MLC Fall 2016

Model Solutions

Written Answer Questions

MLC Fall 2016

Question 1 Model Solution

Learning Outcome: 1(a), 1(b), 1(d)

Chapter References: AMLCR Chapter 8

Notes:

1. *Part (a) was done very well by many candidates. Most gave the definition in words, though a definition using symbols was acceptable, provided it was sufficiently clear. For full credit, it was necessary to refer to the specific model in the question in the explanation part.*

2. *In part (b) a small number of stronger candidates who knew the proof achieved full or nearly full credit; a much larger group achieved a small amount of partial credit by writing down the Kolmogorov equations, and some candidates omitted the part completely.*

3. *Valid proofs that were presented differently than the model were awarded full credit, but candidates who wrote the first and last line of the proof, but put irrelevant detail in between, gained no credit. The graders do read every line.*

4. *Part (c) was done well, with most candidates achieving full credit.*

(a) $_tp_x^{00}$ is the probability that a life who is currently age x and in State 0 is also in State 0 in t years, at age $x + t$.

$_tp_x^{\overline{00}}$ is the probability that a life who is currently age x and in State 0 remains in State 0 throughout the following t years.

In the model shown, these probabilities are not the same because backward transitions between State 1 and State 0 are admitted, so the first probability, $_tp_x^{00}$, includes the possibility that (x) leaves State 0 and then returns to it in the period $(0, t)$, while the second does not.

(b)

$$_{t+h}p_x^{00} = {}_tp_x^{00} \, {}_hp_{x+t}^{00} + {}_tp_x^{01} \, {}_hp_{x+t}^{10}$$

$$= {}_tp_x^{00} \left(1 - \left({}_hp_{x+t}^{01} + {}_hp_{x+t}^{02} \right) \right) + {}_tp_x^{01} \, {}_hp_{x+t}^{10}$$

$$\Rightarrow {}_{t+h}p_x^{00} - {}_tp_x^{00} = {}_tp_x^{01} \, {}_hp_{x+t}^{10} - {}_tp_x^{00} \left({}_hp_{x+t}^{01} + {}_hp_{x+t}^{02} \right)$$

$$\Rightarrow \lim_{h \to 0^+} \frac{{}_{t+h}p_x^{00} - {}_tp_x^{00}}{h} = \lim_{h \to 0^+} \left\{ {}_tp_x^{01} \frac{{}_hp_{x+t}^{10}}{h} - {}_tp_x^{00} \left(\frac{{}_hp_{x+t}^{01}}{h} + \frac{{}_hp_{x+t}^{02}}{h} \right) \right\}$$

$$\Rightarrow \frac{d}{dt} {}_tp_x^{00} = {}_tp_x^{01} \, \mu_{x+t}^{10} - {}_tp_x^{00} \left(\mu_{x+t}^{01} + \mu_{x+t}^{02} \right)$$

(c)

$$_2p_x^{\overline{00}} = 0.165 = \exp\left\{ -\int_0^2 \mu_{x+t}^{01} + \mu_{x+t}^{02} \, dt \right\}$$

$$= \exp\left\{ -\int_0^2 0.5 + kt \, dt \right\}$$

$$= \exp(-(1 + 2k))$$

$$\Rightarrow -\log(0.165) = 1 + 2k$$

$$\Rightarrow k = 0.400905$$

MLC Fall 2016

Question 2 Model Solution

Learning Outcomes: 2(a), 2(b), 3(a), 3(b)

Chapter References: AMLCR Chapters 6, 11

Notes:

1. *Part (a) was done very well by almost all candidates. The two most common mistakes were incorrectly calculating the probability of the second death occurring in the second year, and averaging the 2-year spot rates to 6.6% before doing the EPV calculations.*

2. *Most candidates were less successful on part (b). A common error was to try to use memorized formulas that do not apply here (e.g $(1 + P/d)^2 ({}^2A - A^2))$, instead of working with the 3-point distribution.*

3. *Candidates who recognized that the random interest rate affected all policies did very well on part (c). Many candidates wrongly assumed independence and used $Var[L] = N\,Var[L_{0,j}]$. Others incorrectly used $E[Var[L|Sc]]$ for $Var[L]$ and only received partial credit.*

4. *Most candidates provided essentially the same answer to both (d) and (e); either repeating the mathematical definition of a diversifiable risk or repeating that the risk doesn't decrease by increasing the size of the portfolio. Many candidates commented on the total risk for the portfolio instead of considering the average risk per policy. Few candidates received full credit for these two parts.*

(a) Let $v(t)$ denote the discount function:

$$v(1) = v_{5\%} \text{ under both scenarios}$$

$$v(2) = \begin{cases} v_{6\%}^2 & \text{Scenario 1} \\ v_{7\%}^2 & \text{Scenario 2} \end{cases}$$

Also $p_{65:70} = 0.85500, \quad {}_1q_{65:70} = 1 - p_{65:70} = 0.145,$

and $\quad {}_{1|1}q_{65:70} = p_{65:70} - {}_2p_{76:70} = 0.85500 - 0.71529 = 0.13971$

4

The EPVs of premiums and benefits are as follows, where Sc.1 refers to Scenario 1, and Sc.2 to Scenario 2.

EPV Premiums $= P(1 + p_{65:70}\, v(1)) = 1.81429P$

EPV Benefits|Sc.1 $= 10\,000 \left(q_{65:70}\, v_{5\%} + {}_{1|1}q_{65:70}\, v_{6\%}^2 \right) = 2624.3$

EPV Benefits|Sc.2 $= 10\,000 \left(q_{65:70}\, v_{5\%} + {}_{1|1}q_{65:70}\, v_{7\%}^2 \right) = 2601.2$

$\Rightarrow$ EPV Benefits $= 0.4(2624.3) + 0.6(2601.2) = 2610.5$

$\Rightarrow P = \dfrac{2610.5}{1.81429} = 1438.82$

(b) In terms of the discount function $v(t)$, we have

$$
L_{0,j} = \begin{cases}
10\,000\, v(1) - P & \text{w.p. } q_{65:70} \\
10\,000\, v(2) - P(1 + v(1)) & \text{w.p. } {}_{1|1}q_{65:70} \\
-P(1 + v(1)) & \text{w.p. } {}_{2}p_{65:70}
\end{cases}
$$

So

$$
L_{0,j}|\text{Sc.1} = \begin{cases}
8085.0 & \text{w.p. } 0.14500 \\
6090.1 & \text{w.p. } 0.13971 \\
-2809.2 & \text{w.p. } 0.71529
\end{cases}
$$

which gives

$$
\begin{aligned}
E\left[L_{0,j}|\text{Sc.1}\right] &= 8085.0(0.145) + 6090.1(0.13971) - 2809.2(0.71529) \\
&= 13.88
\end{aligned}
$$

$$
\begin{aligned}
\text{Var}\left[L_{0,j}|\text{Sc.1}\right] &= 8085.0^2(0.145) + 6090.1^2(0.13971) + 2809.2^2(0.71529) - 13.88^2 \\
&= 20\,305\,469 = 4506.2^2
\end{aligned}
$$

Under Scenario 2 we have

$$
L_{0,j}|\text{Sc.2} = \begin{cases}
8085.0 & \text{w.p. } 0.14500 \\
5925.3 & \text{w.p. } 0.13971 \\
-2809.2 & \text{w.p. } 0.71529
\end{cases}
$$

which gives

$$
E\left[L_{0,j}|\text{Sc.2}\right] = -9.25
$$

$$
\text{Var}\left[L_{0,j}|\text{Sc.2}\right] = 20\,027\,618 = 4475.2^2
$$

(c) Let Sc denote the unknown interest rate scenario, either Scenario 1 or Scenario 2. Then

$$L|Sc = \sum_{j=1}^{N} L_{0,j}|Sc \Rightarrow E[L|Sc] = NE[L_{0,j}|Sc]$$

and $\text{Var}[L|Sc] = N\text{Var}[L_{0,j}|Sc]$

Using iterated expectation,

$$E[L] = E\Big[E[L|Sc]\Big] = NE[L_{0,j}]$$

$$\text{Var}[L] = E\Big[\text{Var}[L|Sc]\Big] + \text{Var}\Big[E[L|Sc]\Big]$$

Now

$$E[L|Sc] = \begin{cases} 13.88N & \text{w.p. } 0.4 \\ -9.25N & \text{w.p. } 0.6 \end{cases}$$

$$\text{Var}[L|Sc] = \begin{cases} 4506.2^2 N & \text{w.p. } 0.4 \\ 4475.2^2 N & \text{w.p. } 0.6 \end{cases}$$

So we have

$$E\Big[\text{Var}[L|Sc]\Big] = N\left(0.4(4506.2^2) + 0.6(4475.2^2)\right) = 4487.6^2\, N$$

$$\text{Var}\Big[E[L|Sc]\Big] = 0.4(13.88^2 N^2) + 0.6(9.25^2 N^2) - E[L]^2$$

where $E[L] = 0.4(13.88N) + 0.6(-9.25N) = 0$

$$\Rightarrow \text{Var}\Big[E[L|Sc]\Big] = 128.4N^2$$

$$\Rightarrow \text{Var}[L] = 4487.6^2 N + 128.4N^2$$

(d) The risk is diversifiable if and only if

$$\lim_{N \to \infty} \sqrt{\frac{\text{Var}[L]}{N^2}} = 0$$

In this case we have

$$\begin{aligned}
\lim_{N \to \infty} \sqrt{\frac{\text{Var}[L]}{N^2}} &= \lim_{N \to \infty} \sqrt{\frac{4487.6^2 N + 128.4 N^2}{N^2}} \\
&= \lim_{N \to \infty} \sqrt{\frac{4487.6^2}{N} + 128.4} \\
&= \sqrt{128.4} = 11.3 \neq 0
\end{aligned}$$

Hence, the risk is not diversifiable.

(e) The risk is not diversifiable because the interest scenario applies to all policies identically; all policies experience the same interest rate scenario. It is not diversified by increasing the number of policies.

MLC Fall 2016

Question 3 Model Solution

Learning Outcomes: 2(a), 2(b), 2(c).

Chapter References: AMLCR Chapter 8

Notes:

1. *Overall, candidates did extremely well on parts (a) and (b). A significantly smaller group achieved full credit on (c) and (d); most candidates omitted these parts. Part (e) was particularly challenging, with a handful of candidates achieving full credit, and a slightly larger group achieving partial credit.*

2. *In part (c) many candidates know the formula for part (i), but a much smaller number knew or could derive the formula for part(ii).*

3. *Many candidates omitted parts (d) and (e). No partial credit was given in (d) for candidates using the UDD assumption, as that was already covered in part (a). In part (e), some candidates achieved partial credit by writing down the correct integral formula for the valuation, without any further calculations. The key insight, for those who understood how to set up the integral, was that the $\bar{A}^{id}_{x:\overline{2|}}$ function is a constant, because of the constant force of mortality.*

(a)

$$
\begin{aligned}
EPV =& 100\,000 \left(\frac{d^{(d)}_{35} v + d^{(d)}_{36} v^2 + d^{(d)}_{37} v^3}{l^{(\tau)}_{35}} \right) \\
&+ 50\,000 \left(\frac{d^{(i)}_{35} v + d^{(i)}_{36} v^2 + d^{(i)}_{37} v^3}{l^{(\tau)}_{35}} \right) \\
=& 100\,000 \frac{64v + 64v^2 + 65v^3}{45\,730} + 50\,000 \frac{46v + 43v^2 + 45v^3}{45\,730} \\
=& 362.40 + 125.94 = 488.34
\end{aligned}
$$

(b) The EPV is $488.34 \dfrac{i}{\delta} = 507.62$

(c) (i)

$$
p^{(\tau)}_{35} = \frac{42\,927}{45\,730} = 0.93870
$$

Also, $p^{(\tau)}_{35} = e^{-\mu^{(\tau)}_{35}}$

So, $\mu^{(\tau)}_{35} = -\log(0.93870) = 0.06325$

(ii) $\mu_x^{(j)} = \mu_x^{(\tau)} \left(\dfrac{d_x^{(j)}}{d_x^{(\tau)}} \right)$

So $\mu_{35}^{(d)} = 0.06325 \left(\dfrac{64}{2803} \right) = 0.00144$ and $\mu_{35}^{(i)} = 0.06325 \left(\dfrac{46}{2803} \right) = 0.00104$

(d)

$$EPVDB = 100\,000 \int_0^1 {}_t p_{35}^{(\tau)} \mu_{35+t}^{(d)} e^{-\delta t}\, dt$$

$$= 100\,000 \int_0^1 e^{-t(\mu_{35}^{(\tau)}+\delta)} \mu_{35+t}^{(d)} dt$$

$$= 100\,000(0.00144) \frac{1 - e^{-(0.06325+0.076961)}}{0.06325 + 0.076961} = 134.76$$

Similarly for the disability benefit

$$EPVDis = 50\,000 \int_0^1 {}_t p_{35}^{(\tau)} \mu_{35+t}^{(i)} e^{-\delta t}\, dt$$

$$= 50\,000 \int_0^1 e^{-t(\mu_{35}^{(\tau)}+\delta)} \mu_{35+t}^{(i)} dt$$

$$= 50\,000(0.00104) \frac{1 - e^{-(0.06325+0.076961)}}{0.06325 + 0.076961}$$

$$= 50\,000(0.000969) = 48.43$$

The total EPV is 183.18.

(e) The EPV of the additional benefit in integral form is

$$50\,000 \int_0^1 {}_t p_{35}^{(\tau)} \mu_{35+t}^{(i)} e^{-\delta t}\, \bar{A}^{id}_{35+t:\overline{2|}}$$

where the $\bar{A}^{id}_{35+t:\overline{2|}}$ function is the EPV, at the moment of transition to disability, of the death benefit after disability[1]. Because the force of mortality after disability is

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[1]We are using the multiple state notation here because the model is no longer a pure multiple decrement model.

9

constant, this function does not depend on t:

$$\bar{A}^{id}_{35+t:\overline{2}|} = \int_0^2 {}_tp^{ii}_{35+t+r}\,\mu^{id}_{35+t+r}e^{-\delta\,r}dr$$

$$= \int_0^2 e^{-(0.5+\delta)r}(0.5)dr$$

$$= 0.5\left(\frac{1-e^{-2(0.5+0.076961)}}{0.5+0.076961}\right) = 0.59328$$

Hence the value of the extra DB after disability is

$$50\,000(0.59328)(0.000969) = 28.73$$

MLC Fall 2016

Question 4 Model Solution

Learning Outcomes:3(a), 3(b)

Chapter References: AMLCR Chapter 7

Notes:

1. *Overall, candidates did a good job with this question. The majority of candidates got full marks on parts (a), (b), and (d).*
2. *For part (b), candidates who lost marks generally gave the probability that $L_0 < 0$, instead of the probability that $L_0 > 0$ as directed.*
3. *In part (c), candidates who lost points often failed to recognize that $E[L_0] = 0$ since the premium is determined using the equivalence principle, and/or were unable to calculate $E[L_0^2]$ correctly.*
4. *Candidates who were unable to calculate $E[L_0^2]$ correctly often did not incorporate the probability of survival into their calculation, or incorrectly calculated the second moment using $(PVFB)^2 - (PVFP)^2$ instead of $(PVFB - PVFP)^2$.*
5. *In part (e), many candidates wrongly assumed that the variance for each policyholder covered is the same.*
6. *Most candidates got some marks on part (f), but few candidates achieved full marks, for which candidates needed to explain the effect of reduced diversification.*

Working in $000's

(a)

$$P = \frac{500 q_{[77]} v + 300 p_{[77]} q_{78} v^2 + 100 \, {}_2p_{[77]} q_{79} v^3}{1 + p_{[77]} v + {}_2p_{[77]} v^2}$$

$$= \frac{35.245}{2.7444} = 12.8427$$

That is, the premium is $12\,843 to the nearest $1.

(b) *L* will be greater than zero if the life dies during the term (check: death in third year gives $L = 100 \, v^3 - P(1 + v + v^2) = 49.104$).

The probability is

$${}_3q_{[77]} = 1 - {}_3p_{[77]} = 1 - 0.97 * 0.94 * 0.933 = 0.14930$$

(c) We have

$$
L_0 = \begin{cases}
500\,v - P & \text{w.p. } q_{[77]} \\
300\,v^2 - P(1+v) & \text{w.p. } p_{[77]}q_{78} \\
100\,v^3 - P(1+v+v^2) & \text{w.p. } {}_2p_{[77]}q_{79} \\
-P(1+v+v^2) & \text{w.p. } {}_3p_{[77]}
\end{cases}
$$

That is:

$$
L_0 = \begin{cases}
462.157 & \text{w.p. } 0.03 \\
245.706 & \text{w.p. } 0.0582 \\
49.103 & \text{w.p. } 0.061091 \\
-36.635 & \text{w.p. } 0.850709
\end{cases}
$$

We know that $E[L_0] = 0$ from the equivalence principle.

So

$$
\mathrm{Var}[L_0] = 462.157^2(0.03) + 245.706^2(0.0582) + 49.103^2(0.061091) + 36.635^2(0.850709)
$$
$$
= 11\,210.3
$$
$$
\Rightarrow SD[L_0] = \sqrt{11210.3} = 105.879
$$

That is, the standard deviation of L_0 is 106,000 to the nearest 1000.

(d)

$$
E[L] = 0
$$
$$
\mathrm{Var}[L] = 8000\,\mathrm{Var}[L_0] = 89\,683\,000
$$
$$
\Rightarrow SD[L] = 9\,470.1
$$

The 90% quantile of the standard normal distribution is 1.282, so the approximate 90th percentile of the loss is

$$
1.282(9\,470.1) = 12\,141
$$

Which is $12.141 million.

(e) Let L_0^m denote the random loss at issue variable for each multiple policy.

$$
E[L_0^m] = 5E[L_0] = 0
$$

$$
\mathrm{Var}[L_0^m] = 5^2\,\mathrm{Var}[L_0] = 280\,252
$$

So $E[L^*] = 0$ and

$$\text{Var}[L^*] = 7000\,\text{Var}[L_0] + 200\,\text{Var}[L_0^m] = 11\,598^2$$

And the revised 90th percentile estimate is

$$1.282 * 11\,598 = 14\,869$$

That is \$14.869 million.

(f) L^* is less diversified than L, with the same total sum insured spread around fewer policyholders. Since diversification reduces the relative uncertainty, less diversification means greater uncertainty, as confirmed by the larger 90th percentile.

MLC Fall 2016

Question 5 Model Solution

Learning Outcomes: 4(a), 4(c), 4(g)

Chapter References: AMLCR Chapter 7, 12

Notes

1. *Parts (a) and (b) were done reasonably well, with a majority of candidates achieving full credit.*
2. *In part (a), some candidates included expenses in the FPT reserve or premium, which is a significant error, as the FPT is a net premium reserve method.*
3. *The most common error in part (b) was to assume that the withdrawal and death probabilities given were independent, rather than dependent probabilities.*
4. *In part (c) the most common errors were (i) not applying survival probabilities correctly to the Pr_t terms, (ii) discounting for the wrong number of years and (iii) using the wrong discount rate.*
5. *Part (d) was omitted by most candidates, but those who persisted scored well.*
6. *Part (e) was more challenging, and many candidates omitted this also. Those who did attempt it generally did not work through to the correct final answer, but did achieve partial credit in general. A common mistake was to forget the change to Pr_0 arising from the premium change.*
7. *Many candidates who skipped parts (d) and or (e) attempted part (f), and some did well. The most common error was assuming that the change would make no difference, as the cash flows were the same (partial credit).*

(a) $_2V^{FPT} = 100\,000 q_{72} v - P^*$ where

where $P^* = 100\,000 \dfrac{q_{71} v + p_{71} q_{72} v^2}{1 + p_{71} v} = 1892.2$

$\Rightarrow {}_2V^{FPT} = 98.23$

(b)

$Pr_2 = 2200 * 0.9 * 1.06 - 0.018(100\,000) - (1 - 0.018 - 0.1) * {}_2V^{FPT}$

$\quad = 212.16$

(c)

$NPV = -660 + 115 v_{10\%} + 212.16 \, p_{70}^{(\tau)} v_{10\%}^2 + 340 \, {}_2p_{70}^{(\tau)} \, v_{10\%}^3$

where $_2p_{70}^{(\tau)} = 0.82 * (1 - 0.018 - 0.1) = 0.72324$

so $NPV = -226.9$

14

(d)

$$\text{Pr}_1^* = 0.48G(1.06) - q_{70}^{(d)}(100\,000) - p_{70\ 1}^{(\tau)}V^{FPT}$$

$$\text{Pr}_1 = (0.48)(2200) - q_{70}^{(d)}(100\,000) - p_{70\ 1}^{(\tau)}V^{FPT}$$

$$\Rightarrow \text{Pr}_1^* - \text{Pr}_1 = (0.48)(1.06)(G - 2200) = 0.5088(G - 2200)$$

$$\Rightarrow \text{Pr}_1^* = 0.5088(G - 2200) + \text{Pr}_1$$

(e) Express all Pr_t^* in terms of Pr_t as follows, similarly to the case in (d):

$$\text{Pr}_0^* = -.3G = -0.3(G - 2200) + \text{Pr}_0$$

$$\text{Pr}_1^* = 0.5088(G - 2200) + \text{Pr}_1 \qquad \text{from (d)}$$

$$\text{Pr}_2^* = 0.9G(1.06) - q_{71}^{(d)}(100\,000) - p_{71\ 2}^{(\tau)}V^{FPT}$$

$$\qquad = 0.954(G - 2200) + \text{Pr}_2$$

$$\text{Pr}_3^* = 1.06(G - 2200) + \text{Pr}_3$$

Let $D = G - 2200$, then if NPV^* and NPV are the Net Present Values respectively for premium G and premium 2200, we have

$$NPV^* = -0.3D + 0.5088Dv_{10\%} + 0.954Dp_{70}^{(\tau)}v_{10\%}^2 + 1.06D_2p_{70}^{(\tau)}v_{10\%}^3 + NPV$$

$$\qquad = 1.3850D - 226.9$$

Set $NPV^* = 100$ for $D = 236$ which gives $G = 2436.$.

(f) The total expenses are unchanged, and the timing of the payments is effectively unchanged. However, for expenses deemed paid at the start of year 1 we allow for interest at 6%, and then discount at 10%. For pre-contract expenses there is no interest or discounting. The overall effect is that moving expenses from beginning year 1 to pre-contract will decrease the NPV.

MLC Fall 2016

Question 6 Model Solution

Learning Outcomes: 5(a), 5(b), 5(c), 5(d), 5(f)

Chapter References: AMLCR Chapter 10

Notes:

1. *Overall, candidates did not score very well on this question. Only a small number of candidates omitted the question completely, but most omitted several parts.*
2. *The question was more complex than previous exams, as it incorporated mid-year exits and a withdrawal benefit in part (b), and a bridge benefit in part (c).*
3. *Some candidates lost marks on part (a) because they did not explain their example in sufficient detail.*
4. *In part (b) many candidates correctly calculated the normal contribution for Ken, but not for Tom. A very common error was to omit the withdrawal benefit.*
5. *In part (c) there were many omitted answers and few complete answers. Most candidates who attempted this part received substantial partial credit, with common errors generally arising from miscalculation of the bridge benefit.*

(a) For both Traditional Unit Credit (TUC) and Projected Unit Credit (PUC) the valuation does not take into consideration any future service. For PUC, the valuation does allow for expected future salary increases to retirement. For TUC, the valuation does not allow for future salary increases, but uses the final average earnings calculation applied as at the valuation date.

Example: Consider a DB benefit based on final year's salary. For a current member with salary S_x in the year of age x to $x + 1$, we would value the retirement benefit payable from age 65, say, under PUC using the final salary S_{64}.

For the TUC valuation, at age 35, say, we would use S_{34}.

(b) **For Tom**, the Actuarial Liability at the valuation date (AL_0) is the sum of the EPV of the retirement benefit and the withdrawal benefit.

$$AL_0 = 2 * 65000 * (1.03)^{24} * 0.013 * 0.94 * v_{6.5\%}^{25} * \ddot{a}_{65}^{(12)}$$

$$+ 2 * 65000 * 0.013 * 0.06 * v_{6.5\%}^{25} * \ddot{a}_{65}^{(12)}$$

$$= 6421.5 + 201.6$$

$$= 6623.1$$

16

The EPV at the valuation date of the actuarial liability at time 1 allows for staying in the plan or the cost of withdrawal:

$$= 3 * 65000 * (1.03)^{24} * 0.013 * 0.94 * v^{25}_{6.5\%} * \ddot{a}^{(12)}_{65}$$

$$+ 3 * 65000 * 0.013 * 0.06 * v^{25}_{6.5\%} * \ddot{a}^{(12)}_{65}$$

$$= 9632.3 + 302.4$$

$$= 9934.7$$

So $NC^t = 9934.7 - 6623.1 = 3311.6$.

For Ken the NC just one year's accrual.

$$AL_0 = 5 * 80\,000(1.03)^{14} * 0.013 * v^{15}_{6.5\%} * \ddot{a}^{(12)}_{65} = 29359.7$$

$$NC^K = \frac{29359.7}{5} = 5871.94$$

(c) The AL at 2017 of the normal retirement age benefit is

$$0.5 \left(6 * 80\,000 * (1.03)^{14} * 0.013 * v^{14}_{6.5\%} * \ddot{a}^{(12)}_{65} \right)$$

$$= 0.5(37521.7) = 18760.9$$

The AL of the early retirement benefit is

$$0.79 \Bigg\{ 0.5 \left(6 * 80\,000 * (1.03)^{11} * 0.013 * v^{11}_{6.5\%} * \ddot{a}^{(12)}_{62} \right)$$

$$+ 0.5 \left(15 * 12 * 6 * v^{11}_{6.5\%} * \ddot{a}^{(12)}_{62:\overline{3}|} \right) \Bigg\}$$

$$= 0.79 \left\{ 0.5(44502.6 + 1458.6) \right\} = 18154.7$$

The AL of the total benefits is $18760.9 + 18154.7 = 36915.5$.

Since all the benefits are proportional to service, the NC is $AL/6 = 6152.6$.