# GIIRR Model Solutions
# Fall 2016

**1.** **Learning Objectives:**
  4.  The candidate will understand trending procedures as applied to ultimate claims, exposures and premiums.

**Learning Outcomes:**
(4a)  Identify the time periods associated with trending procedures.
(4e)  Choose trend rates and calculate trend factors for exposures.

**Sources:**
Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 26.

**Commentary on Question:**
*This question tests the candidate's understanding of premium trend analysis.*

**Solution:**
(a)  Compare the written and earned premium approaches for determining the trending period.

  Written premiums are trended from the average written date of the premiums earned in the experience period to the average written date in the forecast period.

  Earned premiums are trended from the average earned date in the experience period to the average earned date in the forecast period.

(b)  Explain why you would use each approach identified in part (a).

  Written premiums reflect shifts in the mix of exposures more quickly than earned premiums.

  Earned premiums are used in most areas of ratemaking and are therefore more convenient to use.

# 1. Continued

(c)     Calculate the trending periods for both of the written and earned premium approaches.

Written premiums approach:

Average written date in the experience period = January 1, 2015
Average written date in the forecast period = July 1, 2017
Trending Period = 2.5 years

Earned premiums approach:

Average earned date in the experience period = July 1, 2015
Average earned date in the forecast period = January 1, 2018
Trending Period = 2.5 years

(d)     Provide two reasons why exposures are preferable to premiums when analyzing shifts in the mix of business.

It avoids the need to adjust premiums to current rate levels.

It eliminates distortions in the average premium values that may be due to shifts in various rating factors.

**2.** **Learning Objectives:**
2. The candidate will understand how to calculate projected ultimate claims and claims-related expenses.

**Learning Outcomes:**
(2b) Estimate ultimate claims using various methods: development method, expected method, Bornhuetter Ferguson method, Cape Cod method, frequency-severity methods, Berquist-Sherman methods.

**Sources:**
Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 16.

**Commentary on Question:**
*This question tests the candidate's understanding of estimating ultimate claims using the expected method.*

**Solution:**
(a) Define an exposure base.

An exposure base is a measure that is known or accurately estimated in advance and that varies directly with the quantity being estimated.

(b) Define a leading indicator.

A leading indicator is a measure that is not known in advance, but is directly correlated with the quantity being measured.

(c) State an example of a leading indicator used for estimating ultimate claims.

Reported claim counts are a leading indicator of ultimate claims.

(d) Explain why a pure premium approach is preferred over an expected claim ratio approach when developing expected claims for self-insurers.

For self-insurers, the pure premium approach is used instead of the expected claims ratio approach since a self-insurer does not typically have earned premiums in the same way that an insurer does.

**2.** **Continued**

(e) Calculate the trended on-level reported claim ratios at the cost and rate level of 2015 for accident years 2010-2014.

| | (1) | (2) | (3) | (4) | (5) | (6) = [(2)(4)(5)] / [(1)(3)] |
|---|---|---|---|---|---|---|
| | | Projected Ultimate | | Claim Adjustment Factors | | |
| Accident Year | Earned Premiums (000) | Reported Claims (000) | Premium On-Level Factor | Trend at 2.0% | Tort Reform | Claim Ratios |
| 2010 | 9,300 | 6,805 | 1.025 | 1.104 | 0.75 | 0.591 |
| 2011 | 9,800 | 6,460 | 1.065 | 1.082* | 0.75 | 0.502 |
| 2012 | 9,410 | 5,650 | 1.045 | 1.061 | 1.00 | 0.610 |
| 2013 | 9,700 | 5,830 | 1.035 | 1.040 | 1.00 | 0.604 |
| 2014 | 8,300 | 5,810 | 1.020 | 1.020 | 1.00 | 0.700 |

* e.g., $1.082 = 1.02^4$

(f) Recommend an expected claim ratio for accident year 2015. Justify your recommendation.

Recommend the average of accident years 2010, 2012, 2013 = 0.602.

Justification: exclude high and low outliers.

(g) Estimate the expected ultimate claims for accident year 2015.

Expected ultimate claims for accident year 2015 = $0.602 \times 8,950 = 5,388$.

**3.** **Learning Objectives:**
1. The candidate will understand the key considerations for general insurance actuarial analysis.

2. The candidate will understand how to calculate projected ultimate claims and claims-related expenses.

**Learning Outcomes:**
(1l) Adjust historical earned premiums to current rate levels.
(2b) Estimate ultimate claims using various methods: development method, expected method, Bornhuetter Ferguson method, Cape Cod method, frequency-severity methods, Berquist-Sherman methods.
(2d) Explain the effect of changing conditions on the projection methods cited in (2b).

**Sources:**
Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapters 12, 14, 16, 17, 18, and 20.

**Commentary on Question:**
*This question tests the candidate's understanding of analyzing actual versus expected claims at interim evaluations or future evaluations where interpolation may be needed. This question also tests the candidate's understanding of adjusting historical premiums to current rate levels for different purposes.*

**Solution:**
(a) State the two primary assumptions of the development method.

Historical experience is predictive of future experience.

Activity observed to date is relevant for projecting future activity.

(b) Explain how the Bornhuetter Ferguson method and the Cape Cod method each reflect a change in claim experience.

The Cape Cod method is more responsive to a change in claim experience because the actual reported claims of all years enters the calculation of expected claims.

In the Bornhuetter Ferguson method, expected claims are based on the a priori estimate which does not change (unless the actuary deliberately makes a change).

**3.** **Continued**

(c) Compare actual reported claims with expected reported claims as of September 30, 2015.

The evaluation age is 21 months.
The interpolated cumulative percent reported at 21 months = 72.5%.
Expected reported claims at 21 months = 63,945 = (126,000 × 70% × 72.5%).
Actual reported claims of 63,000 (given) are lower than expected reported claims of 63,945 at 21 months.

(d) Calculate estimates of ultimate pure premium from the Bornhuetter Ferguson method and the expected method as of September 30, 2015.

Ultimate pure premium from expected method = 84.0 = (126,000 × 70%) / 1,050.
IBNR factor at 21 months = 27.5% = 100% – 72.5% from part (c).
Ultimate claims using Bornhuetter Ferguson method = 87,255 = 63,000 + (27.5% × 126,000 × 70%).
Ultimate pure premium from Bornhuetter Ferguson method = 83.10 = 87,255 / 1,050.

(e) Estimate IBNR reserves as of December 31, 2015 using the Bornhuetter Ferguson method.

IBNR factor at 24 months = 20% = 100% – (50% + 30%).
IBNR at 24 months = 17,640 = (20% × 126,000 × 70%).

(f) Estimate reported claims between December 31, 2015 and December 31, 2016 using the a priori expected claims ratio.

Incremental reporting pattern between December 31, 2015 and December 31, 2016 (i.e., 24-36 months) = 15%.
Reported claims between December 31, 2015 and December 31, 2016 = 13,230 = (15% × 126,000 × 70%).

(g) Calculate the on-level premium needed for ratemaking for policies written on or after January 1, 2017.

The earned rate level for accident year 2014 is 1.00 since it is prior to any rate changes.
The current rate level is 1.05.
Therefore, the on-level factor to current rate level = 1.05 = 1.05 / 1.00.
2014 on-level premium = 132,300 = 1.05 × 126,000.

**3.** **Continued**

(h)     Calculate the used-up on-level earned premium as of December 31, 2015 needed to apply the Cape Cod method.

The earned rate level for accident year 2014 is 1.00 since it is prior to any rate changes.
The earned rate level for accident year 2015 is 1.025 = [(50% × 1.00) + (50% × 1.05)].
Therefore, the accident year 2014 on-level factor to accident year 2015 = 1.025 = (1.025 / 1.00).
Also, the cumulative reported percentage as of 24 months is 80%.
Therefore, the used-up on-level earned premium = 103,320 = (1.025 ×126,000 × 80%).

**4.** **Learning Objectives:**

4.     The candidate will understand trending procedures as applied to ultimate claims, exposures and premiums.

5.     The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

7.     The candidate will understand the nature and application of catastrophe models used to manage risks from natural disasters.

**Learning Outcomes:**
(4a)    Identify the time periods associated with trending procedures.
(4c)    Choose trend rates and calculate trend factors for claims.
(4e)    Choose trend rates and calculate trend factors for exposures.
(5d)    Calculate loadings for catastrophes and large claims.
(5f)    Calculate overall rate change indications under the claims ratio and pure premium methods.
(7b)    Apply catastrophe models to insurance ratemaking, portfolio management, and risk financing.

**Sources:**
Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapters 25, 26, 30, and 31.

Catastrophe Modeling: A New Approach to Managing Risk, Grossi, P. and Kunreuther, H., Chapter 5.

**Commentary on Question:**
*This question tests the candidate's understanding of basic ratemaking, including the application of a loading for catastrophes in ratemaking.*

**Solution:**
(a)    Explain why two trend adjustments must be made to the modeled expected catastrophe claims in order to calculate the catastrophe loading for ratemaking.

Past adjustment: modeled catastrophe claims must be trended from September 1, 2014 to July 1, 2015 for exposure trend to reflect in-force exposures as of July 1, 2015.

Future adjustment: modeled catastrophe claims must be trended from July 1, 2015 to midpoint of future rating period for severity trend to reflect the cost level in the future rating period.

**4.** **Continued**

(b) Calculate the catastrophe loading to be used for ratemaking, as a claim ratio.

| | |
|---|---|
| Midpoint of future rating period | April 1, 2018 |
| Exposure trend period (months): September 1, 2014 to July 1, 2015 | 10 |
| Exposure trend $= 1.02^{(10/12)}$ | 1.017 |
| Severity trend period (months): July 1, 2015 to April 1, 2018 | 33 |
| Severity trend $= 1.04^{(33/12)}$ | 1.114 |
| Trended modeled catastrophe claims $= 150 \times 1.017 \times 1.114$ | 169.94 |
| Catastrophe loading $= 169.94 / 9,440$ | 1.80% |

(c) Calculate the indicated rate level change.

Using formula 31.8:
Total claim ratio including ULAE $= (0.64 + 0.018) \times 1.12 = 0.737$

$$\frac{ECR + \dfrac{F}{R_C}}{1 - V - Q} - 1 = \frac{0.737 + 0.07}{1 - 0.20 - 0.04} - 1 = 6.18\%$$

(d) Calculate the profit and contingencies to premium ratio implied by increasing rates by 2%.

$$\frac{ECR + \dfrac{F}{R_C}}{1 - V - Q} - 1 = 2\% \qquad \rightarrow Q = 1 - V - \frac{ECR + \dfrac{F}{R_C}}{1.02} = 1 - 0.2 - \frac{0.737 + 0.07}{1.02} = 0.88\%$$

(e) Describe an approach that would increase your confidence in the estimate of expected earthquake claims.

Running alternative catastrophe models would increase confidence in the estimate of expected earthquake claims.

(f) State two measures from the earthquake model that could be used to determine a risk load.

Calculating the standard deviation.

Getting a percentile from the exceedance curve.

**5.**

**Learning Objectives:**
3.    The candidate will understand financial reporting of claim liabilities and premium liabilities.

**Learning Outcomes:**
(3a)    Describe the key assumptions underlying ratio and count-based methods for estimating unpaid unallocated loss adjustment expenses.

(3b)    Estimate unpaid unallocated loss adjustment expenses using ratio and count-based methods.

**Sources:**
Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 22.

**Commentary on Question:**
*This question tests the candidate's understanding of estimating unpaid ULAE using two different projection methods.*

**Solution:**
(a)    Estimate the unpaid ULAE as of December 31, 2015 using a paid-to-paid method.

| Calendar Year | (1) Paid ULAE | (2) Paid Claims | (3)=(1)/(2) Paid ULAE/ Paid Claims |
|---|---|---|---|
| 2013 | 71,520 | 765,000 | 9.3% |
| 2014 | 55,420 | 635,000 | 8.7% |
| 2015 | 35,630 | 450,000 | 7.9% |
| | | 3 year average: | 8.6% |

| | (4) Unpaid Claims | (5) Multiplier | (6) Selected Paid-to-Paid Ratio | (7)=(4)(5)(6) Unpaid ULAE |
|---|---|---|---|---|
| IBNR Reserves | 101,250 | 100% | 8.6% | 8,708 |
| Case Reserves | 270,000 | 70% | 8.6% | 16,254 |
| Total Reserves | 371,250 | | | 24,962 |

**5. Continued**

(b) Estimate the unpaid ULAE as of December 31, 2015 using the Wendy Johnson count-based method.

Historical Counts

| Calendar Year | (1) Paid ULAE | (2) Newly Reported During the Year | (3) Open at End of the Year | (4) Closed During the Year | (5) Weighted Total | (6)=(1)/(5) Average ULAE per Weighted Count |
|---|---|---|---|---|---|---|
| 2013 | 71,520 | 86 | 43 | 93 | 60.9* | 1,174 |
| 2014 | 55,420 | 67 | 32 | 78 | 47.1 | 1,177 |
| 2015 | 35,630 | 44 | 19 | 57 | 30.3 | 1,176 |
| | | | | | 3 year average: | 1,176 |
| | Weights | 30% | 60% | 10% | | |

* e.g., $60.9 = 86 \times 0.3 + 43 \times 0.6 + 93 \times 0.1$

Forecast Counts

| Calendar Year | (7) Newly Reported During the Year | (8) Open at End of the Year | (9) Closed During the Year | (10) Weighted Total | (11) = (6)$_{Total}$×(10) Estimated Unpaid ULAE |
|---|---|---|---|---|---|
| 2016 | 19 | 7 | 31 | 13.0 | 15,288 |
| 2017 | 6 | 1 | 12 | 3.6 | 4,234 |
| 2018 | 1 | 0 | 2 | 0.5 | 588 |
| | | | | | 20,110 |

(c) Recommend the unpaid ULAE from either part (a) or (b) to management. Justify your recommendation.

**Commentary on Question:**
*Other justifications may be acceptable.*

Recommend the part (b) count-based method:
- This method provides a more accurate/detailed analysis of costs based on type of transaction.
- Exposure is declining significantly (near run-off state) and ratio method is not appropriate in the case of significantly changing exposure.

**6.** **Learning Objectives:**
2.      The candidate will understand how to calculate projected ultimate claims and claims-related expenses.

**Learning Outcomes:**
(2d)    Explain the effect of changing conditions on the projection methods cited in (2b).

**Sources:**
Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 20.

**Commentary on Question:**
*This question tests the candidate's understanding of changing conditions on age-to-age development factors and different projection methods.*

**Solution:**
(a)     Describe what effect the change in claim department processing would have had on the reported claim age-to-age development factors.

        The case reserve strengthening will increase age-to-age development factors in recent evaluations and decrease age-to-age development factors in later evaluations, following a diagonal of increased development factors at all evaluations when the change occurred (i.e., three years ago).

(b)     Explain which of the following two methods is likely to produce a more accurate estimate of ultimate claims in recent accident years under this scenario:

        (i)     the reported Bornhuetter Ferguson method, or

        (ii)    the reported Cape Cod method.

        The Bornhuetter Ferguson method will be more accurate than the Cape Cod method.  Both methods will reflect similar distortions from higher age-to-age development factors, but the Cape Cod method will also reflect the higher reported claims in its expected claim ratio which will increase the distortion even more than the Bornhuetter Ferguson method.

(c)     Describe what effect an increase in policy limits would have had on the reported claim age-to-age development factors.

        Age-to-age development factors would increase for the latest three accident years.

(d)     Describe an approach to selecting an appropriate tail factor under this scenario.

        Use industry data (benchmarks).

**7.**  **Learning Objectives:**
5.	The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

**Learning Outcomes:**
(5j)	Perform individual risk rating using standard plans.
(5k)	Calculate rates for claims-made coverage.

**Sources:**
Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapters 34 and 35.

**Commentary on Question:**
*This question tests the candidate's understanding of individual risk rating.*

**Solution:**
(a)	Explain which coverage would be preferable for BOO: claims-made or occurrence.

Claims-made coverage would be preferable because there would be no coverage gap.  Note that prior acts coverage would be needed to avoid a coverage gap.

(b)	Demonstrate, with a hypothetical numerical example, how individual designer claims experience and exposure can be used to allocate the 1,000,000 to designers within the East region.

**Commentary on Question:**
*There are other examples that can be used.*

Assume two designers and allocation of the full 1,000,000.  The allocation should consider a balance of experience and exposure.  One designer has twice as much exposure and exposure gets a 60% weight.  Experience gets 40% weight and both are equal.  Thus, the allocation is 600,000 for the larger firm and 400,000 for the smaller.

(c)	Describe how an actuary can use credibility in the allocation procedure in part (b).

Credibility can be applied to either experience or exposure.  The larger firm's experience could be given greater credibility than the smaller firm's experience.

**7.** **Continued**

(d)     Provide an argument to support the same pure premium for all regions.

       **Commentary on Question:**
*Other answers are possible.*

       One year of experience is not enough to change the initial assumption.

(e)     Provide an argument to support different regional pure premiums.

       **Commentary on Question:**
*Other answers are possible.*

       The large disparity in first-year claims should be given some credibility.

**8.** **Learning Objectives:**
3. The candidate will understand financial reporting of claim liabilities and premium liabilities.

**Learning Outcomes:**
(3e) Describe the components of premium liabilities in the context of financial reporting.
(3f) Evaluate premium liabilities.

**Sources:**
Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 24.

**Commentary on Question:**
*This question tests the determination of premium liabilities.*

**Solution:**
(a) Calculate the unearned premium as of December 31, 2015.

| | | (1) | (2) Deposit Premium | (3) Locations | (4) = (2)×1.1 Ultimate Premium | (5) Unearned Years | (6) = (4)(5) Unearned Premium |
|---|---|---|---|---|---|---|---|
| Policy | Inception | | | | | | |
| A | Apr. 1, 2015 | | 24,000 | 30 | 26,400 | 0.25 | 6,600 |
| B | Jul. 1, 2015 | | 40,000 | 50 | 44,000 | 0.50 | 22,000 |
| C | Oct. 1, 2015 | | 36,000 | 40 | 39,600 | 0.75 | 29,700 |
| Total | | | 100,000 | | 110,000 | | 58,300 |

(b) Calculate the premium deficiency reserve or equity in the unearned premium as of December 31, 2015.

| | (7) = 600×(3)×(5) |
|---|---|
| Policy | Expected Claims |
| A | 4,500 |
| B | 15,000 |
| C | 18,000 |
| Total | 37,500 |

| | |
|---|---|
| (8) Expected ULAE [(7)$_{total}$ × 20%] | 7,500 |
| (9) Maintenance Expenses [(6)$_{total}$ × 5%] | 2,915 |
| (10) Total claims and expenses = (7)$_{total}$ + (8) + (9) | 47,915 |
| (11) Equity in unearned premium [(6)$_{total}$ – (10)] | 10,385 |

**8.** **Continued**

(c)  Calculate the reported deferred policy acquisition expense (DPAE) as of December 31, 2015.

| | |
|---|---:|
| Deferred acquisition expenses as % unearned premium | 15% |
| DPAE = UPR × Acquisition Cost as % Premiums = $(6)_{total}$ × 15% | 8,745 |
| Reported DPAE = Min(DPAE, Equity) | |
| = Min(8,745, 10,385) | 8,745 |

(d)  Recalculate the premium deficiency reserve or equity in the unearned premium as of December 31, 2015, allowing for this legislative change.

| Policy | (5) Unearned Years | (6) Unearned Premium | (12) Pre-Change Unearned Years | (13) = (5) – (12) Post-Change Unearned Years | (14) Expected Claims |
|---|---|---|---|---|---|
| A | 0.25 | 6,600 | 0.25 | 0.00 | 4,500 |
| B | 0.50 | 22,000 | 0.25 | 0.25 | 18,750 |
| C | 0.75 | 29,700 | 0.25 | 0.50 | 24,000 |
| Total | | 58,300 | | | 47,250 |

Note: (14) = 600×(3)×{(12) + (13)×[1 + 0.50]}

| | |
|---|---:|
| (15) Expected ULAE [$(14)_{total}$ × 20%] | 9,450 |
| (16) Maintenance Expenses [$(6)_{total}$ × 5%] | 2,915 |
| (17) Total claims and expenses = $(14)_{total}$ + (15) + (16) | 59,615 |
| (18) Premium deficiency reserve [$(17)$ – $(6)_{total}$] | 1,315 |
| Note: Premium deficiency reserve since (17) > $(6)_{total}$ | |

(e)  Recalculate the reported DPAE as of December 31, 2015.

| | |
|---|---:|
| DPAE = UPR × Acquisition Cost as % Premiums = $(6)_{total}$ × 15% | 8,745 |
| Reported DPAE = Min(DPAE, Equity) = Min(8,745, 0) | 0 |
| Note: Equity = 0 since there is a premium deficiency | |

**8.** **Continued**

(f) Explain, but do not calculate, how the following expected changes during 2016 will affect your calculations of year-end 2015 premium liabilities:

   (i) Premiums will be increased by 30% upon renewal.

   (ii) Claims department expenses are expected to fall by 20% in 2016.

   (i) Change in future renewal premiums will not have direct impact on policy liabilities for existing policies.

   (ii) Expected decrease in 2016 claims department expenses should be reflected in ULAE factor used to estimate costs for unexpired risk period of policies in force at year-end 2015.

**9.** **Learning Objectives:**
5.    The candidate will understand how to apply the fundamental ratemaking
      techniques of general insurance.

**Learning Outcomes:**
(5k)   Calculate rates for claims-made coverage.

**Sources:**
Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 34.

**Commentary on Question:**
*This question tests the candidate's understanding of claims-made ratemaking.*

**Solution:**
(a)   State two lines of business for which claims-made coverage is prevalent.

      **Commentary on Question:**
      *Other answers are possible.*

      - Legal professional liability
      - General liability

(b)   State one reason why claims-made is a preferred form of coverage for the lines of
      business in part (a) from the insurer's perspective.

      **Commentary on Question:**
      *Either of the following are acceptable.*

      - Often claims do not emerge for years after the date of occurrence.
      - Pricing can be more accurate with claims-made coverage.

(c)   Describe two methods of individual risk rating that can apply to the rating of
      small groups of physicians.

      Schedule rating depending upon such items as office staffing, cleanliness, risk
      management, etc.

      Prospective experience rating based on prior claims history of the small group.

**9.**  **Continued**

    (d)    Calculate the following items:

        (i)      The ratio of the mature claims-made rate to the occurrence rate

        (ii)     The tail factor for mature claims-made coverage

        (iii)    The tail factor for a new physician who plans to change careers after three years of claims-made coverage

        (i)      The ratio of the mature claims-made rate to the occurrence rate = 1.0

        (ii)     Mature claims-made tail factor = (1+…+9)/10 = 4.5

        (iii)    Third-year claims-made tail factor = (9+8+7)/3 = 8.0

    (e)    Explain how a coverage gap can exist when the insured switches:

        (i)      Between claims-made insurers

        (ii)     From claims-made to occurrence coverage

        (iii)    Between occurrence insurers

        (iv)    From occurrence to claims-made coverage

        (i)      Claims-made to claims-made: a gap can occur if there is no tail policy purchased from the prior insurer and no nose coverage purchased from the new insurer.

        (ii)     Claims-made to occurrence: a gap can occur as the tail of claims-made is not covered by occurrence coverage unless it is purchased separately.

        (iii)    Occurrence to occurrence: a gap can occur if there is a timing issue between expiration date of prior policy and effective date of new policy.

        (iv)    Occurrence to claims-made: usually no issue, unless there is a timing issue between expiration date of the prior policy and effective date of new policy.

**10.** **Learning Objectives:**
2.  The candidate will understand how to calculate projected ultimate claims and claims-related expenses.

3.  The candidate will understand financial reporting of claim liabilities and premium liabilities.

**Learning Outcomes:**
(2b)  Estimate ultimate claims using various methods: development method, expected method, Bornhuetter Ferguson method, Cape Cod method, frequency-severity methods, Berquist-Sherman methods.
(3d)  Evaluate the estimates of ultimate claims to determine claim liabilities for financial reporting.

**Sources:**
Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapters 14 and 23.

**Commentary on Question:**
*This question tests the development method of estimating ultimate claims, as well as determining claim liabilities.*

**Solution:**
(a)  Define downward development.

Downward development is represented by age-to-age development factors less than 1.

(b)  Identify two reasons why downward development may occur.

**Commentary on Question:**
*Any two of the following are acceptable.*

- Claims settle for less than anticipated in case estimates
- There have been recoveries
- Miscoded or incorrect data

# 10. Continued

(c)    Select age-to-age factors for 6-12 months and 12-18 months and justify your
       selection.

|                                         | 6-12  | 12-18 |
|-----------------------------------------|-------|-------|
| Average age-to-age development factors: |       |       |
| Simple All-1                            | 1.054 | 0.989 |
| Simple All-2                            | 1.104 | 0.988 |
| Simple All                             |       | 0.989 |
| Selected                                | 1.104 | 0.989 |

For the period 6-12, use the simple average of 2012-2, 2013-2, 2014-2, i.e. 1.104
(use the second halves of the years because the projection for the latest period is
for 2015-2).

For the period 12-18, use 0.989.  A simple average of all factors is appropriate
given the stability in the numbers and lack of seasonality shown.

(d)    Explain why an analysis of frequency and severity might be appropriate in this
       case.

To test for the influence of seasonality in trended severities and test for the
influence of seasonality in trended frequencies.

(e)    Calculate the ultimate claims for accident year 2015.

|                      | 6-12  | 12-18 | 18-24 |
|----------------------|-------|-------|-------|
| Development factors: |       |       |       |
| Selected Age-to-Age  | 1.104 | 0.989 | 1.001 |
| Age-to-Ultimate      | 1.093 | 0.990 | 1.001 |

|                     | (1)    | (2)      | (3)      | (4) = (2)(3) |
|                     |        |          | Age-to-  |              |
|                     | Paid   | Reported | Ultimate | Ultimate     |
| Accident Half-Year  | Claims | Claims   | Factor   | Claims       |
|---------------------|--------|----------|----------|--------------|
| 2015-1              | 20,100 | 24,500   | 0.990    | 24,255       |
| 2015-2              | 15,700 | 22,100   | 1.093    | 24,155       |
|                     |        |          |          | 48,410       |

# 10. Continued

(f)     Calculate the accident year 2015 unpaid claims using the ultimate claims calculated in part (e), showing the case estimate and indicated IBNR separately.

| Accident Half-Year | (5) = (4) – (1) Unpaid Claims | (6) = (2) – (1) Case Estimate | (7) = (5) – (6) IBNR |
|---|---|---|---|
| 2015-1 | 4,155 | 4,400 | –245 |
| 2015-2 | 8,455 | 6,400 | 2,055 |
| | 12,610 | 10,800 | 1,810 |

# 11.

**Learning Objectives:**

2. The candidate will understand how to calculate projected ultimate claims and claims-related expenses.

**Learning Outcomes:**

(2b) Estimate ultimate claims using various methods: development method, expected method, Bornhuetter Ferguson method, Cape Cod method, frequency-severity methods, Berquist-Sherman methods.

**Sources:**

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 19.

**Commentary on Question:**

*This question tests the candidate's ability to estimate ultimate claims using Berquist-Sherman adjustments when there has been a change in settlement rates.*

**Solution:**

(a) Calculate the disposal ratio triangle.

Disposal ratios = closed counts / selected ultimate counts

| Accident Year | 12 | 24 | 36 |
|---------------|-----|-----|-----|
| 2013 | 40% | 50% | 90% |
| 2014 | 35% | 45% | |
| 2015 | 30% | | |

(b) Explain how this disposal ratio triangle does or does not provide evidence of delays in claims processing.

There is a decrease down each column that confirms evidence of delays in claims processing.

## 11. Continued

(c) Calculate the adjusted paid claims triangle.

| Accident Year | 12 | 24 | 36 |
|---|---|---|---|
| Selected disposal ratios: | 30% | 45% | 90% |

Adjusted closed counts = (disposal ratio)(selected ultimate counts)

| | | | |
|---|---|---|---|
| 2013 | 60 | 90 | 180 |
| 2014 | 72 | 108 | |
| 2015 | 101 | | |

| | |
|---|---|
| Severity trend: | 5% |

Average claim cost (e.g., 104.76 = 110 / 1.05)

| | | | |
|---|---|---|---|
| 2013 | 99.77 | 99.77 | 99.77 |
| 2014 | 104.76 | 104.76 | |
| 2015 | 110.00 | | |

Adjusted Paid Claims = (Adjusted closed counts)(Average claim cost)

| | | | |
|---|---|---|---|
| 2013 | 5,986 | 8,979 | 17,959 |
| 2014 | 7,543 | 11,314 | |
| 2015 | 11,110 | | |

(d) Explain whether or not a Berquist-Sherman adjustment would be appropriate for a new line of business.

It would not be appropriate as Berquist-Sherman adjustments depend on a significant volume of credible experience.

**12.** **Learning Objectives:**

1. The candidate will understand the key considerations for general insurance actuarial analysis.

**Learning Outcomes:**
(1k) Estimate written, earned and unearned premiums.
(1l) Adjust historical earned premiums to current rate levels.

**Sources:**
Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapters 11 and 12.

**Commentary on Question:**
*This question tests the candidate's understanding of data patterns related to earned exposures and earned premiums. The candidate also needs to understand earned premiums adjusted to current rate level.*

**Solution:**
(a) Identify two possible anomalies related to earned vehicles.

The number of earned vehicles increased significantly in 2012 and then decreased the next year.

The number of earned vehicles increased significantly in 2015.

(b) Provide a likely explanation for each anomaly identified in part (a).

Calendar year 2012 – This may well be a data error for 2012 given that trend reverts back to where it had been.

Calendar year 2015 – Check whether company was planning on increasing sales or whether it is a data error.

**12.** **Continued**

(c) Calculate the average on-level earned premiums for each calendar year.

| Calendar Year (CY) | (1) Earned Vehicles | (2) Earned Premiums (000) | (3) Premium On-Level Factor to CY 2015 | (4) = (2)(3) / (1) Average On-Level Earned Premium |
|---|---|---|---|---|
| 2010 | 60,700 | 17,250 | 1.070 | 304 |
| 2011 | 60,900 | 17,500 | 1.050 | 302 |
| 2012 | 64,800 | 18,900 | 1.036 | 302 |
| 2013 | 61,100 | 18,900 | 1.023 | 316 |
| 2014 | 61,300 | 19,200 | 1.010 | 316 |
| 2015 | 63,600 | 20,100 | 1.000 | 316 |

(d) Identify a possible anomaly related to the average on-level earned premiums calculated in part (c).

Calendar years 2010 – 2012 at a lower level than calendar years 2013 – 2015.

(e) Provide two likely explanations for the anomaly identified in part (d).

There is a possible shift in type of insureds (i.e., mix of business).

There are possible changes in policy deductibles and limits.

**13.** **Learning Objectives:**

4. The candidate will understand trending procedures as applied to ultimate claims, exposures and premiums.

5. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

**Learning Outcomes:**
(4c) Choose trend rates and calculate trend factors for claims.
(4e) Choose trend rates and calculate trend factors for exposures.
(5f) Calculate overall rate change indications under the claims ratio and pure premium methods.

**Sources:**
Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 25, 26 and 31.

**Commentary on Question:**
*This question tests the candidate's understanding of basic ratemaking, including making claim and premium trend adjustments.*

**Solution:**
(a) Explain whether an increase or decrease in profit for this line of business should be expected over the next three years under the base scenario.

- Claims trend $= (1 - 0.01)(1 + 0.07) - 1 = 5.93\%$
- Premium trend $= 2\%$
- Since claims trend is greater than premium trend, claims will grow faster than premium in the future if there are no premium increases, thereby suggesting a decrease in profit over the next three years.

(b) Calculate the forecasted profit for calendar year 2018.

|   |   |   |
|---|---|---:|
| (1) | Premium $= 1{,}900 \times 6{,}760 \times (1 + 0.02)^3$ | 13,630,156 |
| (2) | Fixed expenses $= 60 \times 6{,}760$ | 405,600 |
| (3) | Variable expenses $= (1) \times 0.12 = 13{,}630{,}156 \times 0.12$ | 1,635,619 |
| (4) | Claims $=$ Exposures $\times$ Trended frequency $\times$ Trended severity $\times\,(1 + \text{ULAE})\ = 6{,}760 \times 0.08 \times (1 - 0.01)^3 \times 17{,}000$ $\times\,(1 + 0.07)^3 \times (1 + 0.05)$ | 11,474,448 |
| (5) | Profit $= (1) - (2) - (3) - (4)$ | 114,489 |

# 13. Continued

(c)    State two actions the company can take in order to increase profits in 2018.

**Commentary on Question:**
*Any two of the following are acceptable.*

- Increase rates
- Decrease expenses
- Decrease claims (e.g., changing mix of business, better risk selection)

# 14.

**Learning Objectives:**

2. The candidate will understand how to calculate projected ultimate claims and claims-related expenses.

3. The candidate will understand financial reporting of claim liabilities and premium liabilities.

4. The candidate will understand trending procedures as applied to ultimate claims, exposures and premiums.

6. The candidate will understand the need for monitoring results.


**Learning Outcomes:**

(2b) Estimate ultimate claims using various methods: development method, expected method, Bornhuetter Ferguson method, Cape Cod method, frequency-severity methods, Berquist-Sherman methods.

(3d) Evaluate the estimates of ultimate claims to determine claim liabilities for financial reporting.

(4a) Identify the time periods associated with trending procedures.

(4c) Choose trend rates and calculate trend factors for claims.

(6b) Analyze actual claims experience relative to expectations.

**Sources:**

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapters 15, 23, 25, and 36.

**Commentary on Question:**

*This question tests the development-based frequency-severity method of estimating ultimate claims, as well as the candidate's understanding of comparing actual vs. expected claims.*

## 14. Continued

**Solution:**

(a)     Estimate the ultimate claims for accident years 2014 and 2015.

| Accident Year (AY) | (1) Projected Ultimate Severity based on Development Method | (2) = $(1.05)^{(2015-AY)}$ Severity Trend at 5.0% | (3) = (1)(2) Trended Severity |
|---|---|---|---|
| 2011 | 7,800 | 1.2155 | 9,481 |
| 2012 | 8,400 | 1.1576 | 9,724 |
| 2013 | 8,900 | 1.1025 | 9,812 |
| 2014 | 9,200 | 1.0500 | 9,660 |
| 2015 | 8,500 | 1.0000 | 8,500 |
| Average 2011-2014 | | | 9,669 |
| Selected Severity at 2015 cost level | | | 9,669 |

| Accident Year (AY) | (4) = 9,669 / (2) Selected Ultimate Severity | (5) Ultimate Counts | (6) = (4)(5) Ultimate Claims |
|---|---|---|---|
| 2011 | | | |
| 2012 | | | |
| 2013 | | | |
| 2014 | 9,209 | 1,300 | 11,971,529 |
| 2015 | 9,669 | 1,200 | 11,603,175 |

(b)     Explain why the frequency-severity method may not provide an appropriate selection of ultimate claims for accident years 2011 through 2013.

**Commentary on Question:**
*Other explanations are possible.*

These are more mature periods and the frequency-severity method is typically used for immature periods.

# 14. Continued

(c) Calculate the difference between the actual and expected reported claims from December 31, 2015 through March 31, 2016 for accident year 2015, using linear interpolation.

Expected reported % as of March 31, 2016 = 0.75×77% + 0.25×87% = 79.50%

$$\text{Expected reported} = \left(11{,}603{,}175 - 9{,}500{,}000\right)\frac{\left(79.5\% - 77\%\right)}{\left(1 - 77\%\right)} = 228{,}606$$

Actual reported = 9,800,000 – 9,500,000 = 300,000
Difference = 300,000 – 228,606 = 71,394

(d) Describe what the answer to part (c) implies about the selection of the development-based frequency-severity method as the basis for the estimate of ultimate claims.

It implies that the development-based frequency-severity method may understate the ultimate claims (the implied ultimate from development method = 9,500,000 / 0.77 = 12,337,662, which is much higher than the frequency-severity estimate of 11,603,175).

## 15.

**Learning Objectives:**

2. The candidate will understand how to calculate projected ultimate claims and claims-related expenses.

**Learning Outcomes:**

(2b) Estimate ultimate claims using various methods: development method, expected method, Bornhuetter Ferguson method, Cape Cod method, frequency-severity methods, Berquist-Sherman methods.

(2e) Assess the appropriateness of the projection methods cited in (2b) in varying circumstances.

**Sources:**

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapters 14, 17, 18, 19, 20, and 21.

**Commentary on Question:**

*This question tests the candidate's understanding of the effect of changing conditions on different projection methods and the evaluation and selection of estimated ultimate claims under changing conditions.*

**Solution:**

(a) State one weakness of the Benktander method.

There is no clear guidance with respect to the appropriate number of iterations to perform.

(b) Explain the challenge reinsurers face when using the Cape Cod method to estimate ultimate claims.

The Cape Cod method relies on on-level earned premiums by year. However, for reinsurers, detailed information about premium rate changes is not always available.

(c) Describe two situations where Berquist-Sherman adjustments are most commonly implemented.

- Where there has been a change in case reserve adequacy.
- Where there has been a change in claim settlement pattern.

## 15. Continued

(d)     Provide the following for accident years 2010, 2011 and 2015 separately:

    (i)     The column letter from the table above that corresponds to your selection of ultimate claims, and

    (ii)    Justification for your selection.

**Commentary on Question:**
*Other answers may be acceptable if proper justification for the selection is provided.*

Accident year 2010 – (C) Reported Bornhuetter Ferguson method because it makes the most use of current case information without a distortion from the large open claim.

Accident year 2011 – (A) Reported development method because it makes the most use of current case information and there is nothing unusual in this period at this development age.

Accident year 2015 – (B) Paid development method because paid methods are not distorted by the change in case reserve adequacy, but will be responsive to the claim deterioration in recent years.

**16.** **Learning Objectives:**
5.     The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

**Learning Outcomes:**
(5b)     Calculate expenses used in ratemaking analyses including expense trending procedures.

**Sources:**
Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 29.

**Commentary on Question:**
*This question tests the candidate's understanding of the expenses for ratemaking, including adjusting fixed expenses for trend.*

**Solution:**
(a)     Identify two sources of data that an insurer can use in determining expenses for ratemaking.

- Insurer historical expenses
- Budgeted or planned expenses

(b)     Describe when it is appropriate to use each source identified in part (a).

Insurer historical expenses can be used if historical values are representative of future expenses. The actuary should consider whether trending procedures are required to adjust historical expenses to reflect the cost level anticipated for the forecast period.

Budgeted or planned expenses can be used during times of change such as the implementation of a major IT system or the introduction of a new commission structure. Budgeted expenses would also be necessary when pricing a new line of business.

# 16. Continued

(c)  Recommend a fixed expense ratio as a percentage of premiums to be used for ratemaking purposes. Justify your recommendation.

| | (1) | (2) | (3) |
|---|---|---|---|
| | Average Incurred Date | | Trending |
| | Experience | Forecast | Period in |
| Half Year | Period | Period | Months |
| 2015-1 | Apr. 1, 2015 | Apr. 1, 2018 | 36 |
| 2015-2 | Oct. 1, 2015 | Apr. 1, 2018 | 30 |
| 2016-1 | Apr. 1, 2016 | Apr. 1, 2018 | 24 |
| 2016-2 | Oct. 1, 2015 | Apr. 1, 2018 | 18 |

| | (4) | (5) | (6) | (7) = (4)(5)/(6) |
|---|---|---|---|---|
| | | | Trended | |
| | | Fixed | Earned | Trended Ratio |
| | | Expenses | Premium at | Fixed |
| | Fixed | Trend at | Current Rate | Expenses to EP |
| Half Year | Expenses | 1.25% | Level | at CRL |
| 2015-1 | 110,000 | 1.038 | 1,692,000 | 6.75% |
| 2015-2 | 125,000 | 1.032* | 1,667,000 | 7.74% |
| 2016-1 | 119,000 | 1.025 | 1,750,000 | 6.97% |
| 2016-2 | 122,000 | 1.019 | 1,768,000 | 7.03% |

* e.g., $1.032 = 1.0125^{(30/12)}$

Recommend ratio of 7%.
Rationale: the most recent two periods are similar so 7.0% is expected to be more reflective of future.

**17.** **Learning Objectives:**
7.   The candidate will understand the nature and application of catastrophe models used to manage risks from natural disasters.

**Learning Outcomes:**
(7a)   Describe the structure of catastrophe models.
(7b)   Apply catastrophe models to insurance ratemaking, portfolio management, and risk financing.

**Sources:**
Catastrophe Modeling: A New Approach to Managing Risk, Grossi, P. and Kunreuther, H., Chapter 4.

**Commentary on Question:**
*This questions tests the candidate's understanding of epistemic uncertainty with respect to catastrophe modeling.*

**Solution:**
(a)   Provide, for each module, either:

(i)   An explanation of why it may not be possible to reduce epistemic uncertainty by collecting more data, or

(ii)   A hurricane or earthquake example of how collecting more data may reduce epistemic uncertainty.

Hazard: Not possible.  For a given region historical data is likely available and there is no more additional data.
Hazard: Possible.  One can get more information about geology, topography, and soil for earthquake models.

Inventory: Not possible.  The structures are set, so there cannot be any more structures.
Inventory: Possible.  One can add structures if current model is incomplete (applies to any catastrophe type).

Vulnerability: Possible.  More data can be obtained relating damage to structure and hazard.

Loss: Possible.  More information can be collected with regard to repair and business interruption costs (applies to any catastrophe type).

# 17. Continued

(b)     Provide, for each module, either:

(i)     An explanation of why it may not be possible to reduce epistemic uncertainty by improving the accuracy of the data or model, or

(ii)    A hurricane or earthquake example of how improving the accuracy of the data or model may reduce epistemic uncertainty.

Hazard: Possible.  There are several ways to use the data for earthquake models, such as magnitude-frequency and characteristic models.  There is always room for improvement; for example, one can improve accuracy and modeling of geology, topography, and soil for earthquake models.

Inventory: Possible.  More accurate information can be obtained on construction type, age, height, value, and location (applies to any catastrophe type).

Vulnerability: Possible.  There is not full understanding of how the nature of the hurricane/earthquake combines with the structure details to produce damage. There can be better models constructed.

Loss: Possible.  Better models can be constructed to estimate demand surge and indirect costs related to business interruption (applies to any catastrophe type).

**18.** **Learning Objectives:**
  5.    The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

**Learning Outcomes:**
(5g)   Calculate risk classification changes and territorial changes.

**Sources:**
Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 32.

**Commentary on Question:**
*This question tests the candidate's understanding of risk classification.*

**Solution:**
(a)    Explain how a minimum bias method improves the accuracy of the risk classification analysis.

Minimum bias improves accuracy by ensuring balance for each risk class within a risk characteristic.

(b)    Determine whether or not there is distributional bias between these risk territories and classes.

To test for distributional bias, need to check for consistent exposure ratios across each dimension:

| Ratio of Class 1 to Class 2: | | Ratio of Territory A to Territory B: | |
|---|---|---|---|
| Territory | Exposures | Class | Exposures |
| A | 0.667 | 1 | 0.833 |
| B | 0.667 | 2 | 0.833 |

There is no distributional bias since ratios are the same for each territory and each class.

# 18. Continued

(c) Determine whether or not there is dependence between these risk territories and classes.

To test for dependence, need to check for consistent pure premium ratios across each dimension:

| Ratio of Class 1 to Class 2: | | | Ratio of Territory A to Territory B: | |
|---|---|---|---|---|
| Territory | Pure Premium | | Class | Pure Premium |
| A | 0.800 | | 1 | 1.200 |
| B | 1.250 | | 2 | 1.875 |

There is dependence since ratios are not the same for each territory (or each class).

(d) Recommend an approach that will best replicate observed pure premiums for this case. Justify your recommendation.

Since both the one-way and the minimum bias methods will not replicate the observed pure premiums, a model that reflects dependence such as a GLM is needed.

(e) Explain how the use of relativities from the one-way analysis instead of relativities from the minimum bias analysis creates the potential for adverse selection.

- All level A pure premiums from minimum bias are *higher* than the one-way pure premiums.
- All level C pure premiums from minimum bias are *lower* than the one-way pure premiums.
- This would create a cross-subsidy between level A and level C risks.

(f) Critique your colleague's recommendation.

A lower absolute error suggests increased precision, but in practice the actuary would consider more than one measure before accepting or rejecting a classification system.

# 19.

**Learning Objectives:**

5.  The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

**Learning Outcomes:**

(5h)    Calculate deductible factors, increased limits factors, and coinsurance penalties.

**Sources:**

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 33.

**Commentary on Question:**

*This question tests the candidate's understanding of how policy limits are applied with allocated loss adjustment expenses, as well as how to calculate increased limits factors with censored data.*

**Solution:**

(a)    Calculate the amount the insurer would pay if the policy limit applies only to indemnity.

ALAE is not included in policy limit, so insurer pays the amount of the indemnity limited to 500,000, plus ALAE = 500,000 + 150,000 = 650,000.

(b)    State why an insurer would want ALAE included in the policy limit.

**Commentary on Question:**

*Either one of the following is acceptable.*

- When ALAE is a large portion of total costs
- When there are increasing legal costs

**19.** **Continued**

(c)     Calculate the limited average severity for the 100,000 to 250,000 layer.

*Limited Average Severity (LAS) in the layer 100,000 to 250,000:*

| Claim Range | | | Counts | Capped Claims | 100,000 to 250,000 Layer |
|---|---|---|---|---|---|
| *250,000 Limit Policies* | | | | | |
| 0 | – | 100,000 | 820 | 53,300,000 | 0 |
| 100,000 | – | 250,000 | 1,070 | 171,200,000 | 64,200,000 |
| | | | | | |
| *500,000 Limit Policies* | | | | | |
| 0 | – | 100,000 | 870 | 53,070,000 | 0 |
| 100,000 | – | 250,000 | 630 | 95,760,000 | 32,760,000 |
| 250,000 | – | 500,000 | 270 | 89,100,000 | 40,500,000 |
| | | | | | |
| Total (in layer) | | | 1,970 | | 137,460,000 |
| LAS for the layer, given a claim in the layer | | | | | 69,777 |

Probability of a claim in the layer =     $\dfrac{1,970}{3,660}$ =     0.5383

LAS for the 100,000 to 250,000 layer:     37,557

Notes:  64,200,000 = 171,200,000 – 100,000×1,070
   32,760,000 = 95,760,000 – 100,000×630
   40,500,000 = 150,000×270
   69,777 = Total Amount in Layer / Counts in Layer
   37,557 = LAS, given claim in layer × Probability in layer

*Alternatively*, $37,557 = \dfrac{137,460,000}{3,660}$

# 19. Continued

(d)     Calculate the increased limit factor at the 500,000 limit, assuming the basic limit of liability is 100,000.

First, need to calculate the LAS(100,000):

| Claim Range | | | Counts | Capped Claims |
|---|---|---|---|---|
| 0 | – | 100,000 | 3,690 | 244,370,000 |
| 100,000+ | | | 1,970 | 197,000,000 |
| | | | 5,660 | 441,370,000 |
| LAS(100,000) = 441,370,000 / 5,660 = | | | | 77,981 |

*LAS in the 250,000 to 500,000 layer:*

| Claim Range | | | Counts | Capped Claims | 250,000 to 500,000 Layer |
|---|---|---|---|---|---|
| *500,000 Limit Policies* | | | | | |
| 0 | – | 100,000 | 870 | 53,070,000 | 0 |
| 100,000 | – | 250,000 | 630 | 95,760,000 | 0 |
| 250,000 | – | 500,000 | 270 | 89,100,000 | 21,600,000 |
| | | | 1,770 | | 21,600,000 |

| LAS for the layer: | = 21,600,000 / 1,770 = | 12,203 |
|---|---|---|

LAS(500,000) = LAS(100,000) + LAS for the 100,000 to 250,000 Layer + LAS for the 250,000 to 500,000 layer
         = 77,981 + 37,557 + 12,203 = 127,741

ILF(500,000) = LAS(500,000) / LAS(100,000) = 127,741 / 77,981 = 1.638