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Risks & Rewards

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FIXED INCOME INVESTMENT STRATEGIES IN RESPONSE TO QE TAPERING

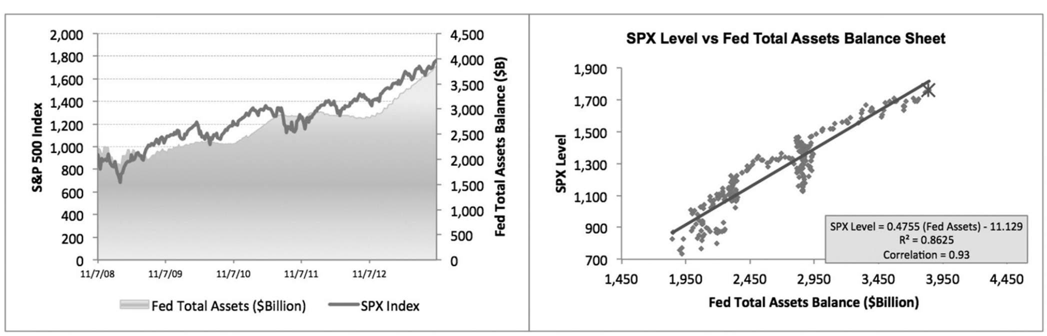
By Larry Zhao

The Federal Reserve's balance sheet, as a gauge of its lending to the financial system, reached the level of $3.8 trillion as of Nov. 8, 2013, exactly five years after the announcement of its first Quantitative Easing (QE) program.

The impact of the Fed's stimulus on the financial markets has been enormous and profound. The S&P 500 index is 93 percent correlated to the level of the Fed's balance sheet. With the Fed now owning more than one-third of the Treasury markets, the Fed's massive buying has significantly lowered the interest rates and bond yields. Also, the Fed has drastically reduced the cost of credit default protection, especially for the U.S. high yield corporate. The high yield credit default swap spread fell by more than 80 percent from the height of the financial crisis. While the Fed would like to unwind its QE programs quietly, with all the asset classes so correlated to the level of the Fed's balance sheet, the impact of the removal of the stimulus will be equally enormous and profound, and may be disruptive.

FIGURE 1: IMPACT OF FEDERAL RESERVE TOTAL ASSETS ON EQUITY AND CREDIT MARKETS

(Source: Bloomberg, Federal Reserve Bank of St. Louis)



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Nino A. Boezio, Newsletter Editor
(Chief Editor of this issue)
Segal Rogerscasey Canada
65 Queen St. West, Suite 2020
Toronto, ON M5H 2M5
ph: 416.642.7790 f: 416.304.0570

Joseph Koltisko, Newsletter Editor
(Chief Editor of next issue)
New York Life Insurance Co
51 Madison Avenue rm 1113
New York, N.Y. 10010
ph: 212.576.5625 f: 212.447.4251

Next Submission deadline May 5

CHAIRPERSON'S CORNER

INFORMATION DISCOVERY AND KNOWLEDGE MANAGEMENT: HOW EBSCO CAN HELP YOU BUILD YOUR OWN INFORMATION PYRAMID

By Larry Zhao

As investment professionals, we all know that information is costly to discover, collect and implement. On the other hand, we can be too easily overwhelmed by noisy information and risk not being able to see the forest for the trees as a result. Therefore, we need a balanced approach that can help us both discover information and manage our knowledge bases efficiently.

In the past, investment professionals primarily relied on business newspapers (such as WSJ, FT and Barron's), books and libraries. But today, as technology has revolutionized the media itself, as well as how people consume and utilize information (e.g., smartphone apps, YouTube, wikis). The supply and demand for information have shifted to a new equilibrium where the best minds and thought leaders interact more directly with their readership via new platforms (e.g., blogs, tweets) that offer faster, wider and more immediate impact. We need to be cognizant of these new emerging trends and markets. The first layer of my information pyramid includes blogs, tweets, equity analyst reports, investment bank research reports and updates, as well as Bloomberg terminal news and market colors.

As trained and credentialed actuaries, we have gone through a rigorous curriculum when preparing for our examinations that greatly helped us build a solid foundation and knowledge base. I still regularly rely on lots of these materials. These textbooks and research papers, along with global body of investment knowledge (GBIK) from the CFA program and the FRM (Financial Risk Manager) program, serve as the third layer of my information pyramid.

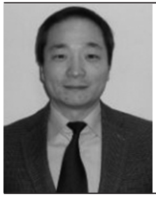
For the second layer, when researching topics that require deeper understanding but have not yet been formalized in textbooks, I used to use Google Scholar, a freely accessible search engine that indexes scholarly literature across diverse formats and databases. The frequent issues I have with Google Scholar, however, are its accessibility, quality and inefficiency. By accessibility, I mean that the full texts of articles in Google Scholar are quite often not freely available, because the hosting sites require either subscription or purchase. By quality, I mean that a whole range of qualities are associated with search results, from poor, to good, to excellent, which subsequently leads to inefficiency in that much time is spent without a good return in finding the most relevant and useful articles.

Recently I switched to EBSCO Business Source Corporate Plus (BSC+), a portal that provides full-text access to thousands of journals, magazines and newspapers. Due to a joint effort spearheaded by our past chairperson, Tom Anichini, and the Society of Actuaries (SOA) last year, Investment Section members now enjoy free access to numerous investment-related periodicals via EBSCO. Our main goal is to help improve the return on investment (ROI) of your time and help increase productivity by simplifying your research process.

Features	EBSCO (BSC+)	Google Scholar
Number of items	Hundred of thousand articles published in journals, newspapers, magazines and ebooks.	Billion of articles or webpages created by individuals and organizations
Ease of use	Easy	Easy
Access to publication	Free access to hundred of thousands peer reviewed, investment related articles.	No automatic free access to publications at subscription sites.
Publication quality	Very good to excellent.	Quality varies widely from poor, to good, to excellent; sometimes inadequate and less often updated.
Vulnerability to advertising	No such vulnerability. Clean scholarly work.	Vendors potentially can include advertising and product brochures among the search results.
Vulnerability to spam	No such vulnerability. Clean, reliable, authoritative, scholarly work.	Citation counts, impact factors and ranking algorithms can be gamed such that nonsensical articles can be indexed and included in search results.
Search settings	Can limit search to peer-reviewed only articles. Can create search profiles and retrieve search history.	No such capabilities.

Based on experience using both EBSCO and Google Scholar, I have found that, if used properly, the former is far more efficient. For example, when I type in "two factors Hull White," EBSCO immediately delivers the paper I want in full-page PDF, while Google Scholar points me to many red herrings. EBSCO has since become a useful tool for building the second layer of my information pyramid.

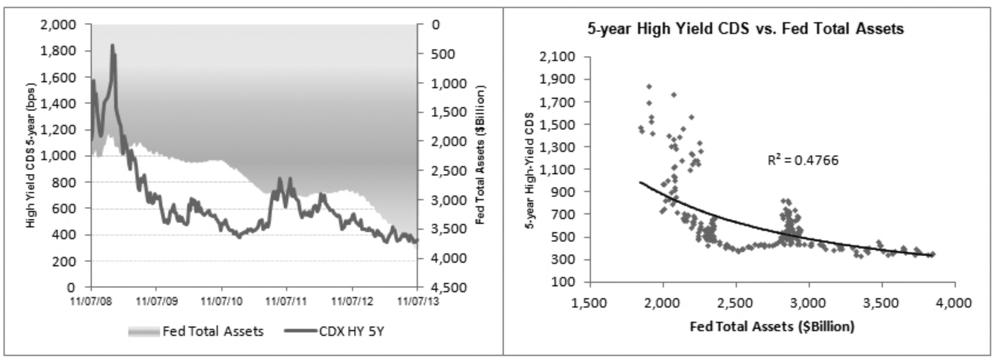
Information is costly to discover, collect and implement, yet with the right tools and technology, the process can be made relatively easier. Inevitably, the information "arms race" simply ratchets up to another level. The issue of efficiency and competitive advantage will never go away. Be prepared to seek out better solutions and continually adapt.



Larry Zhao, FSA, CERA, CFA, FRM, PhD, is an associate vice president at Nationwide Financial. He can be reached at zhaol1@nationwide.com.

FIGURE 1: IMPACT OF FEDERAL RESERVE TOTAL ASSETS ON EQUITY AND CREDIT MARKETS – CONTINUED

(Source: Bloomberg, Federal Reserve Bank of St. Louis)



IMPACT ON INSURANCE COMPANY INVESTMENT STRATEGIES

1. The artificially low interest rates and bond yields created greater asset and liability management problems for insurers. As the higher yielding bonds mature and are replaced with lower yielding bonds, the problem gradually but surely bleeds into the book yield on the investment portfolio.

2. With the Fed buying $85 billion Treasury and mortgage-backed securities (MBS) on a monthly basis and with the federal government cutting budgets, the QE program has effectively reduced the supply of fixed income securities available to institutional investors like insurers.

Responding to the changes in the capital markets, many investment departments of insurers have adapted in their investment strategies, including:

- Increased allocations to higher yield fixed income securities such as MBS, bank loans and high yield corporate bonds

- Increased allocations to less liquid assets such as infrastructure and real estate, picking up illiquidity premium while matching long-dated liability profiles.

- Extended duration and rode down the steepened yield curve.

- Increased allocations to alternative investment assets such as private equities.

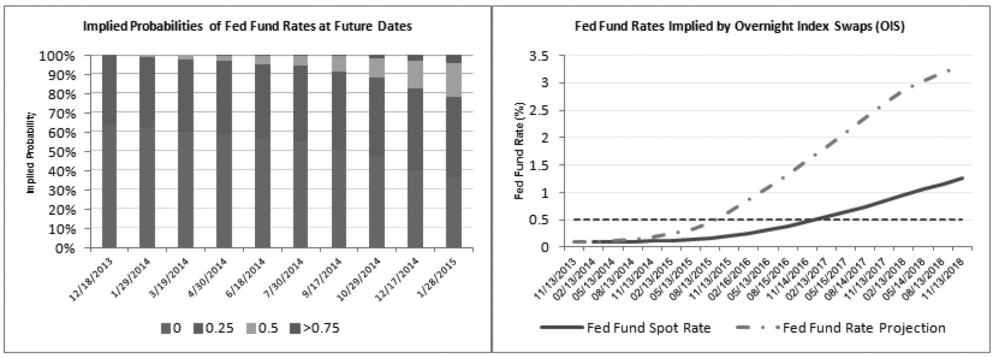
Many of the strategies have performed quite well in the low rate and low yield environment so far.

TIME TO ADAPT AGAIN

If something cannot go on forever, it will stop. With the macro economy getting better, tapering and unwinding the QE programs are inevitable. The capital markets have gradually priced in the unwinding of this unconventional monetary policy. As shown in Figure 2 (as of Nov. 8, 2013) the probability of a 25 basis point hike in January 2015, implied by the Fed fund futures market, is about 20 percent; while

FIGURE 2: MARKET IMPLIED PROBABILITY AND TIMING OF THE UNWINDING OF THE QE PROGRAMS

(Source: Bloomberg)



the overnight index swaps (OIS) market currently prices in a similar rate rise around the late second-half of 2015.

Since May 2013, the Fed's communication on tapering has greatly impacted the fixed income markets, increasing more than 100 basis points in the 10-year U.S. Treasury notes, as well as increasing volatilities across markets. Higher rates, higher volatility and steepening swap spreads are expected to be hallmarks of the fixed income markets for the next few years. Accordingly, investment strategies need to adapt to the new environment again.

INVESTMENT STRATEGIES UNDER FED TAPERING AND UNWINDING

The following strategies are just personal opinions, and do not, in any way, shape or form, represent any institutional views.

Shorten Duration

This strategy is to avoid locking in low yields and to mitigate extension risk. It is recommended to focus on the short end of the yield curve and create laddered portfolios that will allow for maturing shorter bonds to be reinvested at higher interest rates over time. This is also a strategy advocated by PIMCO, one of the largest bond fund managers in the United States.

When the rate rises, MBS are particularly susceptible to extension risk, because the prepayment will slow accordingly. Under such scenarios, shorter maturity MBS such as the 15-year FNMA will perform much better than longer maturity MBS such as 30-year FNMA. Figure 3 illustrates (on the next page) this idea. While the two MBS (30-year FNCL 4.5 vs. 15-year FNCI 3) have similar average lives in the base scenario (5.09 vs. 4.78), they behave quite differently under different rate scenarios: when the rate rises 200 bps, the 30-year MBS extends by 3.8 years and has an annualized total return of -18.3 percent, while the 15-year MBS extends by only 0.7 years and has an annualized total return of -14.3 percent.

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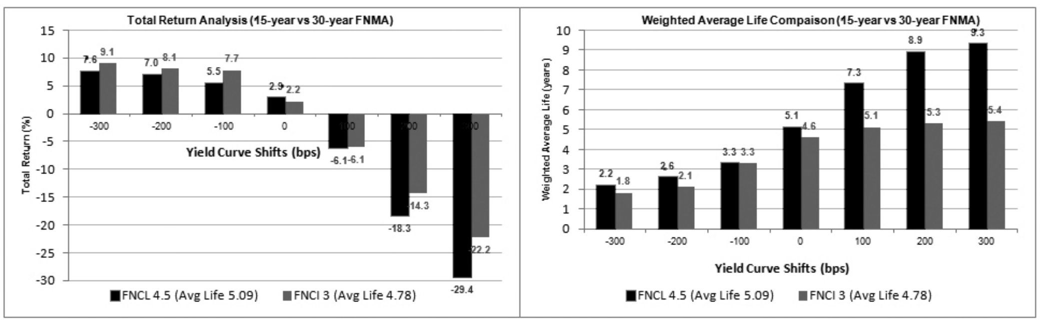
FIGURE 3: DURATION MANAGEMENT AND TOTAL RETURN ANALYSIS OF MORTGAGE-BACKED SECURITIES

(Source: Bloomberg)



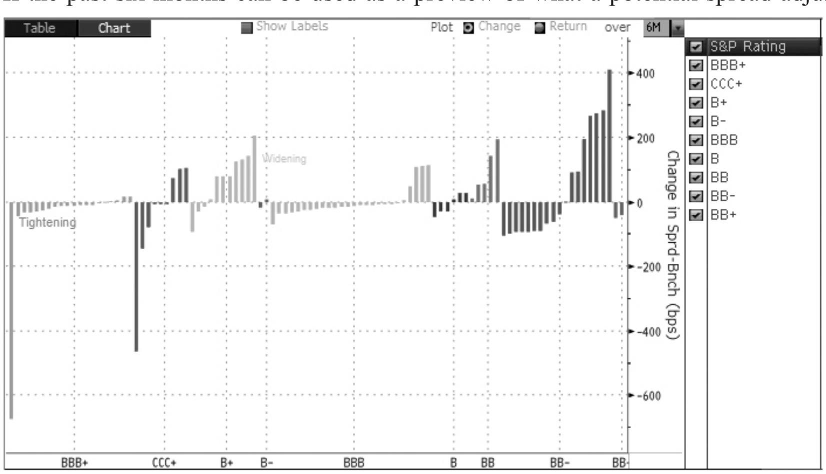
Reduce High Yields

The high yield bonds have been one of the hottest areas since the start of the QE. The average junk bond yield is now hovering around 6 percent, which is only 350 basis points above the yield on the 10-year U.S. Treasury bond, in comparison to a historical average spread of 500 basis points. Since the QE taper talk in May 2013, investment grade bonds have outperformed junk bonds, based on a quantitative analysis over 967 bonds (U.S. domiciled, issue amount $550+ million, duration less than five years, and S&P ratings between BBB+ and CCC+). As shown in Figure 4, more than one-half of the issues rated BB+/BB-/B+/B/B- saw their spreads relative to equivalent benchmarks widened, while more than four-fifths of the issues rated BBB+/BBB enjoyed their spreads tightened.

FIGURE 4: CREDIT PERFORMANCE COMPARISON: INVESTMENT GRADE AND HIGH YIELD

(Source: Bloomberg)

If the past six months can be used as a preview of what a potential spread adjustment might look like when the



tapering actually starts, reducing allocations to high yield bonds may be a prudent move.

Rotate Sectors

Based on a quantitative study using 970 bonds (U.S. domiciled, issue amount $800+ million, duration less than five years, and S&P ratings BBB or above), bonds in the financial, consumer discretionary, staples, technology and industrial sectors are more likely to benefit, while U.S. Treasuries and bonds in utilities/telecoms are more likely to suffer, from a QE tapering.

Since the introduction of QE 2, the net interest margin (NIM), the primary driver of earnings and profits for banks and depository institutions, has compressed to multiyear lows. Rising rates will significantly increase the lending profits and improve credit spreads for the financial sector.

The consumer discretionary, staples, technology and industrial sectors might benefit from the expectation of strengthening economy, increased spending, improving employment, and wealth effects. As the economy escapes from disinflation, businesses holding large cashes and liquidity will increasingly find the need and benefits on capital spending such as upgrading software and hardware, systems, equipment and facilities. This will benefit technology and industrial sectors.

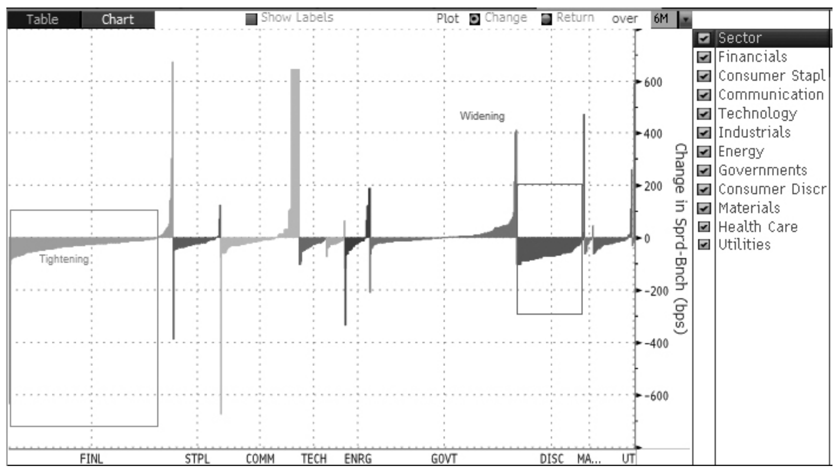
As a traditional defensive play, utilities and telecom sectors have performed extraordinarily well in the past few years, because they typically pay above-market dividends (about 1.5 to 2.5 times that of the S&P 500 index) with a lower level of volatility, which was outstanding in a low rate and low yield environment. However, rising rates will increase the interest burden and financing costs because the sectors tend to be capital intensive and heavily indebted.

Use Interest Rate Derivatives

Interest rate derivatives can be versatile and valuable in

FIGURE 5: CREDIT PERFORMANCES AMONG SECTORS

(Source: Bloomberg)



hedging different interest rate risk profiles.

A pure bond funds portfolio is long duration. One simple and practical way to neutralize interest rate risk is to use interest rate swaps and hedge dynamically. Figure 6 illustrates how a hedged portfolio, where a vanilla $10 million notional 10-year pay-fixed par swap is used to hedge a generic $10 million 10-year U.S. Treasury bond, performs under different interest rate scenarios.

From the perspective of asset and liability management, it is the net exposure that determines the interest rate risk profiles. For example, liabilities of the pension funds and long-term care are roughly equivalent to a giant *short* position in fixed income and have a much longer duration than the fixed income assets in their investment portfolios. Thus, their net exposure is short fixed income. In the past few years, some companies have implicitly adopted a wait-and-

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FIGURE 6: CHANGES OF MARKET VALUES OF A HEDGED PORTFOLIO UNDER DIFFERENT RATE SCENARIOS

(Source: Bloomberg)

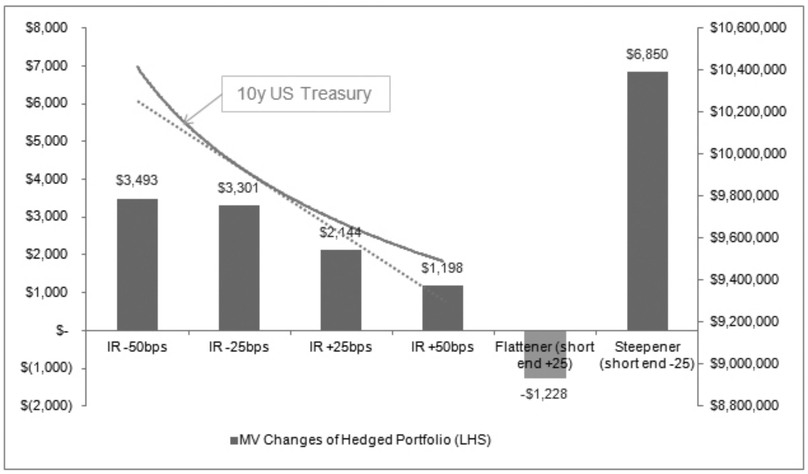
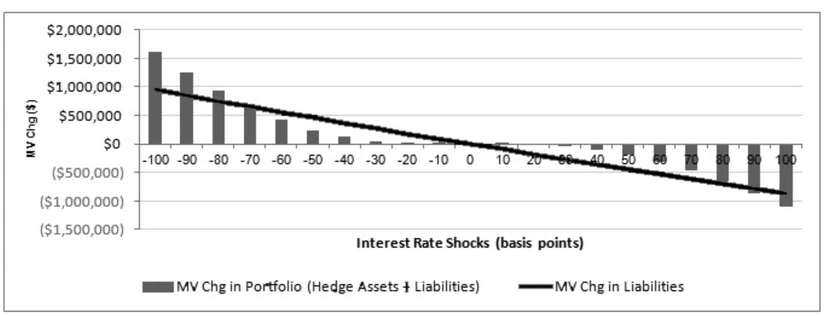


FIGURE 7: USING A SWAPTION COLLAR TO PROTECT DOWNSIDE RISK COST EFFICIENTLY

(Source: Bloomberg)



see strategy because they were hesitant to buy long duration in the low interest rate environment.

The rising rates and rising volatility might offer a good opportunity using derivatives to hedge against a *decrease* in interest rates if the QE tapering turns out to be disruptive and drives up the fight-to-safety plays, domestically and internationally. Figure 7 illustrates how to use a conditional interest rate swap hedge strategy to protect this downside risk in a cost-efficient way.

The conditional interest rate swap strategy writes a six-month 10-year payer swaption at the strike 3.5 percent and at the same time buys a six-month 10-year receiver swaption at the strike 2.4 percent to hedge the net short fixed income (approximated using a 10-year U.S. Treasury). The portfolio is constructed in such a way that both the portfolio duration and the option premium paid upfront are close to zero.

In this particular example, if swap rate is over 3.5 percent at expiration, the receiver swaption expires worthless but the payer swaption is exercised by counterparty in the money. However, the negative market value is offset by the decrease in the net present value of liabilities due to increased discounting rates. If rate is below 2.4 percent at expiration, the payer swaption expires worthless but the receiver swaption is in the money and its cash value can help offset the increase in the present value of liabilities.

SUMMARY

The impact of the Fed's QE program on the financial markets has been enormous and profound. The tapering and the eventual removal of the stimulus will be equally enormous and profound, and may be disruptive. To adapt to the potential regime change in the near future, some investment strategies have been proposed: (1) shorten duration to focus on the short end of the yield curve and create laddered portfolios; (2) reduce allocation to high yield bonds; (3) rotate out interest rate sensitive sectors such as utilities; and (4) use interest rate derivatives to hedge the interest rate uncertainties lying ahead.

On the other hand, contrarians who see the QE program continuing may stay the course and consider: (1) adding duration; (2) gaining exposure to high yield bonds; and (3) increasing allocation to utilities and telecoms.

Larry Zhao, FSA, CERA, CFA, FRM, PhD, is an associate vice president at Nationwide Financial. He can be reached at zhaol1@nationwide.com.

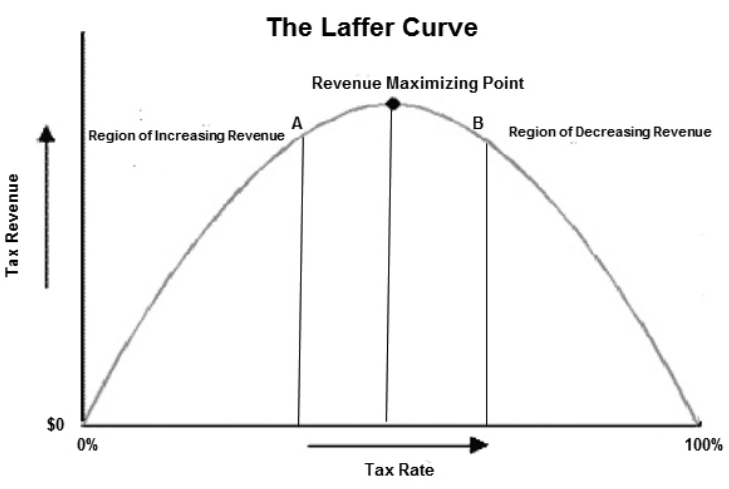
The views expressed herein are those of the author and do not reflect the views of Nationwide Financial.

TAKING STOCK: HOW REAGANOMICS LED THE UNITED STATES TO ITS CURRENT FISCAL DILEMMA

By Nino Boezio

I recall watching a particular episode of ABC's "World News Tonight" in August 1985. Canadian-born anchor Peter Jennings announced the news "bombshell" that President Reagan's Budget Director David Stockman resigned. My father and I were devout "Reaganites" even though we lived in Canada, and we always liked to keep up on what was happening in U.S. politics. We also liked Ronald Reagan's tough stance against Iran following the Islamic Revolution of 1979 and the "evil empire" of the Soviet Union. I did not completely understand what was going on in the U.S. economy, even though I did like the idea of smaller government, reduced taxes and an expanded military.

The following year Stockman wrote a book called *The Triumph of Politics: Why the Reagan Revolution Failed*.[1] It identified the perils of having too big a welfare state, high federal spending and excessive subsidies. It argued that government should be smaller and explained why cutting taxes could help the economy. It also explained why various reforms could possibly increase tax revenue and why U.S. government spending should be cut in order to cover any potential tax revenue shortfall.



I always found the Laffer Curve intriguing. Arthur Laffer was the chief economist at the Office of Management and Budget from 1970 to 1972. He was associate professor at the University of Chicago when the curve was developed, and it looked somewhat similar to the graph in the lower left hand corner below (supposedly, the original was sketched on a napkin in 1974 when Laffer met with Nixon/Ford officials). The graph suggested that after a certain level of taxation, tax revenue falls as the disincentives produced by higher taxation outweigh the benefits for agents/individuals to produce additional income. On the other hand, if taxation remains or is reduced to below a certain level, the incentives to foster a growing economy become significant.

In 2006, I sat at Laffer's table at an industry event where he was to speak. Laffer was on Reagan's Economic Policy Advisory Board during 1981 to 1989. I was awestruck—I was getting to meet someone who was one of the major icons of U.S. economic policy (incidentally I also had some interaction with Stockman in the 1990s when I did some work for a major private equity fund he headed—it is a small world after all). Part of the incentive for the Reagan tax cuts was the postulation that the U.S. taxation system (as it was in 1980) could be on the right side of the Revenue Maximizing Point (such as at point B), and thus this level of taxation provided a disincentive to economic growth. If taxes are cut from a point on the right side of the curve and not below point A, the tax cuts could then pay for themselves.

Reaganomics rolled out as a tax-cut and spending-cut strategy. Coupled with the principles of supply-side economics, it was supposed to put the U.S. economy into a very strong economic and financial position. Stockman mentioned how some characterized the Reagan Revolution as the biggest economic policy development since the "New Deal" of the 1930s. However, as the early 1980s progressed, I do admit I was getting a bit confused by the entire strategy. The probable reason for the confusion was that Reaganomics was being morphed into something different from what it was initially portrayed.

Stockman resigned after the Reagan Revolution became derailed—politicians eventually went around the country saying they would cut taxes and the tax cuts would pay for themselves (a "free lunch" as Stockman put it), and the political pundits put their own agenda ahead of the country's best interests and ignored the matter of spending cuts. They sold the easy story to the American public and ignored the hard story. Cutting spending in areas that included various types of government and social programs was still not popular and therefore would not be pleasant for politicians to undertake. Stockman's book explained how the political and government bureaucracy ultimately undermined an otherwise noble ideology intended to bring about many positive reforms.

The United States was never on the right side of the Laffer Curve, as the growing U.S. economy did not make up for the losses in government revenue due to tax cuts. (Data we have seen since the early 1980s confirms this view.) Stockman from the beginning was skeptical about the Laffer Curve's applicability to America as it stood at the dawn of the Reagan era, even though senior politicians and many others bought into its premises quickly. In my view, the Laffer Curve is intuitive but as with any theory, assumptions are required and attaching point values to parts of the curve is very subjective, especially given the complexity of most tax systems. In addition, once a theory is put into practice and the outcomes are not working out as planned, we must be ready to admit the mistakes, make adjustments, and adopt remedies very quickly to fix the undesirable effects. One of the insurance policies against the Laffer Curve going wrong was to adopt spending cuts in tandem—something that was subsequently dropped off by politicians as unnecessary, inconvenient or unpleasant as the train left the station.

In the past 30 years, we have continued to see various U.S. politicians singing the "tax cuts will pay for themselves" mantra despite the evidence to the contrary, since such a slogan resonates well with voters. Sometimes tax cuts have been promoted with the idea that the new deficits will indirectly "squeeze" other politicians to adopt the required spending cuts later on—but they rarely do so and therefore this political strategy also fails. Stockman predicted that with the new fiscal/tax positioning of the government in the 1980s due to the insufficient reforms, the United States would have deficits running into perpetuity because of the new imbalance created between tax revenue and spending. In the years that followed, this actually bore itself out as the U.S. government failed to avoid budget deficits (with the exception of part of the Clinton era, when interest rates fell and some taxes were raised or introduced) and the federal debt continued to escalate.

The contents of Stockman's book went mostly unheeded as the 1980s later bloomed to a great era of economic prosperity (I entered the actuarial profession during that time). The burgeoning economic growth was considered something that would help tax revenues catch up eventually. Stockman faded into history once he left politics, but he has been resurrected occasionally through media interviews, as some drifted back to look at the Reagan era with nostalgia, or wanted to seek out some of Stockman's updated insights.

This may seem to be a bit of interesting history lesson but some may not yet see the connection to the current U.S. and global environment. U.S. federal debt is huge at $17 trillion and will continue to grow, while easy monetary policy and cheap money is everywhere. Understanding the past can help us understand the direction of interest rates, inflation, debt, risk and its implications for investment management and the U.S. dollar. Stockman had some insights and identified behavioral traits that are still with us today.

CONTINUED ON **PAGE 12**

Comparing Debt Ratios (%)

Rank	Country	Category	Rating	Outlook	2012	2013*	2014*	2015*	2016*
1	Japan	Advanced	AA-	Negative	233.4	238.0	242.4	246.7	250.5
2	Greece	Advanced	CC	Negative	157.7	157.0	152.5	149.4	145.5
3	Ireland	Advanced	BBB+	Negative	121.5	125.8	125.0	123.5	121.5
4	Italy	Advanced	A+	Negative	120.0	119.7	119.3	118.7	118.0
5	U.S.	Advanced	AA+	Negative	102.9	105.6	107.5	109.4	111.9
6	Belgium	Advanced	AA+	Negative	97.4	97.8	98.2	98.3	98.8
7	Portugal	Advanced	BBB-	Negative	94.6	97.5	100.8	103.7	106.5
8	Iceland	Advanced	BBB-	Negative	97.1	92.1	85.3	80.9	73.8
9	Singapore	Advanced	AAA	Stable	91.3	88.5	86.1	83.9	82.9
10	France	Advanced	AAA	Stable	86.9	87.6	87.1	85.9	84.1
11	U.K.	Advanced	AAA	Stable	86.5	87.4	86.5	84.4	81.3
12	Canada	Advanced	AAA	Stable	83.1	81.0	78.5	75.6	72.6
13	Germany	Advanced	AAA	Stable	79.4	77.9	75.8	73.8	71.9
14	Hungary	Emerging	BBB-	Negative	76.9	77.1	76.8	76.0	75.3
15	Austria	Advanced	AAA	Stable	70.7	70.9	70.5	70.1	69.8
16	Spain	Advanced	AA	Negative	67.1	69.9	72.1	74.1	75.9
17	India	Emerging	BBB-	Stable	69.9	67.5	65.4	63.5	61.8
18	Israel	Advanced	A	Stable	70.0	67.5	64.8	62.6	60.6
19	Netherlands	Advanced	AAA	Stable	66.5	66.7	66.5	65.7	64.4
20	Brazil	Emerging	BBB-	Positive	65.0	63.8	62.1	60.3	58.6
21	Jordan	Emerging	BB	Negative	62.1	61.9	61.7	61.3	60.5
22	Poland	Emerging	A-	Stable	57.3	57.5	57.4	56.5	55.5
23	Malaysia	Emerging	A-	Stable	55.8	56.5	56.9	57.2	57.3
24	Finland	Advanced	AAA	Stable	52.7	55.0	57.3	59.2	61.1
25	Norway	Advanced	AAA	Stable	54.3	54.3	54.3	54.3	54.3
26	Morocco	Emerging	BBB-	Stable	53.8	54.0	53.8	53.0	51.7
27	Switzerland	Advanced	AAA	Stable	51.2	49.7	48.3	47.0	45.7
28	Kenya	Emerging	B+	Stable	50.9	49.6	48.9	48.3	47.0
29	Pakistan	Emerging	B-	Stable	50.9	49.5	47.8	44.5	41.2
30	Lithuania	Emerging	BBB	Stable	45.4	47.7	49.5	50.9	52.1
31	Slovenia	Advanced	AA	Negative	44.9	46.7	48.0	49.3	50.6
32	Slovak Repu	Advanced	A+	Stable	46.2	46.5	46.8	46.2	45.7
33	Denmark	Advanced	AAA	Stable	46.5	46.3	45.1	43.0	40.2
34	Czech Repub	Advanced	A	Positive	43.4	44.5	45.6	46.5	47.6
35	Philippines	Emerging	BB	Stable	46.0	44.3	42.8	41.4	40.1
36	Thailand	Emerging	BBB+	Stable	44.0	43.9	43.8	43.5	43.0
37	South Africa	Emerging	BBB+	Stable	42.8	43.8	43.5	42.0	38.7
38	Mexico	Emerging	BBB	Stable	42.1	42.0	41.9	41.6	41.4
39	Ukraine	Emerging	B+	Stable	43.5	41.5	39.5	35.0	30.7
40	Latvia	Emerging	BB+	Positive	41.0	39.8	38.1	36.8	36.2
41	Romania	Emerging	BB+	Stable	37.7	37.5	36.8	36.0	35.0
42	New Zealan	Advanced	AA+	Negative	36.4	36.5	36.1	33.7	31.7
43	Turkey	Emerging	BB	Positive	37.6	36.4	35.4	34.7	34.0
44	Argentina	Emerging	B	Stable	36.7	35.8	35.3	34.3	31.4
45	Colombia	Emerging	BBB-	Stable	35.4	34.0	32.7	31.5	29.9
46	Sweden	Advanced	AAA	Stable	34.9	32.2	29.3	26.2	22.8
47	Korea	Advanced	A	Stable	26.9	25.1	23.3	21.5	19.8
48	Australia	Advanced	AAA	Stable	24.8	23.3	23.6	21.8	20.6
49	Indonesia	Emerging	BB+	Positive	24.0	22.8	21.8	21.1	19.9
50	Nigeria	Emerging	B+	Stable	18.5	21.1	18.6	16.7	14.8
51	Bulgaria	Emerging	BBB	Stable	20.0	19.7	18.8	17.3	15.2
52	Peru	Emerging	BBB-	Positive	20.7	19.3	18.1	17.0	16.1
53	Kazakhstan	Emerging	BBB	Stable	14.1	15.3	16.2	17.4	18.1
54	China	Emerging	AA-	Stable	16.3	15.0	13.4	10.8	9.7
55	Chile	Emerging	A+	Positive	10.2	9.6	9.0	8.5	8.0
56	Russia	Emerging	BBB	Stable	8.8	9.5	11.3	13.5	15.9
57	Saudi Arabia	Emerging	AA-	Stable	7.1	6.2	5.3	4.4	3.7
58	Estonia	Advanced	A	Positive	6.0	5.7	5.4	5.2	4.9
59	Hong Kong	Advanced	AAA	Stable	4.0	3.8	3.6	3.4	3.2

Source: Standard & Poor's (long-term credit & outlook); International Monetary Fund (debt), * are estimates
http://online.wsj.com/article/SB10001424052748703789104576272891515344726.html

Stockman's charges against politicians for their inability to cut spending or unwillingness to raise taxes have been frequently validated by the regular Washington gridlock and the arguments over the debt ceiling. Since the 1980s, we have continued to see U.S. politicians promoting tax cuts, and even implying that future economic growth will somehow pay for those cuts. On the other hand, we have also seen politicians strongly entrenched in a position of not wanting to cut spending on anything. We have seen an anti-tax culture develop in the United States that is very detrimental to balancing the U.S. budget. We also see a culture of entitlement and a large welfare state, where no one wants their benefits reduced, claiming that they earned the rights to those benefits even though the country cannot afford these promises any longer, and such promises when granted were never prepaid. These attitudes and problems are not unique to the U.S. environment, but perhaps are more visible to the world than those of countries in similar dilemmas, as the U.S. political infighting is on big display globally through the various forms of media.

When we watch the political infighting in Washington, any potential resolutions barely reach an amount that comes close to 5 percent of the $17 trillion debt (they appear to be "nickel and dime" solutions). Most heavily debt-laden economies are also hoping that their economies can grow them out of their debt burdens—Japan, for example, is hoping that inflation will come to the rescue, increasing both tax revenue and potentially reducing the real value of debt. Nevertheless, it is becoming evident that we need big solutions to avoid an economic malaise at best and some sort of financial collapse at worst, as various countries drown under their own debt. As seen in the table to the left, debt-to-GDP levels have become very high. I believe that the low level of interest rates is the primary reason that many economies have avoided crises in the past few years. If these debt levels were achieved before the financial crisis of 2008 to 2009 when interest rates were substantially higher, a number of countries would have been in trouble before the crisis.

In a television interview given three years ago, Stockman suggested an idea that initially seemed bizarre but was rational in hindsight. He highlighted that the richest 5 percent of Americans have increased their wealth from $8 trillion to about $40 trillion (between 1985 and 2010) and that an excise tax of 15 percent on these individuals would raise $6 trillion in revenue, contributing significantly to federal debt reduction.[2] He remarked: "If [these politicians] were all put into a room on penalty of death to come up with how much they could cut, they couldn't come up with $50 billion, when the problem is $1.3 trillion,"[2] which just highlights the polarization of the problem. In a more recent account, it was remarked that "Stockman would subject the nation's top 10% of households to a levy equal to 30% of their wealth, payable over a decade"[3] in order to bring the United States away from the brink of a financial disaster. Stockman's newest book *The Great Deformation—The Corruption of Capitalism in America* further signals that there is serious trouble ahead, in part because of all the attempts to control and manipulate the financial system to achieve certain ends. In the past year, we have been hearing much about the rise in interest rates globally, and the paradigm used for the progression of investment returns is strongly based on historical data and analyses (which incidentally did help get us into the financial crisis, as financial models could not envision tail events greater than those that occurred in the past). From my view, the real jeopardy for the bond market is not rising interest rates due to an improving economy or Federal Reserve tapering as the headlines suggest, but the possibility that no one will want to buy the debt anymore. Political and economic analysts have argued for some time about when foreign governments will stop buying U.S. Treasury debt or even dump what they already own. This debate dates back to the 1980s when the United States started to run huge deficits and increased debt levels, and this tirade of criticism has been repeated so often that it is not taken too seriously anymore. However, we may get to a point that there is too much simply out there to buy, even if a central bank continues to step in. Major segments of the population down the road are going to have to make sacrifices if big solutions are not adopted now, and the victims could include the average person who will have assets eroded by high inflation. Investment activity and performance are going to be a confusing exercise, to say the least.

The primary lesson we need to understand from all this is that taxes need to be raised, spending has to be cut, or both—and that governments globally need to make big adjustments very soon, since the financial issues have been neglected for too long (of course, this approach is going to be painful for any economy to sustain, but it may be the only way out). This was the same principle understood at least by some during the Reagan years, but was subsequently ignored. If nothing is done to fix the current problems, then eventually Ms. Market or Mr. Inflation will have to make the adjustments, and these two do not represent one of the nicest pair I would want to reckon with. Unfortunately, a number of major countries are in similar circumstances, and it will be frightening to see them all punished at the same time.

ENDNOTES

1. Stockman, David A. 1986. *The Triumph of Politics: Why the Reagan Revolution Failed.* Harper & Row.
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The views expressed herein are those of the author and do not necessarily reflect the views of Segal Rogerscasey.

Nino Boezio, FSA, CFA, FCIA, is with Segal Rogerscasey Canada. He can be contacted at nboezio@ segalrc.com.

MORE TECHNIQUES FOR BETTER ATTRIBUTIONS

By Mark Evans

Joshua Boehme, in the August 2013 issue of Risks and Rewards, describes the Aumann-Shapley technique and applies it to attribution. This article further elaborates on theoretical and practical considerations of Aumann-Shapley and compares it to similar methods.

Boehme presents the Aumann-Shapley formula:

$$a_i = (v_i - u_i)\int_0^1 \frac{\delta f\{(1-z)u + zv\}}{\delta x_i} dz$$

The subscript *i* represents the ith component attribution, so a_i is the portion of change attributed to component *i*, and *u* and *v* are the vectors of the attribution components at the beginning and end of the attribution period.

Boehme goes on to mention several advantages of using this as an attribution tool. This will be expanded here, but first consider the following formula:

$$a_i = (v_i - u_i)\int_0^P \frac{\delta f(t)}{\delta x_i} dt$$

Here, instead of integrating with respect to *z*, we integrate with respect to *t,* which represents time where we are integrating over the attribution period (0 *P*). This integral follows the path of the security (or portfolio) over time. This produces the theoretically correct attribution. This will be referred to as the time-based method. Aumann-Shapley assumes implicitly that the attribution factors evolve in a linear fashion in unison. Actually, they follow various paths, and depending on the factor may be very non-linear and fall well outside the boundary set by the beginning and ending values during the attribution period. As it turns out, that is usually not that bad of a simplification, but it does introduce a source of error with the Aumann-Shapley approach.

The time-based method also has some practical limitations. In theory, it requires continuous observation of all attribution factors, which is usually impossible. As with Aumann-Shapley, this is normally solved with approximate integration. With Aumann-Shapley, approximate integration involves some sort of discrete summation on the variable *z*. For the time-based method, discrete summation is by time. Given that the time-based method is the theoretically correct approach, using more time steps in the attribution is more productive than using smaller increments of *z* in the Aumann-Shapley approximate integration, assuming observations of the attribution components are available.

For many attribution components, there is a legitimate cross term. This can be captured by the step-through approach by shocking variables one at a time and then in combination. The change due to shocking the combination that differs from the sum of the individual shocks is due to cross terms, and it is appropriate to show those as a separate attribution component. Time is unique in that it should not have a true cross term. Assuming various market parameters move according to Brownian motion, there should not be a correlation between those components and time. Time can be addressed by using Aumann-Shapley or other methods to determine its contribution without leaving a cross term. Any cross term mechanically produced where time is involved should theoretically mean revert to zero over the long term. An implication of this is that we can lump all market-driven parameters into a single category, leave theta in its own category, and then calculate a separate theta attribution component. Now, it turns out that Aumann-Shapley, average step-through, and average partial derivatives will produce about the same result in most instances. The reason for this can be shown mathematically. For small changes in financial related instruments, the first and second order terms are the most important. Given that, approximate *f* with the following formula:

$$f = \sum_{i=1}^{n} B_i x_i + \sum_{i=1}^{n}\sum_{j=1}^{i} B_{ij} x_i x_j$$

Then

$$\frac{\delta f}{\delta x_i} = B_i + B_{ii}x_i + \sum_{j=1}^{n} B_{ij}\, x_j$$

We can think of the various B as partial derivatives of the first and second order (B_{ii} corresponds to 1/2 times the second partial derivative). Now, regardless of whether we use Aumann-Shapley with exact integration, Aumann-Shapley with Simpson's rules, average step-through, or average partial derivative, we get the following result:

$$a_i = (v_i - u_i)\left\{B_i + \frac{1}{2}B_{ii}(u_i + v_i) + \frac{1}{2}\sum_{j=1}^{n} B_{ij}\,(u_j + v_j)\right\}$$

With two variables for component 1 we get:

$$a_1 = (v_1 - u_1)\left\{B_1 + B_{11}(u_1 + v_1) + \frac{1}{2}B_{12}(u_2 + v_2)\right\}$$

We can see an interesting verbal interpretation of the above. If we do a linear transformation so that $u = 0$ and v is then the change over the time period, we see that a_1 is just Taylors's with an added term equal to half the cross term. The proofs for these are pretty straightforward with the possible exception of the average step-through. There we will get a cross term for each step for a_i. The key to the proof is recognizing that for any given cross component j, half the step's contribution to the average will be based on u_j and half will be based on v_j.

So we see that Aumann-Shapley with exact integration, Aumann-Shapley with Simpson's rule, average step-through, and average partial derivative only differ to the extent higher order moments matter. They all have the common advantages that they recognize the state of the attribution at the beginning and end of the period and capture first and second order moments. They all share a disadvantage that if the underlying components follow a significantly non-linear path and/or fall far outside the bounds of the beginning and ending value, they can perform poorly. Usually, however, the advantages will outweigh the disadvantages.

Extending this to more variables, consider three categories of attribution components: time(theta), market variable, and contractual. Market variable would include equity levels (and other assets) and interest rates. Within equity, for example, we have delta, gamma, vega, and perhaps some higher order greeks. Also, equity might be subdivided by index or, depending on the application, even individual stocks. There would also be correlations between them, so we care about their cross terms. Contractual would include entries and exits via new business, new deposits, surrenders, death claims, partial withdrawals, transfers, etc. With a daily attribution, the contractual items normally occur at the end of the day and therefore can be dealt with last in a step-through process. They should not be included in any averaging. This leads to a general approach. Assuming a daily attribution, use one of the four methods above to separate theta and market variables impact. Each market variable could then be further subdivided. Equity, for example, could be separated into delta, gamma, cross terms, and other. Other would theoretically include higher order greeks and some noise created by the separation of theta and the market variable attribution components.

Consider a six-month European call option struck at the money. Assume constant volatility of 20 percent, level interest rate of 3 percent and a constant dividend of 2 percent. This produces an option cost of 5.8 percent of the strike. We study the period Dec. 31, 2007 through Dec. 31, 2012, issuing 10 S&P 500 options, one each on the last business day of every June and December. We calculate a daily partial derivative attribution and monthly attributions based on the step-through, partial derivatives, Aumann-Shapley (based on many points), Aumann-Shapley based on three points, and average partial derivative methods. Approximate integration for Aumann-Shapley is based on

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Simpson's rule as described by Boehme. Boehme describes all but the average partial derivative method, which is the same as the partial derivative method except the average of the beginning and ending partial derivatives is used instead of the beginning partial derivatives.

With our simplifying assumptions, the two attribution components are equity index level and time. This gives us delta and theta. For the daily partial derivative and monthly partial derivative methods, we also use gamma. The step-through approach uses the averaging technique described by Boehme. With 10 consecutive options of six months each, this gives us 60 months of data to compare these methods. While this is not a huge sample, we will see the results are quite conclusive and furthermore provide justification and explanation for the results.

We sum the results for the daily attribution to get monthly totals. This allows us to compare each of the monthly attributions to the daily and see which performs best.

The following table shows the daily attribution for January 2008:

Date	Index	Time to Expiry	Chg from Delta	Chg from Gamma	Chg from Theta	Chg from Other	Option Value	Delta	Gamma	Theta
12/31/2007	1468.36	0.498289					85.1969	0.536501	0.001895	-86.886089
1/2/2008	1447.16	0.492813	-11.3738	0.4259	-0.4758	0.0080	73.7811	0.495485	0.001944	-86.256485
1/3/2008	1447.16	0.490075	0.0000	0.0000	-0.2362	-0.0003	73.5446	0.495283	0.001950	-86.492251
1/4/2008	1411.63	0.487337	-17.5974	1.2308	-0.2368	0.0174	56.9586	0.425105	0.001973	-82.806428
1/7/2008	1416.18	0.479124	1.9342	0.0204	-0.6801	-0.0080	58.2251	0.432950	0.001991	-84.103801
1/8/2008	1390.19	0.476386	-11.2524	0.6723	-0.2303	0.0099	47.4247	0.380918	0.001972	-80.005480
1/9/2008	1409.13	0.473648	7.2146	0.3538	-0.2190	-0.0077	54.7662	0.418077	0.001999	-83.498522
1/10/2008	1420.33	0.470910	4.6825	0.1254	-0.2286	-0.0046	59.3408	0.440120	0.002008	-85.349529
1/11/2008	1401.02	0.468172	-8.4987	0.3743	-0.2337	0.0075	50.9902	0.400945	0.002002	-82.584717
1/14/2008	1416.25	0.459959	6.1064	0.2322	-0.6783	-0.0222	56.6283	0.430342	0.002030	-85.702886
1/15/2008	1380.95	0.457221	-15.1911	1.2647	-0.2346	0.0115	42.4788	0.358696	0.001989	-79.434624
1/16/2008	1373.2	0.454483	-2.7799	0.0597	-0.2175	0.0043	39.5455	0.342720	0.001974	-77.863995
1/17/2008	1333.25	0.451745	-13.6917	1.5751	-0.2132	-0.0068	27.2091	0.265873	0.001823	-67.448148
1/18/2008	1325.19	0.449008	-2.1429	0.0592	-0.1847	0.0060	24.9466	0.250528	0.001784	-65.153953
1/22/2008	1310.5	0.438056	-3.6803	0.1925	-0.7135	0.0453	20.7906	0.221472	0.001707	-60.849916
1/23/2008	1338.6	0.435318	6.2234	0.6739	-0.1666	-0.0021	27.5192	0.270913	0.001867	-69.629897
1/24/2008	1352.07	0.432580	3.6492	0.1694	-0.1906	-0.0089	31.1382	0.295722	0.001933	-73.660736
1/25/2008	1330.61	0.429843	-6.3462	0.4451	-0.2017	0.0097	25.0452	0.254473	0.001831	-67.398706
1/28/2008	1353.96	0.421629	5.9419	0.4991	-0.5536	-0.0477	30.8850	0.296273	0.001957	-74.762319
1/29/2008	1362.3	0.418891	2.4709	0.0681	-0.2047	-0.0062	33.2130	0.311988	0.001997	-77.277879
1/30/2008	1355.81	0.416153	-2.0248	0.0421	-0.2116	0.0046	31.0233	0.298325	0.001974	-75.590664
1/31/2008	1378.55	0.413415	6.7839	0.5103	-0.2070	-0.0091	38.1014	0.343548	0.002064	-81.960049

The following table shows the daily attribution summed by month for January 2008 through June 2008:

SUM OF DAILY PARTIAL DERIVATIVE

Date	Index	Time to Expiry	Chg from Delta	Chg from Gamma	Chg from Theta	Chg from Other	Option Value	Delta	Gamma	Theta
12/31/2007	1468.36	0.498289	0.0000	0.0000	0.0000	0.0000	85.1969	0.536501	0.001895	-86.886089
1/31/2008	1378.55	0.413415	-49.5722	8.9943	-6.5180	0.0003	38.1014	0.343548	0.002064	-81.960049
2/29/2008	1330.63	0.334018	-19.6830	6.3108	-6.1853	-0.0985	18.4454	0.220341	0.001921	-70.354232
3/31/2008	1322.7	0.249144	-10.8846	8.5686	-5.4607	0.3648	11.0335	0.164651	0.001874	-67.374585
4/30/2008	1385.59	0.167009	8.0150	5.5672	-7.9109	0.1605	16.8654	0.257295	0.002844	-112.188237
5/30/2008	1400.38	0.084873	-1.0771	5.3010	-10.7737	-0.2839	10.0316	0.220218	0.003628	-145.018470
6/30/2008	1280	0.000000	-11.5709	5.9933	-4.4095	-0.0446	0.0000	0.000000	0.000000	0.000000
			-84.7727	40.7352	-41.2581	0.0988				

The "Chg from Other" column in the above corresponds to the nonzero unexplained amount discussed in Boehme. Here it is small relative to the other attribution components because of the small size (daily) of the attribution period. It is only meaningful on days with large market moves. For example, on Sept. 29, 2008 it was -1.93 due to a 107 point drop in the index. Normally, gamma accurately predicts the change in delta, but for large market moves the higher order terms of Taylor's series matter. In this case, using gamma one would have expected delta to be 0.02 at the end of the day, but it was 0.08. Similarly, the large index movement creates a huge drop in theta causing the change from theta to be inaccurate as well. The only other "Chg from Other" with absolute value greater than 1.00 was June 29, 2009 when it was 1.01. This was the next-to-last day of the option where the option finished slightly in-the-money. When an index finishes close to the strike, gamma is extremely high, causing attributions, hedging, etc. to break down. For the 10 six-month options, the total "Chg from Other" for the six months ranged from -2.12 to 1.83. The average magnitude was 0.71.

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From our sample options, using all the methods discussed, we get the following results:

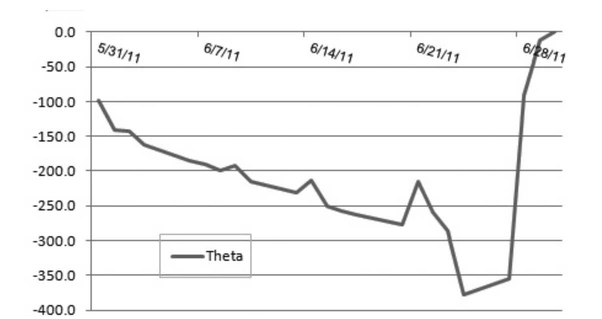
Method	Item	Std Dev Error	June 2011 Theta Error
Sum of Daily Partial Derivative	Other	0.33	
	Theta Adj	0.02	
Partial Derivative	Theta	2.26	10.05
	Other	2.61	
Average Step-Through	Theta	1.74	11.27
	Other	0.00	
Aumann-Shapley	Theta	1.86	11.52
	Other	0.02	
Aumann-Shapley-3 points	Theta	1.78	11.28
	Other	0.22	
Average Partial Derivative	Theta	2.63	14.14
	Other	2.41	

For each method we take the "Chg from Other" for each of the 60 months and determine its standard deviation assuming a mean of zero. For each method we compare the difference between its theta for the month and theta from the Sum of Daily Partial Derivative's theta and determine its standard deviation assuming a mean of zero. For the Sum of Daily Partial Derivative we used Aumann-Shapley-3 points on a daily basis to determine theta. Once we did that, we saw other drop to just 0.02.

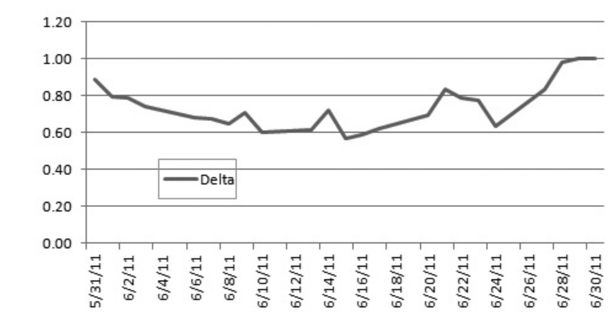
Aumann-Shapley is based on 40 points per month. This results in about the same number of calculations as used for the Sum of Daily Partial Derivative to get an apples-to-apples comparison. Note Average Step-Through, Aumann-Shapley and Aumann-Shapley-3 points all have about the same error in estimating theta. The other is also small in each case, and frankly is irrelevant compared to the theta error. I expected the average partial derivative theta to be more in line with the three other methods as far at theta error, but the "other" error was expected to be more noticeable since that method does not address other like the other three methods.

The theta error is quite large for June 2011 for all five monthly methods. This is the last month of the option period when results are very sensitive. As the graphs below show, both delta and theta took a very non-linear path during June 2011. Furthermore, both greeks frequently were far outside the bounds set at the beginning and ending of the month. The daily attribution handled this well with the other being only 0.41 for the month. That value reduced to 0.004 when using Aumann-Shapley-3 points on a daily basis to calculate theta.

THETA



DELTA



The partial derivative approach happened to do a little better in June 2011, which pulled its standard deviations of error down, but that is misleading since there were 13 months where the theta error was greater than 2.00 while the other four monthly methods had three to five such months.

In summary, Average Step-Through, Aumann-Shapley and Aumann-Shapley-3 points were expected to perform the best given the same time period, and that is the result. In conclusion, attribution should be done with fairly small time steps as opposed to using a lot of points in the Aumann-Shapley calculation, but using Aumann-Shapley-3 points as a last step has some advantages.

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Mark Evans, FSA, MAAA, FLMI/M, is president of Applied Stochastic, LLC. He can be contacted at: mark@appliedstochastic.com.

PRESENT VALUES, INVESTMENT RETURNS AND DISCOUNT RATES

By Dimitry Mindlin

Editor's Note: The first half of this article was published in the August 2013 issue of Risks & Rewards. In case you missed it, below it is reproduced in its entirety. To read only the second half, resume at section 4.

The concept of *present value* lies at the heart of finance in general and actuarial science in particular. The importance of the concept is universally recognized. Present values of various cash flows are extensively utilized in the pricing of financial instruments, funding of financial commitments, financial reporting and other areas.

A typical funding problem involves a *financial commitment* (defined as a series of future payments) to be funded. A financial commitment is funded if all payments are made when they are due. *A present value of a financial commitment is defined as the asset value required at the present to fund the commitment.*

Traditionally, the calculation of a present value utilizes *a discount rate*—a deterministic return assumption that represents investment returns. If the investment return and the commitment are certain, then the discount rate is equal to the investment return and the present value is equal to the sum of all payments discounted by the compounded discount rates. The asset value that is equal to this present value and invested in the portfolio that generates the investment return will fund the commitment with certainty.

In practice, however, perfectly certain future financial commitments and investment returns rarely exist. While the calculation of the present value is straightforward when returns and commitments are certain, uncertainties in the commitments and returns make the calculation of the present value anything but straightforward. When investment returns are uncertain, a single discount rate cannot encompass the entire spectrum of investment returns; hence the selection of a discount rate is a challenge. In general, the asset value required to fund an uncertain financial commitment via investing in risky assets—the present value of the commitment—is uncertain (stochastic).[1]

While the analysis of present values is vital to the process of funding financial commitments, uncertain (stochastic) present values are outside of the scope of this paper. This paper assumes that *a present value is certain (deterministic)*—a present value is assumed to be a number, not a random variable in this paper. The desire to have a deterministic present value requires a set of assumptions that "assume away" all the uncertainties in the funding problem.

In particular, it is generally necessary to assume that all future payments are perfectly known at the present. The next step is to select a proper measurement of investment returns that serves as the discount rate for present value calculations. This step—the selection of the discount rate—is the main subject of this paper.

One of the main messages of this paper is *the selection of the discount rate depends on the objective of the calculation*. Different objectives may necessitate different discount rates. The paper defines investment returns and specifies their relationships with present and future values. The key measurements of investment returns are defined in the context of return series and, after a concise discussion of capital market assumptions, in the context of return distributions.

The paper concludes with several examples of investment objectives and the discount rates associated with these objectives.

1. INVESTMENT RETURNS

This section discusses one of the most important concepts in finance—investment returns. Let us define the investment return for a portfolio of assets with known asset values at the beginning and the end of a time period. If PV is the asset value invested in portfolio P at the beginning of a time period, and FV is the value of the portfolio at the end of the period, then the portfolio return R_P for the period is defined as

$$R_P = \frac{FV - PV}{PV} \qquad (1.1)$$

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Thus, given the beginning and ending values, portfolio return is defined (retrospectively) as the ratio of the investment gain over the beginning value. Definition (1.1) establishes a relationship between portfolio return R_P and asset values PV and FV.

Simple transformations of definition (1.1) produce the following formula:

$$FV = PV(1 + R_P) \qquad (1.2)$$

Formula (1.2) allows a forward-looking (prospective) calculation of the end-of-period asset value FV. The formula is usually used when the asset value at the present PV and portfolio return R_P are known (this explains the notation: PV stands for "Present Value"; FV stands for "Future Value").

While definition (1.1) and formula (1.2) are mathematically equivalent, they utilize portfolio return R_P in fundamentally different ways. The return in definition (1.1) is certain, as it is used retrospectively as a measurement of portfolio performance. In contrast, the return in formula (1.2) is used prospectively to calculate the future value of the portfolio, and it may or may not be certain.

When a portfolio contains risky assets, the portfolio return is uncertain by definition. Most institutional and individual investors endeavor to fund their financial commitments by virtue of investing in risky assets. The distribution of uncertain portfolio return is usually analyzed using a set of forward-looking capital market assumptions that include expected returns, risks, and correlations between various asset classes. Later sections discuss capital market assumptions in more detail.

Given the present value and portfolio return, formula (1.2) calculates the future value. However, many investors with future financial commitments to fund (e.g., retirement plans) face a different challenge. Future values—the commitments—are usually given, and the challenge is to calculate present values. A simple transformation of formula (1.2) produces the following formula:

$$PV = \frac{FV}{1 + R_P} \qquad (1.3)$$

Formula (1.3) represents the concept of *discounting procedure*. Given a portfolio, formula (1.3) produces the asset value PV required to be invested in this portfolio at the present in order to accumulate future value FV. It must be emphasized that return R_P in (1.3) is generated by the actual portfolio P, as *there is no discounting without investing*. Any discounting procedure assumes that the assets are actually invested in a portfolio that generates the returns used in the procedure.

Formulas (1.2) and (1.3) are mathematically equivalent, and they utilize portfolio return in similar ways. Depending on the purpose of a calculation in (1.2) or (1.3), one may utilize either a particular measurement of return (e.g., the expected return or median return) or the full range of returns.[2] The desirable properties of the future value in (1.2) or present value in (1.3) would determine the right choice of the return assumption.

Future and present values are, in a certain sense, inverse of each other. It is informative to look at the analogy between future and present values *in the context of a funding problem*, which would explicitly involve a future financial commitment to fund. Think of an investor that has $\$P$ at the present and has made a commitment to accumulate $\$F$ at the end of the period by means of investing in a portfolio that generates investment return R.

Similar to (1.2), the future value of $\$P$ is equal to

$$FV = P(1 + R) \qquad (1.4)$$

Similar to (1.3), the present value of $F is equal to

$$PV = \frac{F}{1+R} \qquad (1.5)$$

The shortfall event is defined as failing to accumulate $F at the end of the period:

$$FV < F \qquad (1.6)$$

The shortfall event can also be defined *equivalently* in terms of the present value as $P being insufficient to accumulate $F at the end of the period:

$$P < PV \qquad (1.7)$$

In particular, the shortfall probability can be expressed in terms of future *and* present values:

$$\textit{Shortfall Probability} = \Pr(FV < F) = \Pr(PV > P) \qquad (1.8)$$

If the shortfall event happens, then *the shortfall size* can also be measured in terms of future and present values. The future shortfall $F - FV$ is the additional amount the investor will be required to contribute at the end of the period to fulfill the commitment. The present shortfall $PV - P$ is the additional amount the investor is required to contribute at the present to fulfill the commitment.

Clearly, there is a fundamental connection between future and present values. However, this connection goes only so far, as there are issues of great theoretical and practical importance that distinguish future and present values. As demonstrated in a later section, similar conditions imposed on future and present values lead to different discount rates.

Uncertain *future values* generated by the uncertainties of investment returns (and commitments) play no part in financial reporting. In contrast, various actuarial and accounting reports require calculations of *present values*, and these present values must be deterministic (under current accounting standards, at least). Therefore, there is a need for a deterministic discounting procedure.

Conventional calculations of deterministic present values usually utilize a single measurement of investment returns that serves as *the discount rate*. Since there are numerous measurements of investment returns, the challenge is to select the most appropriate measurement for a particular calculation. To clarify these issues, subsequent sections discuss various measurements of investment returns.

2. MEASUREMENTS OF INVESTMENT RETURNS: RETURN SERIES

This section discusses the key measurements of series of returns and relationships between these measurements. Given a series of returns $r_1,\ldots, r_n$ it is desirable to have a measurement of the series—a single rate of return—that, in a certain sense, would reflect the properties of the series. The right measurement always depends on the objective of the measurement. The most popular measurement of a series of returns $r_1,\ldots, r_n$ is its *arithmetic average A* defined as the average value of the series:

$$A = \frac{1}{n}\sum_{k=1}^{n} r_k \qquad (2.1)$$

As any other measurement, the arithmetic average has its pros and cons. While the arithmetic average is an unbiased estimate of the return, the probability of achieving this value may be unsatisfactory. As a predictor of future returns, the arithmetic average may be too optimistic.

Another significant shortcoming of the arithmetic return is it does not "connect" the starting and ending asset values. The starting asset value multiplied by the compounded

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arithmetic return factor (1 + A) is normally greater than the ending asset value.[3] Therefore, the arithmetic average is inappropriate if the objective is to "connect" the starting and ending asset values. The objective that leads to the arithmetic average as the right choice of discount rate is presented in Section 5.

Clearly, it would be desirable to "connect" the starting and ending asset values—to find a single rate of return that, given a series of returns and a starting asset value, generates the same future value as the series. This observation suggests the following important objective.

Objective 1: To "connect" the starting and ending asset values.

The concept of *geometric average* is specifically designed to achieve this objective. If A_0 and A_n are the starting and ending asset values correspondingly, then, by definition,

$$A_0 (1+r_1) \ldots (1+r_n) = A_n \quad (2.2)$$

The geometric average G is defined as the single rate of return that generates the same future value as the series of returns. Namely, the starting asset value multiplied by the compounded return factor $(1 + G)^n$ is equal to the ending asset value:

$$A_0 (1+G)^n = A_n \quad (2.3)$$

Combining (2.2) and (2.3), we get the standard definition of the geometric average G:

$$G = -1 + \prod_{k=1}^{n} (1+r_k)^{1/n} \quad (2.4)$$

Let us rewrite formulas (1.2) and (1.3) in terms of present and future values. If A_n is a future payment and $r_1,\ldots, r_n$ are the investment returns, then the present value of A_n is equal to the payment discounted by the geometric average:

Thus, the geometric average connects the starting and ending asset values (and the arithmetic average does not). Therefore, *if the primary objective of discount rate selection is to connects the starting and ending asset values, then the geometric average should be used for the present value calculations*.

$$A_0 = \frac{A_n}{(1+r_1)\ldots(1+r_n)} = \frac{A_n}{(1+G)^n} \quad (2.5)$$

To present certain relationships between arithmetic and geometric averages, let us define variance V as follows:[4]

$$V = \frac{1}{n}\sum_{k=1}^{n} (r_k - A)^2$$

If V = 0, then all returns in the series are the same, and the arithmetic average is equal to the geometric average. Otherwise (if $V > 0$), the arithmetic average is greater than the geometric average ($A > G$).[5]

There are several approximate relationships between arithmetic average A, geometric average G, and variance V. These relationships include the following relationships that are denoted (R1) – (R4) in this paper.

$$G \approx A - V/2 \quad \text{(R1)}$$

$$(1+G)^2 \approx (1+A)^2 - V \quad \text{(R2)}$$

$$1+G \approx (1+A)\exp\left(-\frac{1}{2}V(1+A)^{-2}\right) \quad \text{(R3)}$$

$$1+G \approx (1+A)\left(1+V(1+A)^{-2}\right)^{-1/2} \quad \text{(R4)}$$

These relationships produce different results, and some of them work better than the others in different situations. Relationship (R1) is the simplest, popularized in many publications, but usually suboptimal and tends to underestimate the geometric return.[6] Relationships (R2) – (R4) are slightly more complicated, but, in most cases, should be expected to produce better results than (R1).

The geometric average estimate generated by (R4) is always greater than the one generated by (R3), which in turn is always greater than the one generated by (R2).[7] Loosely speaking,

$$(R2) < (R3) < (R4)$$

In general, "inequality" (R1) < (R2) is not necessarily true, although it is true for most practical examples. If $A > V/4$, then the geometric average estimate generated by (R1) is less than the one generated by (R2).[8]

There is some evidence to suggest that, for historical data, relationship (R4) should be expected to produce better results than (R1) – (R3). See Mindlin [2010] for more details regarding the derivations of (R1) – (R4) and their properties.

<u>*Example 2.1*</u>.

$n = 2$, $r_1 = -1\%$, $r_2 = 15\%$. Then arithmetic mean A, geometric mean G, and variance V are calculated as follows.

$$A = \frac{1}{2}(-1\% + 15\%) = 7.00\%$$

$$G = \sqrt{(1-1\%)(1+15\%)} - 1 = 6.70\%$$

$$V = \frac{1}{2}\sum_{k=1}^{2}(r_k - A)^2 = 0.64\%$$

Note that $(1+G)^2 = (1+A)^2 - V$, so formula (R2) is exact in this example. Given \$1 at the present, future value FV is

$$FV = 1 \times (1-1\%)(1+15\%) = 1.1385$$

If we apply arithmetic return A to \$1 at the present for two years, we get

$$(1+7\%)^2 = 1.1449$$

which is greater than future value FV = 1.1385.

If we apply geometric return G to \$1 at the present for two years, we get

$$(1+6.70\%)^2 = 1.1385$$

which is equal to future value FV, as expected.

Given \$1 in two years, present value PV is

$$PV = \frac{1}{(1-1\%)(1+15\%)} = 0.8783$$

If we discount \$1 in two years using geometric return G, we get

$$\frac{1}{(1+6.70\%)^2} = 0.8783$$

which is equal to present value PV, as expected.

If we discount \$1 in two years using arithmetic return A, we get

$$\frac{1}{(1+7.00\%)^2} = 0.8734$$

which is less than present value PV = 0.8783.

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3. CAPITAL MARKET ASSUMPTIONS AND PORTFOLIO RETURNS

This section introduces capital market assumptions for major asset classes and outlines basic steps for the estimation of portfolio returns.

It is assumed that the capital markets consist of n asset classes. The following notation is used throughout this section:

m_i mean (arithmetic) return;
s_i standard deviation of return;
c_{ij} correlation coefficient between asset classes i and j.

A portfolio is defined as a series of weights $\{w_i\}$, such that $\sum_{i=1}^{n} w_i = 1$. Each weight w_i represents the fraction of the portfolio invested in the asset class i.

Portfolio mean return A and variance V are calculated as follows:

$$A = \sum_{i=1}^{n} w_i m_i \qquad (3.1)$$

$$V = \sum_{i,j=1}^{n} w_i w_j s_i s_j c_{ij} \qquad (3.2)$$

Let us also define *return factor* as $1 + R$. It is common to assume that the return factor has *lognormal* distribution (which means $\ln(1 + R)$ has *normal* distribution). Under this assumption, parameters μ and σ of the lognormal distribution are calculated as follows:

$$\sigma^2 = \ln\left(1 + V(1+A)^{-2}\right) \qquad (3.3)$$

Using σ calculated in (3.3), parameter μ of the lognormal distribution is calculated as follows:

$$\mu = \ln(1 + A) - \frac{1}{2}\sigma^2 \qquad (3.4)$$

Given parameters μ and σ, the P^{th} percentile of the return distribution is equal to the following:

$$R_P = \exp(\mu + \sigma\Phi^{-1}(P)) - 1 \qquad (3.5)$$

where Φ is the standard normal distribution. In particular, if $P = 50\%$, then $\Phi^{-1}(P) = 0$. Therefore, the median of the return distribution under the lognormal return factor assumption is calculated as follows.

$$R_{0.5} = \exp(\mu) - 1 \qquad (3.6)$$

<u>*Example 3.1*</u>. Let us consider two uncorrelated asset classes with mean returns 8.00% and 6.00% and standard deviations 20.00% and 10.00% correspondingly. If a portfolio has 35% of the first class and 65% of the second class, its mean and variance are calculated as follows.

$$A = 8.00\% \times 35\% + 6.00\% \times 65\% = 6.70\%$$

$$V = (20.00\% \times 35\%)^2 + (10.00\% \times 65\%)^2 = 0.9125\%$$

It is interesting to note that the standard deviation of the portfolio is 9.55% $(= \sqrt{0.9125\%})$, which is lower than the standard deviations of the underlying asset classes (20.00% and 10.00%). Assuming that the return factor of this portfolio has lognormal distribution, the parameters of this distribution are

$$\sigma = \sqrt{\ln\left(1 + \frac{0.9125\%}{(1+6.70\%)^2}\right)} = 0.0893$$

$$\mu = \ln(1 + 6.70\%) - \frac{0.0893^2}{2} = 0.0609$$

From (3.5), the median return for this portfolio is

$$R_{0.5} = \exp\left(0.0609 + 0.0893 \cdot \Phi^{-1}(0.5)\right) - 1 = 6.27\%$$

From (3.5), the 45th percentile for this portfolio is

$$R_{0.45} = \exp\left(0.0609 + 0.0893 \cdot \Phi^{-1}(0.45)\right) - 1 = 5.09\%$$

4. MEASUREMENTS OF INVESTMENT RETURNS: RETURN DISTRIBUTIONS

The previous section presented the relationships between the arithmetic and geometric averages defined for a series of returns. This section develops similar results when *return distribution R is given*.

In this case, the *arithmetic* average (mean) return A is defined as the expected value of R:

$$A = E(R) \tag{4.1}$$

The *geometric* average (mean) return G is defined as follows:

$$G = \exp(E(\ln(1+R))) - 1 \tag{4.2}$$

These arithmetic and geometric average returns are the limits of the arithmetic and geometric averages of appropriately selected series of independent identically distributed returns. Specifically, let $\{r_k\}$ be a series of independent returns that has the same distribution as R. Let us define arithmetic averages A_n and geometric averages G_n for $r_1, \ldots, r_n$:

$$A_n = \frac{1}{n}\sum_{k=1}^{n} r_k \tag{4.3}$$

$$G_n = -1 + \prod_{k=1}^{n}(1+r_k)^{1/n} \tag{4.4}$$

According to the Law of Large Numbers (LLN), A_n converge to E. Also, from (4.4) we have.

$$\ln(1+G_n) = \frac{1}{n}\sum_{k=1}^{n}\ln(1+r_k) \tag{4.5}$$

Again, according to the LLN, $\frac{1}{n}\sum_{k=1}^{n}\ln(1+r_k)$ converges to the expected value $E(\ln(1+R))$. From (4.5), $\ln(1+G_n)$ converges to $E(\ln(1+R))$ as well. Consequently, G_n converge to $\exp(E(\ln(1+R))) - 1$, which, according to (4.2), is equal to G.

To recap, A_n converges to A and G_n. converges to G when n tends to infinity. As discussed above, the approximations (R1) – (R4) are true for A_n and G_n , where V_n is defined as in (2.6):

$$V_n = \frac{1}{n}\sum_{k=1}^{n}(r_k - A_n)^2 \tag{4.6}$$

Since V_n converge to the variance of returns V when n tends to infinity, the approximations (R1) – (R4) are true for A and G as well. As was discussed before, if the primary objective of discount rate selection is to connects the starting and ending asset values, then the geometric mean is a reasonable choice for the discount rate.

This conclusion, however, is valid over relatively long time horizons only. Over shorter time horizons, the geometric average of series $\{r_k\}$ has non-trivial volatility and cannot be considered approximately constant. More importantly, the investor may have objectives other than connecting the starting and ending asset values. All in all, additional conditions of stochastic nature may be required to select a reasonable discount rate. Such conditions are discussed in the next section.

For large n, the Central Limit Theorem (CLT) can be used to analyze the stochastic properties ofthe geometric average. According to the CLT applied to $\frac{1}{n}\sum_{k=1}^{n}\ln(1+r_k)$, the geometric average return factor $1+G_n$ defined as

$$1+G_n = \prod_{k=1}^{n}(1+r_k)^{1/n} = \exp\left(\frac{1}{n}\sum_{k=1}^{n}\ln(1+r_k)\right)$$

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is approximately lognormally distributed. If the mean and standard deviation of $\ln(1+r_k)$ are μ and σ correspondingly, then the parameters of the geometric average return factor are μ and $\frac{\sigma}{\sqrt{n}}$.

Assuming that the return factor has *lognormal* distribution, it can be shown that relationship (R4) is exact:[9]

$$1+G=(1+A)(1+V(1+A)^{-2})^{-1/2} \tag{4.7}$$

An important property of lognormal return factors is *the geometric mean return is equal to the median return*. Indeed, if μ and σ are the parameters of the lognormal distribution, then $\ln(1+R)$ is normal and

$$G=\exp(E(\ln(1+R)))-1=\exp(\mu)-1 \tag{4.8}$$

which is the median of the return distribution according to (3.6).

Thus, if a discount rate were chosen at random (not that this is a great idea), then there would be a 50 percent chance for the discount rate to be greater than the geometric mean and a 50 percent chance to be less than the geometric mean. Similarly, if a present value were calculated using a randomly selected discount rate, then there would be a 50 percent chance that a present value is greater than the present value calculated using the geometric mean.[10]

Given arithmetic mean A and variance V, formula (4.7) produces geometric return G. If there is a need to calculate the arithmetic mean when the geometric mean and the variance are given, then the arithmetic mean is calculated as follows:

$$1+A=(1+G)\sqrt{\frac{1}{2}+\frac{1}{2}\sqrt{1+\frac{4V}{(1+G)^2}}} \tag{4.9}$$

Example 4.1. This example is a continuation of *Example 3.1*. In this example, A = 6.70% and V = 0.9125%. According to (4.7),

$$G=\frac{1+0.067}{\sqrt{1+\frac{0.009125}{(1+0.067)^2}}}-1=6.27\%$$

which is equal to the median return calculated in *Example 3.1*. Note that the geometric returns for the individual asset classes are 6.19 percent and 5.53 percent. It is noteworthy that the geometric return for the portfolio that has 35 percent of the first class and 65 percent of the second class is 6.27 percent, which is higher than the geometric returns of the individual classes.

Let us take a look at the stochastic properties of the geometric average for this portfolio. Under the lognormal return factor assumption, the parameters of the return distribution are μ = 0.0609 and σ = 0.0893 (see *Example 3.1*). If n = 10, then the geometric average return factor $1+G_n$ is approximately lognormally distributed with parameters μ = 0.0609 and σ = 0.283. The mean, median and standard deviation are 6.32 percent, 6.27 percent and 3.00 percent correspondingly. Note significant decreases of the mean and standard deviation of the geometric average compared to the original return distribution (6.32 percent vs. 6.70 percent and 3.00 percent vs. 9.55 percent), while the median remains the same.

5. EXAMPLES OF DISCOUNT RATE SELECTION

As was discussed in the previous section, the investor may have objectives other than connecting the starting and ending asset values. This section discusses and presents three additional examples of such objectives that lead to the selection of discount rates.

Let us consider a simple modification of the funding problem discussed earlier in the paper. Think of an investor that has made a commitment to accumulate $\$F$ at the end of the period by means of investing in a portfolio that generates (uncertain) investment return R. To fund the commitment, the investor wants to make a contribution that is subject to certain conditions.

For convenience, let us recall *Objective 1* introduced in Section 2:

Objective 1: To "connect" the starting and ending asset values.

As was demonstrated in Section 2, the right discount rate for this objective is the geometric return.

Objective 2: To have a "safety cushion."

Let us assume that the investor's objective is to have more than a 50 percent chance that investment returns are greater than the discount rate (the "safety cushion"). For example, if it is required to have a *P*% chance that the investment return is greater than the discount rate, then the discount rate that delivers this safety level is *the (100 - P)th percentile of the return distribution*.

Objective 3: No expected gains/losses ***in the future****.*

Let us assume that the investor's objective is to have neither expected gains nor losses *at the end of the period*. If C_f is the investor's contribution at the present, then this objective implies that the commitment is the mean of the (uncertain) future value of C_f:

$$0 = E(FV) = E(C_f(1+R) - F) \quad (5.1)$$

Equation (5.1) gives the following formula for contribution C_f (subscript f in C_f indicates that the objective is "no expected gains or losses *in the future*"):

$$C_f = \frac{F}{1+E(R)} \quad (5.2)$$

Formula (5.2) shows that the objective "no expected gains or losses in the future" leads to contribution C_f calculated as the present value of the commitment using the arithmetic mean return. Hence, the right discount rate d_f for this objective is the arithmetic mean return:

$$d_f = E(R) \quad (5.3)$$

As discussed in a prior section, there is a certain symmetry and fundamental connection between future and present values. In light of this discussion, the following objective is a natural counterpart to *Objective 3*.

Objective 4: No expected gains/losses ***at the present****.*

At first, this objective looks somewhat peculiar. Everything is supposed to be known at the present, so what kind of gains/losses can exist now? But remember that the asset value required to fund the commitment—the present value of the commitment—is uncertain at the present. Therefore, the objective "today's contribution is the mean of the present value of the commitment" is as meaningful as the objective "the commitment is the mean of the future value of today's contribution" discussed in *Objective 3*.

If C_p is the contribution the investor makes at the present, then the objective "no expected gains/losses at the present" implies the following equation.

$$E(PV) = E\left(\frac{F}{1+R} - C_p\right) = 0 \quad (5.4)$$

Equation (5.4) gives the following formula for contribution C_p (subscript p in C_p indicates that the objective is "no expected gains or losses *at the present*"):

$$C_p = F \cdot E\left(\frac{1}{1+R}\right) \quad (5.5)$$

Formula (5.5) shows that the objective "no expected gains or losses *at the present*" leads to contribution C_p that is equal to the present value of the commitment using discount rate d_p:

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$$C_p = \frac{F}{1+d_p} \qquad (5.6)$$

where d_p is calculated from (5.5) and (5.6) as

$$d_p = \frac{1}{E\left(\frac{1}{1+R}\right)} - 1 \qquad (5.7)$$

Note that Jensen inequality entails

$$E\left(\frac{1}{1+R}\right) > \frac{1}{1+E(R)} \qquad (5.8)$$

Therefore, $d_p < d_f$.

Under the lognormal return factor assumption, we can tell more about discount rate d_p .
Defining ρ_R as

$$\rho_R = 1 + \frac{V}{\left(1+E(R)\right)^2} \qquad (5.9)$$

where V is the variance of return R, it can be shown that the expected value of the reciprocal return factor is

$$E\left(\frac{1}{1+R}\right) = \frac{\rho_R}{1+E(R)} \qquad (5.10)$$

Combining (5.7) and (5.10), we get

$$d_p = \frac{1+E(R)}{\rho_R} - 1 \qquad (5.11)$$

Furthermore, under the lognormal return factor assumption, there is an interesting relationship between the geometric mean return G and discount rates d_p and d_f generated by *Objective 3* and *Objective 4*:

$$1+G = \sqrt{\left(1+d_p\right)\left(1+d_f\right)} \qquad (5.12)$$

Thus, the geometric mean return G is the "geometric midpoint" between the discount rates generated by the objectives of no expected gains/losses in the future and at the present.

Example 5.1. This example is a continuation of *Example 3.1* and *Example 4.1*. As in these examples, A = 6.70% and V = 0.9125% . Then $\rho_R = 1.0080$ and

d_f = 6.70%
d_p = 5.85%

The 45th percentile of the return distribution is $R_{0.45}$ = 5.09% (see *Example 3.1*).

CONCLUSION

The selection of a discount rate is one of the most important assumptions for the calculations of present values. This paper presents the basic properties of the key measurements of investment returns and the discount rates associated with these measurements.

The paper shows that *the selection of the discount rate depends on the objective of the calculation*. The paper demonstrates the selection of discount rates for the following four objectives.

Objective 1: To "connect" the starting and ending asset values. The correct discount rate for this objective is *the geometric mean return*.

Objective 2: To have a certain "safety cushion." The correct discount rate for this objective is *the (100 - P)th percentile of the return distribution* if it is required to have a *P*% chance that the investment return is greater than the discount rate.

Objective 3: No expected gains/losses in the future. The correct discount rate for this objective is *the arithmetic mean return*.

Objective 4: No expected gains/losses at the present. The correct discount rate for this objective is given in formula (5.7).

It is worth reminding that the main purpose of a discount rate is to calculate a deterministic present value. Yet, present values associated with vital funding problems are inherently stochastic. As a result, the presence of a discount rate assumption has significant pros and cons. The primary advantage of a discount rate is the simplicity of calculations. The main disadvantage is a discount rate based deterministic present value cannot adequately describe the present value of an uncertain financial commitment funded via investing in risky assets. This author believes that the direct analysis of present values and their stochastic properties is the most appropriate approach to the process of funding financial commitments, but this subject is outside of the scope of this paper.

This author hopes that the paper would be useful to practitioners specializing in the area of funding financial commitments.

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ENDNOTES

1 There are exceptions, e.g., an inflation-adjusted cash flow with a matching TIPS portfolio.
2 See Mindlin [2009] for more details.
3 That is as long as the returns in the series are not the same.
4 For the purposes of this paper, the concerns that the sample variance as defined in (2.6) is not an unbiased estimate are set aside.
5 This fact is a corollary of the Jencen's inequality.
6 For example, see Bodie [1999], p. 751, Jordan [2008], p. 25, Pinto [2010], p. 49, Siegel [2008], p. 22, DeFusco [2007], p 128, 155.
7 That is, obviously, as long as the returns in the series are not the same and $V > 0$.
8 Mindlin [2010] contains a simple example for which (R1) > (R2).
9 See Mindlin [2010] for more details.
10 The presence of discount rate is critical for these observations. In general, the median of the present value distribution calculated using the full range of returns (and without discount rates) is not equal to the present value calculated using the geometric mean (except when the cash flow contains just one payment). In other words, the median of present value is not the same as the present value at the median return. See Mindlin [2009] for more details regarding stochastic present values.

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Dimitry Mindlin, ASA, MAAA, PhD, is president of CDI Advisors LLC. He can be contacted at dmindlin@cdiadvisors.com.

A Call For Essays

The Investment Section of the Society of Actuaries has issued a call for essays to address "Investment Fallacies."

Over the past decade, the investment world has been buffeted by unprecedented events. Many long-standing beliefs or assumptions held by investment professionals may no longer apply to the new realities. At the same time, many common myths and misconceptions that have been previously debunked continue to influence investors today.

We are seeking essays that identify and expose these fallacies, and plan to share these essays as a resource for investment actuaries and other interested investors later this year. Each essay should describe an investment fallacy and explore its implications, as well as suggest a possible antidote. For example, targeting the pernicious influence of cognitive biases, or the impact of suspect economic theories on investment decisions, are both fair game. Authors may wish to cite existing research or conduct their own analysis. Hence, do the "Dogs of the Dow" really outperform in the following year?

Each essay should be no more than 1,500 words long (or approximately two pages) written in English. You may submit multiple essays, and the collaboration of multiple authors on a jointly-written essay is also permitted.

At the discretion of the reviewing committee, prizes may be awarded to papers of exceptional quality in the following amounts:

1st Place Prize - $500
2nd Place Prize - $300
3rd Place Prize - $200

The final format of the publication of the essays will be an e-book released by the summer 2014.

Feel free to pass along any questions to David Schraub, FSA, CERA, at DSchraub@soa.org, who is coordinating the call for papers and their subsequent publication.

Please submit your essay to Christy Cook at *CCook@soa.org* no later than **March 31, 2014.**

Larry Zhao, FSA, CERA, CFA, PhD
Zhaol1@nationwide.com

Frank Grossman, FSA, FCIA, MAAA
Craigmore54@hotmail.ca

LESSONS LEARNED AND LINGERING QUESTIONS ABOUT THE EQUITY RISK PREMIUM (ERP)

By Tom Anichini

Editor's Note: This article summarizes a session topic on which the author moderated at the 2013 Annual Meeting.

At the 2013 Annual Meeting I enjoyed the privilege of moderating the Equity Risk Premium session with presenters Victor Modugno, FSA, and Brett Hammond, Ph.D. Hammond and Modugno had already presented together twice on this topic previously—for a webcast this past summer and at the Investment Symposium. Both have written on the topic: Hammond co-edited *Rethinking the Equity Risk Premium* (search *cfa-pubs.org* for "Research Foundation Publications" in 2011); Modugno wrote *Estimating Equity Risk Premiums* (Search *soa.org* for "Completed Research Projects—Pension" in 2012).

In the course of reading the panelists' monographs and exploring their citations I learned some subtleties about the topic I had not previously appreciated, and am left with some lingering questions.

FOUR DIFFERENT DEFINITIONS OF ERP

Pablo Fernandez (cited in Modugno's paper) teaches Finance at University of Navarra in Spain. His working paper, *The Equity Premium in 150 Textbook*, takes textbook authors to task for several sins, including not defining all four ERP types: Historic, Expected, Implied and Required. (Search *ssrn.com*.)

BEST MODEL? DEPENDS ON THE FORECAST HORIZON

In Modugno's paper he compares different models' long horizon (20-, 30-, 40- and 50-year) forecasts of the ERP, using data available beginning some 50 years ago. Keeping in mind it reflects very few data points, I find the visual comparison of the models' accuracy over different horizons enlightening, and question why anyone might rely solely on the Historic ERP.

IS THE ERP A PREMIUM ON A SINGLE RISK, OR ON A MOSAIC OF RISKS?

A theme emerging among risk model vendors (e.g., MSCI, Axioma, Northfield) is to think not of a single ERP but instead of as premia on multiple risk factors, such as Size, Value, Momentum, Credit, Illiquidity, Volatility, etc. Hammond (who is managing director and head of Index Applied Research at MSCI) touched on this briefly. Such factors have long been part of quantitative risk and return models, and they have crept into the investment actuary's world as well. In 2013 the Investment Section awarded the Redington Prize for "LDI in a Risk Factor Framework." In the future perhaps we will regard the concept of a monolithic ERP as an anachronism.

ARE WE TOO OPTIMISTIC RIGHT NOW?

Much ERP literature finds investor sentiment makes for a contrary indicator of future returns. In "Expectations of Returns and Expected Returns" (*Review of Financial Studies*, forthcoming) authors Robin Greenwood and Andrei Shleifer conclude that investors extrapolate recent performance too much to be rational. Does this apply even to actuaries? Prior to the 2013 ERP session the Society of Actuaries (SOA) staff surveyed registered attendees on their expectation of real equity returns over the next 10 years, in a question phrased identically to one we asked attendees to a session in October 2011. In 2011, the mean response was 5 percent; in 2013, it was 6 percent. Perhaps future earnings growth will justify the increase in attendees' optimism.

Thomas M. Anichini, ASA, CFA, is a senior investment strategist at GuidedChoice. He may be reached at tanichini@guidedchoice.com.

2013 INVESTMENT CONTEST WRAP-UP

By Tom Anichini & Frank Grossman

"The party's over
It's time to call it a day
They've burst your pretty balloon
And taken the moon away
It's time to wind up the masquerade
Just make your mind up the piper must be paid"
– Betty Comden & Adolph Green

Hopefully you didn't read the news that the equity party is over, or soon will be over, here first. The recent signs have been unmistakable. Whenever our central bankers suggest that it might—just maybe, when you think about it—be time to terminate their soft dollar policy, otherwise known as quantitative easing, a market sell-off ensues. What's an actuary to do? One alternative is to step back from the punch bowl a little, fix the door that you came in by in the corner of your eye, and start charting a path from your present location out through same doorway for personal use once the music stops. Having attended to that small matter, why not turn our attention to the results of the 2013 Investment Contest.

During the second and third quarters of 2013 the Investment Section hosted an asset allocation contest, which attracted more than 100 entrants. The contest required each entrant to allocate a notional portfolio among the following 10 Exchange Traded Products[1] (ETPs):

Ticker Name	Full Name
GSY	Guggenheim Enhanced Short Duration Bond ETF
TIP	iShares Barclays TIPS Bond ETF
BND	Vanguard Total Bond Market ETF
RWO	SPDR Dow Jones Global Real Estate ETF
DBC	PowerShares DB Commodity Index Tracking Fund
VTI	Vanguard Total Stock Market ETF
EFA	iShares MSCI EAFE Index ETF
EEM	iShares MSCI Emerging Markets Index ETF
ACWI	iShares MSCI ACWI Index ETF
GLD	SPDR Gold Shares

Eligible entrants could win a prize in any one of three categories: the highest cumulative return, lowest annualized volatility, and highest reward/risk ratio. The performance measurement period roughly included half a year, from April through September. Performance was measured as if portfolios were rebalanced to their initial allocation weekly, at a zero transaction cost.

HIGHEST CUMULATIVE RETURN WINNER: MARGUERITE BOSLAUGH

Over such a short time span, it is reasonable to expect that a concentrated portfolio would have the highest return, so we were surprised not to see more single-asset portfolios submitted. Only six entries concentrated their allocations in a single instrument, and all six were unique. Marguerite Boslaugh won the prize for highest cumulative return with a portfolio entirely comprised of VTI. Interestingly, hers was the only single-instrument portfolio with a positive return during the six month contest period.

LOWEST VOLATILITY WINNER: VLADIMIR MARTINAK

Anticipating a flood of entries with a portfolio of 100 percent GSY, we limited allocations to that ETF at 20 percent. As a result, we received a veritable flood of entries (15) each with the same allocation: 20 percent GSY/80 percent BND. This choice turned out to be the lowest volatility combination. Fortunately, as a tie breaker, we required entrants predict their eventual volatility. Vladimir Martinak's prediction of 3.50 percent was closest to the actual volatility of 3.90 percent, measured weekly and annualized.

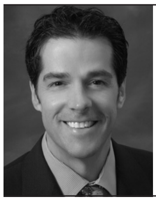
HIGHEST RETURN/VOLATILITY RATIO WINNER: MELISSA KNOPP

Not wishing to entertain quibbles about anybody's connotation of Sharpe Ratio, we simply called this category Ratio of Return/Volatility. It might not be a great surprise to learn that the highest returning portfolio over such a short time span also enjoyed the highest return/volatility ratio. As the contest rules allowed each entrant only one prize, this category's prize went to Melissa Knopp, whose five-instrument portfolio of BND, RWO, DBC, VTI and ACWI ranked next in line after Marguerite.

Looking ahead, plans are already underway for a follow-up 2014 Investment Contest along the same lines as last year's contest. We will modify the rules to allow a few interim portfolio reallocations and to reflect transaction costs. Watch your email inbox for an invitiation to the 2014 contest.

ENDNOTES

[1] We refer to the choices in this contest as ETPs and not as Exchange Traded Funds (ETFs) because the menu includes both ETFs and DBC, which is an Exchange Traded Note (ETN). An ETN is an unsecured debt instrument for which the sponsor promises to pay the relevant return. Unlike ETFs, ETNs entail counterparty risk. However, for some asset classes and strategies—typically any that involves futures—liquid, affordable ETFs do not exist and ETNs are the only choice available.



Thomas M. Anichini, ASA, CFA, is a senior investment strategist at GuidedChoice. He may be reached at tanichini@guidedchoice.com.

Frank Grossman, FSA, FCIA, MAAA, is a Toronto-based actuary who can be reached at craigmore54@hotmail.ca.

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475 N. Martingale Road, Suite 600
Schaumburg, Illinois 60173
p: 847.706.3500 f: 847.706.3599
w: www.soa.org

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