.
- The Expected Opportunity Loss Factor (EOLF) is 23.
- The Opportunity Loss is defined as the shortfall in bond issuance.

- (b) (4 points) Recommend whether or not to proceed with the consultant's proposal. Show your work.

You recommend using stress testing to determine if it is worthwhile to purchase the new technology.

- (c) (3 points)

- (i) Identify four relevant considerations in developing a stress test for this situation based solely on the information provided above.
- (ii) Design an appropriate stress test.
- (iii) Justify the type of stress test and the rationale of the design in (ii).

12. (7 points) Daisy Life, a small insurance company, has a mix of life insurance, annuities, and P&C business on its books. It is considering switching from the percentile approach to the Market Cost of Capital (MCoC) approach for calculating risk margins.

(a) (1 point) Describe the Market Cost of Capital approach.

The Chief Information Officer (CIO) is concerned that Daisy will not have the computing resources to be able to calculate the Market Value Margins (MVM) required by the MCoC approach.

(b) (2 points) Outline a response to address the CIO's concerns.

Daisy has begun the process to move to the MCoC approach. The Chief Actuary is concerned about how to model the non-hedgeable risks.

(c) (2 points)

- (i) Explain the three-step process Daisy would use to model the capital required for non-hedgeable risks.
- (ii) Explain the circularity problem when estimating non-hedgeable parameter risk.
- (iii) Recommend a method Daisy can use to address the circularity problem.

12. Continued

Daisy uses the Prospective Method to demonstrate how a mortality risk margin could be calculated for a simple 5-year term life product.

Year	Base q_x	Shocked q_x^{\wedge}	V0	V1	Margin	Capital
0			83.00	91.25	(i)	(ii)
1	0.00157	0.00173	70.70	77.74	0.98	
2	0.00172	0.00189	56.47	62.07	0.63	
3	0.00187	0.00205	40.13	44.14	0.34	
4	0.00204	0.00224	21.37	23.56	0.12	
5	0.00222	0.00245	0.00	0.00	0.00	

Face value is 10,000
 Interest rate is 4%.
 α is 0.5
 Return on capital is 6%

- (d) (2 points)
- (i) Calculate the mortality risk margin at year 0.
 - (ii) Calculate the capital for the mortality risk margin at year 0.

13. (7 points)

- (a) (1 point) Discuss characteristics of sample data that warrant the use of the t-distribution versus the normal distribution in calculating confidence intervals.

The Marygold Company wants to know the 90% confidence interval for the average number of sick days that its employees are taking per year.

Assume the number of sick days per employee are normally distributed. Marygold has randomly sampled sick days for 8 employees.

Randomly Sampled Employees	# of Sick Days Taken
Employee 1	2
Employee 2	9
Employee 3	7
Employee 4	8
Employee 5	6
Employee 6	7
Employee 7	3
Employee 8	10

Degrees of Freedom	One-sided t-score for 0.90	One-sided t-score for 0.95
1	3.08	6.31
2	1.89	2.92
3	1.64	2.35
4	1.53	2.13
5	1.48	2.02
7	1.42	1.89
8	1.40	1.80
15	1.34	1.75
27	1.31	1.70
Normal z-Score	1.28	1.64

- (b) (2 points) Calculate the 90% confidence interval.

13. Continued

Mathless 90% Confidence Interval for the Median of Population		
Lower bound: <u> </u> th smallest		
Upper bound: <u> </u> th largest		
Sample Size (n)	Sample Value	Confidence
5	1 st	93.75%
8	2 nd	93.0%
11	3 rd	93.5%

- (c) (1 point) Approximate the 90% confidence interval for sick days using the Mathless approach and the sample data above.
- (d) (2 points)
- (i) Describe differences between using the t-distribution and the Mathless approach for determining confidence intervals.
 - (ii) Describe a common drawback shared by using the t-distribution, normal distribution, and the Mathless approach for determining confidence intervals.

After the success in estimating sick days, Marygold wants to make a quick estimate of the average health claim costs of its employees.

- (e) (1 point) Recommend which method works best among normal distribution, t-distribution and the Mathless approach. Support your recommendation.

****END OF EXAMINATION****
Afternoon Session

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