
SOCIETY OF ACTUARIES
Quantitative Finance and Investment Advanced Exam

Exam QFIADV

MORNING SESSION

Date: Thursday, November 3, 2016

Time: 8:30 a.m. – 11:45 a.m.

INSTRUCTIONS TO CANDIDATES

General Instructions

1. This examination has a total of 100 points. It consists of a morning session (worth 60 points) and an afternoon session (worth 40 points).
 - a) The morning session consists of 9 questions numbered 1 through 9.
 - b) The afternoon session consists of 7 questions numbered 10 through 16.

The points for each question are indicated at the beginning of the question.

2. Failure to stop writing after time is called will result in the disqualification of your answers or further disciplinary action.
3. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions on the exam booklet.

Written-Answer Instructions

1. Write your candidate number at the top of each sheet. Your name must not appear.
2. Write on only one side of a sheet. Start each question on a fresh sheet. On each sheet, write the number of the question that you are answering. Do not answer more than one question on a single sheet.
3. The answer should be confined to the question as set.
4. When you are asked to calculate, show all your work including any applicable formulas. When you are asked to recommend, provide proper justification supporting your recommendation.
5. When you finish, insert all your written-answer sheets into the Essay Answer Envelope. Be sure to hand in all your answer sheets because they cannot be accepted later. Seal the envelope and write your candidate number in the space provided on the outside of the envelope. Check the appropriate box to indicate morning or afternoon session for Exam QFIADV.
6. Be sure your written-answer envelope is signed because if it is not, your examination will not be graded.

Tournez le cahier d'examen pour la version française.

****BEGINNING OF EXAMINATION****
Morning Session

1. (6 points) You are given a lognormal forward LIBOR model (LFM) with the following notation:

- $P(t, T)$ is the price at time t of a zero coupon bond with maturity $T, t \leq T$.
- $F(t; T_1, T_2)$ is the forward rate at time t for the expiry T_1 and the maturity $T_2, t \leq T_1 \leq T_2$.
- $L(T_1, T_2) = F(T_1; T_1, T_2)$ is the LIBOR rate at time T_1 for the maturity $T_2, T_1 \leq T_2$.
- We denote $F(t; T_{k-1}, T_k)$ by $F_k(t)$.

(a) (1 point) Explain one advantage of the LFM model over a short rate model.

One of the first steps in the derivation of Black's formula for the price of caps is to take the risk-neutral expectation of the discounted cap payoff. The next step (shown below) is to assume that the short rate is deterministic so that it can be taken out of the expectation and expressed in terms of the bond price:

$$E \left[\exp \left(- \int_0^{T_2} r_s ds \right) \tau (L(T_1, T_2) - X)^+ \right] = P(0, T_2) \tau E^2 \left[(L(T_1, T_2) - X)^+ \right]$$

(b) (2 points) Explain how the LFM model justifies this step without making a simplifying assumption.

Consider now two consecutive maturity dates T_i and T_j , which are not necessarily in that order, and two positive processes $\sigma_i(t)$ and $\sigma_j(t)$. Under the Q^{T_i} -forward measure, for $t < \max(T_i, T_j)$, we have

$$\begin{aligned} dF_i(t) &= \sigma_i(t) F_i(t) dZ(t), \\ dF_j(t) &= -\mu_j(t) F_j(t) dt + \sigma_j(t) F_j(t) dZ(t), \end{aligned}$$

where $Z(t)$ is a Brownian motion under Q^{T_i} , and where $\mu_j(t) > 0$ for all t .

(c) (2 points) Express $Z^j(t)$, the Q^{T_j} -forward measure Brownian motion, in terms of $Z(t), \mu_j(t)$ and $\sigma_j(t)$.

1. Continued

- (d) (1 point) Explain why, from time t , it is easier to simulate values of $F_j(T_{j-1})$ under the Q^T -forward measure than under the Q^t -forward measure.

2. (6 points) Your investment analyst is studying the historical skew of the FTSE 100 implied volatility and has performed a Principal Component Analysis (PCA).

The PCA was performed on daily changes in skew deviations $\Delta(\sigma_K - \sigma_{ATM})$, or deviations of implied volatility at strike K from at-the-money (ATM) implied volatility.

- (a) (1.5 points) Describe the empirical and theoretical advantages of using PCA on daily changes in skew deviations for modeling implied volatility smiles and skew.

You are given the following results from the PCA of skew deviation $\Delta(\sigma_K - \sigma_{ATM})$, for 1-month implied volatility on the FTSE 100, at day t :

- Gamma response coefficients

$$\gamma_{1,t} = 0.0001$$

$$\gamma_{2,t} = 0.00007$$

$$\gamma_{3,t} = -0.0001$$

- Eigenvectors of Correlation Matrix (Table 1)

FTSE Level	P_1	P_2	P_3
4825	0.866	0.017	-0.433
4925	0.810	-0.017	-0.288
5025	0.941	-0.185	0.068
5125	0.926	-0.228	0.130
5225	0.927	-0.211	0.121

- Regression Coefficient of daily change in ATM volatility against daily change in the FTSE index:

$$\beta_t = -0.00019$$

- Other data:

$$S(t) = 5025$$

$$\sigma_{4825}(t) = 0.35$$

On the following day, $t+1$, the FTSE closes at 5125. You assume that there is no serial correlation in the daily changes in the index or in the daily changes of implied volatilities.

- (b) (2.5 points) Estimate the implied volatility at $t+1$ for an option with strike 4825.

2. Continued

Assume that your firm has access to historical data for the preceding 5 years for strikes listed in Table 1, while it has historical data for strike 5000 for only 1 year.

- (c) (2 points) Describe how historical implied volatility data for the FTSE strikes provided in Table 1 can be used by PCA to create missing historical data for implied volatility of the 5000-strike option.

3. (7 points)

(a) (1 point) Define the following as it relates to a Credit Default Swap (CDS):

- I. Upfront payment
- II. Par spread
- III. Flat quoted spread

Your investment manager is interested in CDS-Bond basis trades and has provided you with the following information about a CDS:

Counterparty: ABC Company
Recovery on default: X%
Notional: 10 million
Premium: 460,000 upfront with no running premium
Term: 5-years
Risk-free rate: 3.00% annual effective
Deliverable on default: Amount of notional lost due to default

Probabilities:

End of year T:	1	2	3	4	5
Probability of defaulting at end of year T:	2.50%	2.50%	2.50%	2.50%	2.50%
Probability of surviving to end of year T:	97.50%	95.06%	92.69%	90.37%	88.11%

- (b) (1 point) Show that the CDS running spread for the above CDS is within the range 1.00% to 1.15%.
- (c) (1 point) Solve for X.

3. Continued

Your investment manager has provided you with the following information:

Bond	Term	Par	Z-spread	Default correlation with ABC Company
A	7 years	12 million	75 bps	95%
B	5 years	10 million	225 bps	75%
C	5.5 years	10 million	180 bps	90%

- (d) (2 points) Describe how you would use each bond shown above in a negative CDS-Bond basis trade paired with the CDS from part (b). Be sure to discuss
- (i) the specific bond transaction used, and
 - (ii) advantages and disadvantages of using that specific bond.

Your investment manager executes a negative CDS-Bond basis trade by purchasing Bond B shown above and purchasing protection using the CDS from part (a). You are given the following additional information about Bond B:

Bond B Details

Coupon	4.00% annual bond equivalent rate
Frequency	Semi-annual coupon payments
Purchase price	8.5 million

One year after the strategy is executed Bond B pays its 12-month coupon and defaults; you only recover 40% of the par value. ABC Company defaults at the same time as the bond. The LGD of the CDS is the same as the bond.

- (e) (1 point) Draw a diagram showing all cash flows, over the lifetime of the strategy, for the CDS and Bond B used in the above CDS-Bond basis trade.
- (f) (1 point) Calculate the amount earned (or lost) by entering into this CDS-Bond basis trade.

4. (8 points) RMK Financial plans to offer, for the first time, either a benefit-responsive GIC product or a Corporate-Owned Life Insurance (COLI) product with no deferral options to its existing institutional clients.

(a) (2 points) Identify key features of each of these two products that could potentially impact RMK's liquidity risk profile:

- (i) Benefit-responsive GIC
- (ii) COLI with no deferral options

RMK decides to issue only the COLI product with no deferral options. Table 1 below is the company's current investment strategy supporting one of its non-COLI product lines with the primary objective of maximizing the crediting rate subject to some constraints (such as earning the target spread). In the initial phase of the COLI product development, the company's pricing actuary assumes that the COLI's investment strategy will be the same as in Table 1 and the required capital will be increased to account for the presence of the product's liquidity risk.

Table 1: RMK Financial Investment Strategy for one of its non-COLI product line

Permitted Asset Classes	Allocation
A-Rated Commercial Mortgages	50%
BBB-Rated Corporate Bonds	10%
AAA-Rated ABS	10%
Real Estate	30%
U.S. Treasury Bonds	0%
Cash	0%

(b) Critique the pricing actuary's COLI pricing assumptions with respect to:

- (i) (2 points) Investment strategy
- (ii) (1 point) The additional required capital to account for the liquidity risk

4. Continued

RMK mandates a minimum liquidity coverage ratio of 1.2 over time periods of four months or less. The projected monthly cash flows for cash sources and cash needs in baseline and stress scenarios in each of the next 3 months are as follows:

\$Millions	Cash Flows	1st Month	2nd Month	3rd Month
Baseline Scenario	Sources	7,000	3,500	8,500
	Needs	1,000	4,500	4,500
Stress Scenario	Sources	6,500	3,200	7,500
	Needs	3,000	5,000	6,200

- (c) (2 points) Determine if the company can meet its minimum coverage ratio requirement in each scenario.
- (d) (1 point) Recommend three derivatives that could help mitigate the Stress Scenario liquidity risk.

5. (6 points) Rebonato's *Volatility and Correlation* describes the features of volatility smiles and smile-modeling approaches.

(a) (2 points) Describe two approaches to obtain direct static market information that can be useful for modeling the equity volatility surface.

(b) (2 points) Describe four categories of models that account for the smiles.

You are given the following model:

$$dS = rSdt + S^y \sigma(t) dW_t$$

where r and y are constants, t is time, W_t is a Wiener process, and $\sigma(t)$ is the percentage volatility of the underlying S .

(c) (1 point) Identify which category this model belongs to and explain why.

(d) (1 point) Discuss the parameters of the model in terms of the sticky smile and floating smile.

6. (6 Points) You have been hired by a major investment firm to assist with understanding counterparty credit risk. You have a meeting with the CFO to discuss advanced credit risk modeling techniques. The CFO provides the following information to you:

Asset Value	100
Debt	50
Asset growth rate per annum	3%
Annual volatility	20%

The CFO has heard about a concept of distance to default and wants to understand that better.

- (a) (1 point) Calculate the distance to default 3 years ahead using the information above.
- (b) (1 point) Describe a drawback of distance to default that makes it difficult to use for regulatory purposes.
- (c) (1 point) Describe the differences between structural and reduced form models, giving examples of each.

The CFO is interested in learning about credit index options as methods of protection against defaults.

- (d) (1 point) Describe a credit index option including the front end protection offered.
- (e) (1 point) Explain why the Black model is not a useful method of pricing credit index options.
- (f) (1 point) Describe the issues faced historically with modeling default recovery rates.

7. (7 points) You have been asked to review the following one-factor CIR++ model of short rate dynamics:

$r(t) = x(t) + \varphi(t)$, where $x(t)$ satisfies the stochastic differential equation:

$$dx(t) = 0.5(0.0125 - x(t))dt + 0.15\sqrt{x(t)}dW(t)$$

and where $\varphi(t)$ is a deterministic function, $W(t)$ is a Wiener process, and $x(0) = 0.0075$.

- (a) (1 point) Describe conditions under which it would be reasonable to use a one-factor short rate model (CIR++), rather than a two-factor model (CIR2++).
- (b) (1.5 points) Describe the following related to the CIR++ model:
- (i) The advantages of using CIR++ over CIR2++
 - (ii) The limitations of using CIR++ over CIR2++
 - (iii) The purpose of the $\varphi(t)$ function.
- (c) (2 points) Show that $\varphi(t)$ is the difference between the market forward curve and the CIR forward curve i.e. $\varphi(t) = f^M(0, t) - f^{CIR}(0, t)$, without using the closed-form solution for the CIR bond price.
- (d) (1.5 points) Write down the formula for $f^{CIR}(0, t)$ using the parameters implied by the CIR++ model above.
- (e) (1 point) Determine if zero is accessible to the process $x(t)$.

8. (8 points) REIT A and REIT B are both bidding to purchase (without debt financing) a newly built office tower which is valued using an opportunity cost of capital (OCC) approach.

Your colleague has provided the following relationships with respect to the REITs:

- Net asset value (NAV) per share = share price
 - $IV_R = MV_P$ (No valuation differential)
- (a) (2 points) Describe each of the four terms within the above two equations as they relate to a REIT.
- (b) (2 points) Critique the relationships your colleague provided, taking into consideration the validity of each equation under varying market conditions.
- (c) (1 point) Identify the advantages of a REIT in generating future cash flows from a given property as compared to a private property owner.
- (d) (2 points) Compare and contrast the opportunity cost of capital between the REIT and private property markets and its impact on valuations within these markets.

The following table shows the average cost of capital for both REITs based on their historical stock market performance and their expected future cash flow from the investment:

REIT A CoC	6%
REIT B CoC	7%
OCC _{MARKET}	8%
Annual E[CF]*	51 million
Investment period*	20 years

* Expected future cash flow and investment period are the same for both REITs

REIT B has made an offer of 500 million to purchase the property.

- (e) (1 point) Assess the amount REIT A should be willing to pay to invest in the property.

9. (6 points) ABC is a Real Estate Investment Trust interested in residential real estate properties. ABC considers the monthly log price increase of 5 types of real estate properties:

- R1: Houses within 30mins of commute time from CBD (Central Business District)
- R2: Houses with commute time from 30mins to 1hr from CBD
- R3: Condominium within 30mins of commute time from CBD
- R4: Condominium with commute time from 30mins to 1hr from CBD
- R5: Townhouses within 1hr of commute time from CBD

The reported price of each property type is calculated as below:

$$P_t^{reported} = \alpha * P_{[t-30,t]}^{avg} + \beta * P_{[t-60,t-30]}^{avg} + (1 - \alpha - \beta) * P_{[t-90,t-60]}^{avg}$$

where

$P_{[s,t]}^{avg}$ represents the average price of all the properties from the same category for the period from time s to time t , $\alpha, \beta > 0$ and $\alpha + \beta < 1$.

- (a) (1 point) Explain why reported real estate prices are generally smooth.

You are hired by ABC to analyze the properties using their correlation matrix. You decide to use Principle Component Analysis (PCA). You are given the monthly reported price from Jan 2000 to Jan 2016.

- (b) (1 point) Describe the data problem(s) you might encounter in PCA statistical factor analysis and how the problem(s) can be addressed.

9. Continued

You complete the PCA and get the following eigenvalue (λ) and eigenvector (ε) pairs:

$$\lambda_1 = 24.0054, \varepsilon_1 = (-0.3994, -0.4640, -0.4887, -0.4622, -0.4157)^T$$

$$\lambda_2 = 4.0984, \varepsilon_2 = (-0.3489, -0.3130, -0.3891, 0.4774, 0.6333)^T$$

$$\lambda_3 = 1.7366, \varepsilon_3 = (0.8615, -0.2242, -0.4506, -0.0667, 0.0027)^T$$

$$\lambda_4 = 1.4829, \varepsilon_4 = (-0.0268, 0.7448, -0.6568, 0.0598, -0.0978)^T$$

$$\lambda_5 = 0.8052, \varepsilon_5 = (-0.3132, 0.2573, 0.1094, -0.6648, 0.6179)^T$$

(c) (2 points) Using the PCA results above:

- (i) Construct a reasonable and efficient orthogonal factor model.
- (ii) Calculate the factor loadings.

You find the resulting factor loadings from part (c) to be quite arbitrary and hard to interpret.

(d) (1 point) Explain why the factor loadings are hard to interpret and describe how you can improve their meaning.

There are about 500,000 properties which ABC is now planning to analyze individually based on the monthly log price changes over the past 16 years.

(e) (1 point) Explain how you can apply PCA techniques to this problem.

****END OF EXAMINATION****
Morning Session

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