GI ADV Model Solutions
Fall 2016

1.

Learning Objectives:

4. The candidate will understand how to apply the fundamental techniques of reinsurance pricing.

Learning Outcomes:

(4c) Calculate the price for a casualty per occurrence excess treaty.

Source:

Basics of Reinsurance Pricing, Clark

Solution:

(a) Calculate the loss cost rate for the treaty.

The loss cost is

$$\left[0.60\cdot 0.60+0.40\cdot 0.70\right]\cdot\left[0.50\cdot\left(0.0500-0.0125\right)+0.50\cdot\left(0.1000-0.0250\right)\right]=3.6\%.$$

(b) Explain how you would handle:

(i) Policy limits

(ii) Discounting

Because workers compensation insurance does not have policy limits, no adjustment is needed. For discounting, loss data should be requested on a full undiscounted basis.

(c) Identify an adjustment (unrelated to policy limits or discounting) you would need to make to:

(i) The historical premium

(ii) The historical losses

1. **Continued**

Commentary on Question:
In each case only one adjustment is required to obtain full credit.

Historical premiums should be adjusted for rate changes and for exposure (payroll) inflation. Historical losses should be adjusted for trend and for development.

2. Learning Objectives:

2. The candidate will understand the considerations in selecting a risk margin for unpaid claims.

Learning Outcomes:

(2a) Describe a risk margin analysis framework.

(2b) Identify the sources of uncertainty underlying an estimate of unpaid claims.

(2c) Describe methods to assess this uncertainty.

Source:
A Framework for Assessing Risk Margins, Marshall, et al.

Solution:

(a) Describe three problems with this approach.

Commentary on Question:
Any three of the following is sufficient to earn full credit.

- Commonly used methods are complex and require a lot of data. The benefits may not outweigh the costs.
- Correlations are heavily influenced by correlations in past data and may not accurately reflect the true values.
- It is difficult to separate past results into independent and systemic components.
- Internal systemic risk cannot be modeled using standard correlation modeling techniques.

(b) Propose a numerical value for the correlation between outstanding claim liabilities and premium liabilities within the motor line of business.

Commentary on Question:
Any value above 0.50, with reasonable explanation, is satisfactory.

The authors of the paper describe 0.75 as representing a high correlation, making it a reasonable choice.

2. Continued

(c) Calculate the internal systemic risk coefficient of variation for the motor line of business.

Commentary on Question:
The calculations below use a correlation of 0.75. If the calculations were done correctly using the proposed value from (b), full credit was awarded.

The variance terms add to $8.5^2(0.15/0.5)^2 + 6.0^2(0.35/0.5)^2 = 24.1425$. The covariance term is $(8.5)(6.0)(0.15/0.5)(0.35/0.5)(0.75) = 8.0325$. The total is 24.1425 + 2(8.0325) = 40.2075. The coefficient of variation is the square root, 6.34%.

(d) Describe two implications of differing lengths of claim run-off when performing internal benchmarking of independent risk.

For outstanding claim liabilities, longer run-off implies higher volatility and hence a higher coefficient of variation. This is due to more time for random effects to have an impact. For premium liabilities, long tails imply a higher coefficient of variation relative to outstanding claim liabilities. This is due to smaller volume.

3. **Learning Objectives:**

3. The candidate will understand excess of loss coverages and retrospective rating.

Learning Outcomes:

(3a) Explain the mathematics of excess of loss coverages in graphical terms.

(3b) Calculate the expected value premium for increased limits coverage and excess of loss coverage.

(3c) Explain and calculate the effect of economic and social inflationary trends on first dollar and excess of loss coverage.

Source:

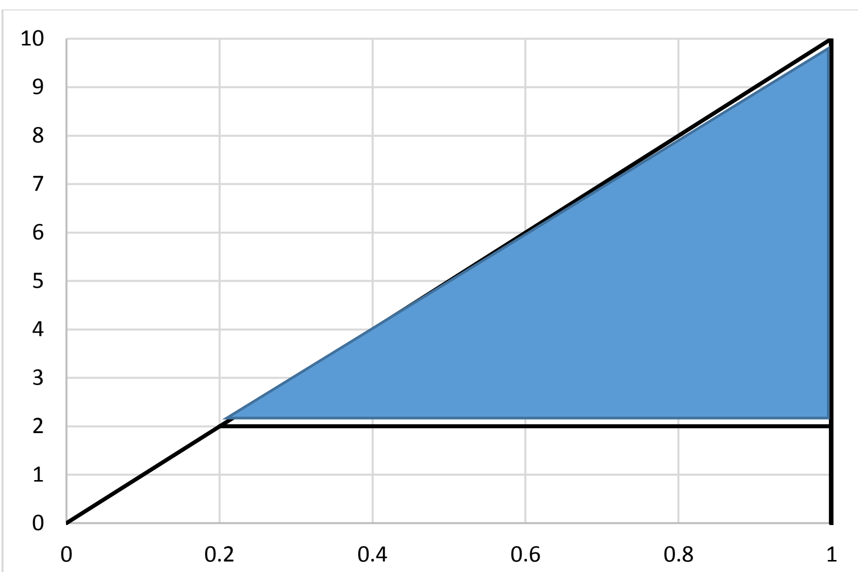
The Mathematics of Excess of Loss Coverages and Retrospective Rating – A Graphical Approach, Lee

Solution:

(a) Demonstrate using calculus that the expected loss in the layer 2 to 10 is 3.2.

$$\int_2^{10} 1-F(x)dx = \int_2^{10} (1-0.1x)dx = x-0.05x^2\Big|_2^{10} = 10-5-2+0.2 = 3.2\,.$$

(b) Demonstrate using a graph that the expected loss in the layer 2 to 10 is 3.2.



The area of the triangle is 0.5(0.8)(8) = 3.2.

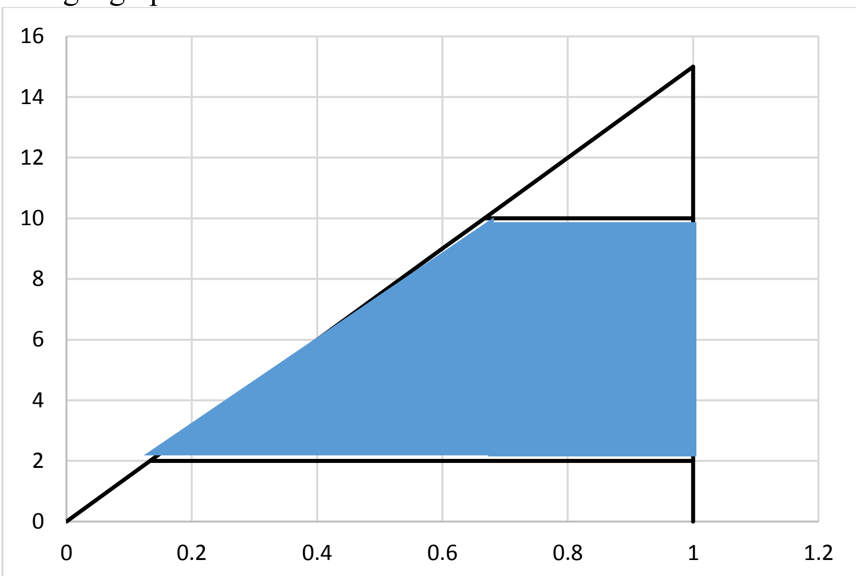
3. **Continued**

(c) Demonstrate using either calculus or a graph that the expected loss in the layer 2 to 10 also increases by 50%.

Using calculus:

$$\int_2^{10} 1-F(x)dx = \int_2^{10} (1-x/15)dx = x - x^2/30\Big|_2^{10} = 10-100/30-2+4/30 = 4.8 = 1.5(3.2).$$

Using a graph:



The area of the trapezoid is 0.5(13/15 + 5/15)(8) = 4.8 = 1.5(3.2).

(d) Explain why the leveraged effect of inflation does not cause an increase greater than 50%.

The leveraging effect of the deductible and the dampening effect of the limit exactly offset each other.

(e) Explain, using words, not calculations, why the increased limit factor at 10 is greater than, less than, or the same as it was before the 50% increase in loss sizes.

From the earlier parts we know that the loss in the layer from 2 to 10 increases by exactly 50%. The loss in the layer from 0 to 2 increases by less than 50% due to the dampening effect of the limit of 2. Therefore, the increased limit factor must be greater than it was before the 50% increase in loss sizes.

4. **Learning Objectives:**

1. The candidate will understand how to use basic loss development models to estimate the standard deviation of an estimator of unpaid claims.

Learning Outcomes:

(1a) Identify the assumptions underlying the chain ladder estimation method.

(1b) Test for the validity of these assumptions.

(1c) Identify alternative models that should be considered depending on the results of the tests.

(1d) Estimate the standard deviation of a chain ladder estimator of unpaid claims.

Sources:

Measuring the Variability of Chain Ladder Reserve Estimates, Mack
Testing the Assumptions of Age-to-Age Factors, Venter

Solution:

(a) Demonstrate that the value of α_5^2 was correctly calculated. (Your calculation need not match to all four decimal places.)

$$\begin{aligned}
\alpha_5^2 &= \frac{1}{7-5-1}\sum_{j=1}^{7-5} C_{j,5}\left(\frac{C_{j,6}}{C_{j,5}} - f_5\right)^2 \\
&= \frac{1}{1}\left[C_{1,5}\left(\frac{C_{1,6}}{C_{1,5}} - f_5\right)^2 + C_{2,5}\left(\frac{C_{2,6}}{C_{2,5}} - f_5\right)^2\right] \\
&= \frac{1}{1}\left[18,910\left(\frac{19,262}{18,910} - 1.01957\right)^2 + 24,827\left(\frac{25,331}{24,827} - 1.01957\right)^2\right] \\
&= 0.0305.
\end{aligned}$$

4. Continued

(b) Demonstrate that the standard error for accident year 4 was correctly calculated.

$$C_{4,7}^2 \sum_{k=7+1-4=4}^{7-1=6} \frac{\alpha_k^2}{f_k^2}\left(\frac{1}{C_{4,k}}+\frac{1}{\sum_{j=1}^{7-k} C_{j,k}}\right) = C_{4,7}^2\left[\frac{\alpha_4^2}{f_4^2}\left(\frac{1}{C_{4,4}}+\frac{1}{\sum_{j=1}^{7-4=3} C_{j,4}}\right)+\frac{\alpha_5^2}{f_5^2}\left(\frac{1}{C_{4,5}}+\frac{1}{\sum_{j=1}^{7-5=2} C_{j,5}}\right)+\frac{\alpha_6^2}{f_6^2}\left(\frac{1}{C_{4,6}}+\frac{1}{\sum_{j=1}^{7-6=1} C_{j,6}}\right)\right]$$

$$= 25,919^2\left[\frac{57.324}{1.04349^2}\left(\frac{1}{23,888}+\frac{1}{18,236+24,701+14,707}\right)+\frac{0.0305}{1.01957^2}\left(\frac{1}{24,927}+\frac{1}{18,910+24,827}\right)\right.$$
$$\left.+\frac{0.00016}{1.01983^2}\left(\frac{1}{25,415}+\frac{1}{19,262}\right)\right]$$

$$= 2,095,309.$$

The square root is 1,448.

(c) Explain, referring to this example, why using the normal approximation may not be reasonable.

Commentary on Question:
Candidates generally did well, providing an explanation that took one of the forms indicated below. Full credit required that the explanation refers to the example.

The normal approximation is symmetric and may allow for more negative movement than is reasonable. In this case, it can be noted either that the standard deviation is close to half the reserve or that there is a significant probability of negative development.

(d) Recommend an approach that may be superior to using the normal approximation. Justify your recommendation.

Commentary on Question:
Either of the justifications below is sufficient for full credit.

A lognormal distribution may be a superior model. It is right skewed and thus provides a lower probability of downward movement. The lognormal distribution assigns probability only to positive values.

4. Continued

(e) Explain why it is necessary to perform a weighted regression.

An unweighted regression assumes a constant variance. In the Mack model the variance is proportional to the developed value and thus a weighted regression is needed.

(f) Describe two other approaches that Venter proposes for comparing or evaluating different models.

Commentary on Question:
Any two of the following are sufficient for full credit.

- Test the significance of the estimated factors.
- Examine the residuals to determine if the model is linear in the specified manner.
- Examine the residuals over time to determine if there are any patterns.

5. **Learning Objectives:**

1. The candidate will understand how to use basic loss development models to estimate the standard deviation of an estimator of unpaid claims.

Learning Outcomes:

(1e) Apply a parametric model of loss development.

(1f) Estimate the standard deviation of a parametric estimator of unpaid claims.

Source:

LDF Curve Fitting and Stochastic Reserving: A Maximum Likelihood Approach, Clark

Solution:

(a) State two of the key assumptions of Clark's method.

Commentary on Question:

Any two of the following are sufficient for full credit.

- Incremental losses are independent and identically distributed.
- The scale parameter is constant.
- Variance estimates are based on the Rao-Cramer lower bound.

(b) Estimate ultimate losses for accident year 2015.

The estimate is

$6,000+12,000(ELR)[1-G(6)]=6,000+12,000(0.7115)e^{-6/7.293}=9,750.$

(c) Estimate the process standard deviation of the accident year 2015 reserve.

The reserve is 9,750 – 6,000 = 3,750.

The estimated process standard deviation is $(3,750\cdot 273)^{0.5}=1,012.$

(d) Estimate the total coefficient of variation of the accident year 2015 reserve.

The estimated coefficient of variation is $(1,012^2+852^2)^{0.5}/3,750=35\%.$

(e) Estimate the expected payments in 2016 for accident year 2015.

The estimate of the expected payments is

$12,000(0.7115)[G(18)-G(6)]=8,538[e^{-6/7.293}-e^{-18/7.293}]=3,027.$

5. Continued

(f) Estimate the total standard deviation of payments in 2016 for accident year 2015.

The estimate of the total standard deviation is $(3,027 \cdot 273 + 512^2)^{0.5} = 1,043$.

(g) Indicate whether this suggests that the reserving model is invalid. Justify your answer.

The observed difference of 3,800 – 3,027 = 773 is less than one standard deviation (1,043) and thus the result is not inconsistent with the model.

6. Learning Objectives:

5. The candidate will understand methodologies for determining an underwriting profit margin.

Learning Outcomes:

(5d) Allocate an underwriting profit margin (risk load) among different accounts.

Source:

An Application of Game Theory: Property Catastrophe Risk Load, Mango

Solution:

(a) Calculate the risk load for each account using the Shapley method.

Var(X) = 17,640,000 + 2,227,500 = 19,867,500
Var(Y) = 78,400 + 22,275 = 100,675
Var(X+Y) = 20,070,400 + 2,695,275 = 22,765,675
Cov(X,Y) = (22,765,675 – 19,867,500 – 100,675)/2 = 1,398,750
Risk load for X: 0.000025(19,867,500 + 1,398,750) = 532
Risk load for Y: 0.000025(100,675 + 1,398,750) = 37

(b) Calculate the risk load for each account using the Covariance Share method.

Commentary on Question:
The key to this method is working with each of the two events separately.

For Event 1, the share for X is (30/32)(20,070,400 – 17,640,000 – 78,400) = 2,205,000. The share for Y is (2/32)(20,070,400 – 17,640,000 – 78,400) = 147,000.
For Event 2, the share for X is (15/16.5)(2,695,275 – 2,227,500 – 22,275) = 405,000 and the share for Y is (1.5/16.5)(2,695,275 – 2,227,500 – 22,275) = 40,500.
Risk load for X: 0.000025(19,867,500 + 2,205,000 + 405,000) = 562
Risk load for Y: 0.000025(100,675 + 147,000 + 40,500) = 7

(c) Evaluate which method is more likely to produce appropriate risk loads to be used in pricing.

The covariance share method is more appropriate because it allocates less of the covariance to smaller accounts, which should have lower risk.

7. Learning Objectives:

5. The candidate will understand methodologies for determining an underwriting profit margin.

Learning Outcomes:

(5b) Calculate an underwriting profit margin using the capital asset pricing model.

Source:
Ratemaking: A Financial Economics Approach, D'Arcy and Dyer

Solution:

(a) Calculate the funds generating coefficient estimate, *k*.

$k = 0.4(0.25) + 0.4(0.5) + 0.2(0.75) = 0.45$.

(b) Calculate the underwriting beta.

Beta = $-(0.45)(-0.2) = 0.09$.

(c) Calculate the underwriting profit margin.

$UPM = -0.45(0.01) + 0.09(0.05) = 0$.

(d) Indicate whether the underwriting profit margin would be higher, lower, or the same if taxes were not ignored.

Any answer is correct becuase it will depend on the relationship between the tax rate on investment income and the tax rate on underwriting income.

(e) Provide two criticisms of models that apply the Capital Asset Pricing Model to insurance.

- The model only covers risk that varies with market returns. As such, it ignores unique insurance risks such as catastrophe.
- The insurance market cannot simply be appended to the stock market.

8. Learning Objectives:

4. The candidate will understand how to apply the fundamental techniques of reinsurance pricing.

Learning Outcomes:

(4a) Calculate the price for a proportional treaty.

Source:

Basics of Reinsurance Pricing, Clark

Solution:

(a) Calculate the expected technical ratio (loss ratio plus commission ratio) for Option 1.

From 30-50% the averages of the two components add to 30% + 40% = 70%.
From 50-70% the averages of the two components add to 25 + 60 = 85%.
From 70-80% the averages of the two components add to 20 + 75 = 95%.
The weighted average is [20(70) + 20(85) + 10(95)]/50 = 81%.

(b) Calculate the expected technical ratio for Option 2.

From 30-60% the averages of the two components add to 27.5 + 45 = 72.5%.
From 60-70% the averages of the two components add to 27.5 + 62.5 = 90%.
From 70-80% the averages of the two components add to 27.5 + 70 = 97.5%.
The weighted average is [30(72.5) + 10(90) + 10(97.5)]/50 = 81%.

(c) Recommend which option Property R Us should choose. Justify your recommendation.

Commentary on Question:

The solution is based on having the same answer for parts (a) and (b). For candidates with different answers, justification based only on that difference received no credit.

Given that the two options have the same technical ratio, the difference lies in their respective variances (ranges could also be used here). Option 1 has the smaller variance (range) and thus is preferred.

8. Continued

(d) Explain how the loss ratio distribution on the surplus share treaty would qualitatively differ from the loss ratio distribution on the quota share treaty.

A surplus share treaty has a larger share of the larger risks and none of the smaller risks. This results in a loss ratio distribution with a larger variance than a quota share treaty.