GI ADV Model Solutions
Spring 2015

1. **Learning Objectives:**

4. The candidate will understand how to apply the fundamental techniques of reinsurance pricing.

Learning Outcomes:

(4c) Calculate the price for a casualty per occurrence excess treaty.

Sources:

Basics of Reinsurance Pricing – Clark

Solution:

(a) Estimate the experience rating loss and ALAE cost as a percentage of the subject premium.

Commentary on Question:

Candidates struggled with applying the trend and development factors at the appropriate points in the calculations. There was also some difficulty in properly allocating ALAE.

Trended loss and ALAE in layer for each claim:

- 400,000 x 1.05^4 = 486,203 < 500,000. Therefore, there is 0 loss in layer and 0 ALAE in layer.
- 750,000 x 1.05^4 = 911,630. Loss in layer is 911,630 – 500,000 = 411,630. ALAE is 100,000 x 1.05^4 = 121,551. ALAE in layer is 121,551 x (411,630/911,630) = 54,884.
- 450,000 x 1.05^3 = 520,931. Loss in layer is 520,931 – 500,000 = 20,931. ALAE is 0, so ALAE in layer is 0.
- 900,000 x 1.05^3 = 1,041,863. Loss in layer is 1,000,000 – 500,000 = 500,000. ALAE is 1,000,000 x 1.05^3 = 1,157,625. ALAE in layer is 1,157,625 x (500,000/1,000,000) = 578,813.
- 500,000 x 1.05^2 = 551,250. Loss in layer is 551,250 – 500,000 = 51,250. ALAE is 100,000 x 1.05^2 = 110,250. ALAE in layer is 110,250 x (51,250/551,250) = 10,250.
- 1,000,000 x 1.05^2 = 1,102,500. Loss in layer is 1,000,000 – 500,000 = 500,000. ALAE is 0, so ALAE in layer is 0.

1. Continued

Developed losses and ALAE for each year:
2012: (411,630 + 54,884) x 1.10 = 513,165
2013: (20,931 + 500,000 + 578,813) x 1.50 = 1,649,616
2014: (51,250 + 10,250 + 500,000) x 3.00 = 1,684,500
Total is 3,847,281

Rate is 3,847,281/30,000,000 = 12.8%.

(b) State an alternative model to experience rating, and identify any additional information you would need to apply this alternative.

Commentary on Question:
Candidates generally did well on this part, but some did not state all the additional information needed.

Exposure rating is an alternative approach. Additional information needed is:

- Expected loss ratio;
- Increased limits factors; and
- An assumption regarding how ALAE relates to the distribution of losses.

(c) Recommend the model to which you would assign greater credibility in this case. Justify your recommendation.

The exposure rating approach should receive greater credibility. There are only one or two losses per year in the layer. This is not enough data to provide a credible experience rate.

2. Learning Objectives:

2. The candidate will understand the considerations in selecting a risk margin for unpaid claims.

Learning Outcomes:

(2a) Describe a risk margin analysis framework.

(2b) Identify the sources of uncertainty underlying an estimate of unpaid claims.

(2c) Describe methods to assess this uncertainty.

Sources:

A Framework for Assessing Risk Margins – Marshall, Collings, Hodson, O'Dowd

Solution:

(a) Define internal systemic risk.

Internal systemic risk refers to the uncertainty arising from the valuation models used being an imperfect representation of the insurance process as it relates to the liabilities.

(b) Calculate the required risk margin.

The combined CoV is $\sqrt{0.048^2 + 0.119^2 + 0.055^2} = 0.140$. The risk margin is 0.140(1.282)(1000) = 179.

(c) Identify two factors that may be causing the differences in CoV between these lines of business.

Factors include differences in size, complexity, inherent risk, and reporting lag.

(d) Explain how the two factors identified in part (c) may be causing the differences in CoV between these lines of business.

Generally, variance is inversely proportional to the size of the line. Thus a larger line of business will have a smaller CoV.

Greater complexity, inherent risk, or reporting lag will increase the standard deviation relative to the mean, and thus lead to a larger CoV.

2. **Continued**

(e) Explain how each of the following changes would be likely to affect your assessment of internal systemic risk:

(i) Including the Bornhuetter Ferguson method

(ii) Including frequency-severity methods

(iii) A significant increase in claims processing times

(iv) Estimating claim liabilities at the subline level

(v) Future unexpected legislative change

Commentary on Question:
Candidates generally did well. At times the change in risk assessment was correct but the explanation not complete.

- Including the B-F method: Adding an additional approach can reduce risk.
- Including F-S methods: Adding an additional approach can reduce risk, and this will likely be a greater reduction than adding B-F.
- Significant increase in claims processing times: The model will need to change and there will be limited information available to build the model, hence increased risk.
- Estimating claim liabilities at the subline level: Risk will be reduced but only to the extent that homogeneity more than offsets the reduced sample size.
- Future unexpected legislative change: This is an external systemic risk, and so would not affect internal systemic risk.

3. **Learning Objectives:**

1. The candidate will understand how to use basic loss development models to estimate the standard deviation of an estimator of unpaid claims.

Learning Outcomes:

(1e) Apply a parametric model of loss development.

(1f) Estimate the standard deviation of a parametric estimator of unpaid claims.

Sources:

LDF Curve Fitting and Stochastic Reserving: A Maximum Likelihood Approach – Clark

Solution:

(a) Provide the fitted triangle of cumulative paid losses.

The fitted values are, by row:
4236, 7174, 7826
4236, 7174
6000
The calculations are:
4236 = 12,000(0.6677)[exp(–0/7.975) – exp(–6/7.975)]
7174 = 12,000(0.6677)[exp(–0/7.975) – exp(–18/7.975)]
7826 = 12,000(0.6677)[exp(–0/7.975) – exp(–30/7.975)]
6000 = 11,347[exp(–0/7.975) – exp(–6/7.975)]

(b) Calculate the reserve for all accident years combined.

2012: 12,000(0.6677) – 7826 = 186
2013: 12,000(0.6677) – 7174 = 838
2014: 11,347 – 6000 = 5,347
Total = 6,371

(c) Identify the number of degrees of freedom associated with the estimate of the scale factor, σ^2.

6 observations – 3 parameters = 3 degrees of freedom.

(d) Calculate the correlation between the estimate of *ELR* and the estimate of ULT_{2014}.

The correlation is 30.0/sqrt(0.00676 x 6,529,363) = 0.143.

3. **Continued**

(e) Provide an expression for the estimate of the parameter covariance between the accident year 2013 reserve and the accident year 2014 reserve using matrix notation. (Do not compute the result.)

Commentary on Question:
Candidates did poorly on this question. The solution provides the steps needed in detail.

The 2013 reserve is (see solutions to (a) and (b)):
$12,000(ELR) - 12,000(ELR)(e^{-0/\theta} - e^{-18/\theta}) = 12,000(ELR)e^{-18/\theta}$
The derivatives with respect to the parameters are (which then form the elements of the first vector, with the parameter estimates inserted):
$d / dELR = 12,000e^{-18/\theta}$

$d / dULT_{2014} = 0$

$d / d\theta = 12,000(ELR)e^{-18/\theta}(18/\theta^2)$
Similarly, the 2014 reserve and derivatives are:
$ULT_{2014} - ULT_{2014}(1 - e^{-6/\theta}) = ULT_{2014}(e^{-6/\theta})$

$d / dELR = 0$

$d / dULT_{2014} = e^{-6/\theta}$

$d / d\theta = ULT_{2014}(e^{-6/\theta})(6/\theta^2)$
The matrix expression for the covariance is then:

$$\begin{pmatrix} 12000 \cdot e^{-\frac{18}{7.975}} & 0 & 8012 \cdot \frac{18}{7.975^2} \cdot e^{-\frac{18}{7.975}} \end{pmatrix} \begin{pmatrix} 0.00676 & 30.0 & 0.0314 \\ 30.0 & 6,529,363 & 2,071 \\ 0.0314 & 2,071 & 2.17 \end{pmatrix} \begin{pmatrix} 0 \\ e^{-\frac{6}{7.975}} \\ 11347 \cdot \frac{6}{7.975^2} \cdot e^{-\frac{6}{7.975}} \end{pmatrix}$$

4. Learning Objectives:

1. The candidate will understand how to use basic loss development models to estimate the standard deviation of an estimator of unpaid claims.

Learning Outcomes:

(1a) Identify the assumptions underlying the chain ladder estimation method.

(1b) Test for the validity of these assumptions.

(1c) Identify alternative models that should be considered depending on the results of the tests.

(1d) Estimate the standard deviation of a chain ladder estimator of unpaid claims.

Sources:

Measuring the Variability of Chain Ladder Reserve Estimates – Mack and Testing the Assumptions of Age-to-Age Factors – Venter

Solution:

(a) Demonstrate that the value of α_{10}^2 was correctly calculated. (Your calculation need not match to all three decimal places.)

$$\alpha_{10}^2 = \frac{1}{12-10-1}\sum_{j=1}^{12-10} C_{j,10}\left(\frac{C_{j,11}}{C_{j,10}} - f_{10}\right)^2 = C_{1,10}\left(\frac{C_{1,11}}{C_{1,10}} - f_{10}\right)^2 + C_{2,10}\left(\frac{C_{2,11}}{C_{2,10}} - f_{10}\right)^2$$

$$= 82,540\left(\frac{82,090}{82,540} - 0.99655\right)^2 + 26,642\left(\frac{26,715}{26,642} - 0.99655\right)^2 = 1.352$$

(b) Demonstrate that the standard error for accident year 3 was correctly calculated.

$$C_{3,12}^2 \sum_{k=12+1-3=10}^{12-1=11} \frac{\alpha_k^2}{f_k^2}\left(\frac{1}{C_{3,k}} + \frac{1}{\sum_{j=1}^{12-k} C_{j,k}}\right) = C_{3,12}^2\left[\frac{\alpha_{10}^2}{f_{10}^2}\left(\frac{1}{C_{3,10}} + \frac{1}{\sum_{j=1}^{12-10=2} C_{j,10}}\right) + \frac{\alpha_{11}^2}{f_{11}^2}\left(\frac{1}{C_{3,11}} + \frac{1}{\sum_{j=1}^{12-11=1} C_{j,11}}\right)\right]$$

$$= 41,348^2\left[\frac{1.352}{0.99655^2}\left(\frac{1}{41,123} + \frac{1}{82,540+26,642}\right) + \frac{0.575}{1.00897^2}\left(\frac{1}{40,981} + \frac{1}{82,090}\right)\right] = 113,242$$

The standard error is the square root, 337.

4. Continued

(c) Explain why the weight $1/C_{i,k}$ is consistent with the variance assumption Mack uses to obtain his standard error estimate.

Commentary on Question:
Candidates had some understanding of the issue, but rarely provided a complete explanation.

Mack's estimate is based on an assumption that the variance is proportional to the previous value. Least-squares estimates should use weights that are proportional to the reciprocal of the variance. Hence, the weights should be the reciprocal of the previous value.

(d) State the formula for the age-to-age factor f_1 that results from one of the other two weights. Verify that the calculated number (1.627 or 1.151) is correct using that formula.

The formulas for weights of 1 and 1/C^2 respectively are:

$$\frac{\sum_{i=1}^{11} C_{i,1}C_{i,2}}{\sum_{i=1}^{11} C_{i,1}^2} \text{ and } \frac{1}{11}\sum_{i=1}^{11} C_{i,2}/C_{i,1}\text{. The calculations are:}$$

$$\frac{17,652(41,350)+\cdots+4,403(4,635)}{17,652^2+\cdots+4,403^2}=1.627 \text{ and}$$

$$\frac{1}{11}\left(\frac{41,350}{17,652}+\cdots+\frac{4,635}{4,403}\right)=1.151.$$

(e) Determine, from this graph, which, if any, of the three models is reasonable. Support your answer.

Commentary on Question:
While none of the lines fits well, the line based on weights of 1 has the steepest slope. This is a better choice than the other two weights. Candidates who selected a model and picked this one received partial credit.

Regardless of weight, all three models lead to a regression line that goes through the origin. However, the scatterplot does not indicate that such a line is appropriate. Hence, none of the models is reasonable.

4. Continued

(f) Describe one such adjustment.

Commentary on Question:
Any one of the three adjustments provides a full-credit answer.

There are three adjustments suggested in the paper:

- Divide by (n – p)^2;
- Multiply by exp(2p/n); and
- Multiply by n^(p/n).

(g) Explain why this method is inappropriate for comparing the three weighted estimates.

Commentary on Question:
Several candidates noted (correctly) that n and p are the same for all three models. This makes adjustments unnecessary (as the same adjustment would be used for all three models) but does not make doing an adjustment inappropriate.

With three different weights, the weighted sums of squares are on very different scales. The adjustments are designed to reflect differing sample sizes or numbers of parameters, not differing measurement scales.

5. Learning Objectives:

5. The candidate will understand methodologies for determining an underwriting profit margin.

Learning Outcomes:

(5d) Allocate an underwriting profit margin (risk load) among different accounts.

Sources:

An Application of Game Theory: Property Catastrophe Risk Load – Mango

Solution:

(a) Calculate the renewal risk loads [using the Marginal Surplus method] for accounts X and Y.

The risk load multiplier is 2.2(0.1)/1.1 = 0.2.

- Risk load for X: The marginal standard deviation is sqrt(64,000,000) – sqrt(16,000,000) = 4000. Multiplying by the risk load multiplier of 0.2 gives the risk load of 800.
- Risk load for Y: The marginal standard deviation is sqrt(64,000,000) – sqrt(25,000,000) = 3000. Multiplying by the risk load multiplier of 0.2 gives the risk load of 600.

(b) Demonstrate that the Marginal Surplus method is not renewal additive.

The total risk load from (a) is 800 + 600 = 1400. The risk load for the combined portfolio is 0.2 x sqrt(64,000,000) = 1600. They are not equal, hence the method is not renewal additive.

(c) Calculate the renewal risk loads [using the Marginal Variance method] for accounts X and Y.

- Risk load for X: 0.000025(64,000,000 – 16,000,000) = 1200.
- Risk load for Y: 0.000025(64,000,000 – 25,000,000) = 975.

(d) Demonstrate that the Marginal Variance method is not renewal additive.

The sum of the individual risk loads is 2175, which does not equal 0.000025(64,000,000) = 1600.

(e) Calculate the renewal risk loads for accounts X and Y using the Shapley method.

Cov(X,Y) = (64,000,000 – 25,000,000 – 16,000,000)/2 = 11,500,000.

- Risk load for X: 0.000025(25,000,000 + 11,500,000) = 912.5.
- Risk load for Y: 0.000025(16,000,000 + 11,500,000) = 687.5.

5. Continued

(f) Demonstrate that the Shapley method is renewal additive.

The total is 1600, which matches the value calculated in part (d).

6. **Learning Objectives:**

3. The candidate will understand how to use a credibility model with parameters that shift over time.

Learning Outcomes:

(3a) Identify the components of a credibility model with shifting risk parameters.

(3c) Estimate the parameters of the model.

(3d) Compare various models that might be used.

Sources:

Credibility with Shifting Risk Parameters – Klugman

Commentary on Question:

Candidates performed poorly on this question. Some were able to state a small number of components for each part.

Solution:

(a) Describe the approach taken by Klugman.

Klugman's approach is based on the linear mixed model. This model allows for the incorporation of both credibility and time series elements. Time series processes are characterized by the autocorrelation structure for observations made at different times within a given group. The model also allows for variation in the overall mean from group to group. This incorporates Bühlmann-Straub credibility. The model has relatively few parameters as the same correlation structure and parameters are assumed for each group. The method of restricted maximum likelihood estimation is used to estimate the parameters. The credibility factors can come directly from the estimates or can be constrained to follow a pre-specified pattern.

(b) Explain why this approach is superior to the one proposed by Warren.

Advantages of the Klugman approach include:

- With common parameters for the time series component there are fewer parameters to estimate. Thus each estimate should be more accurate.
- There is an explicit model for the probability distributions and the relationships of observations between groups and time periods.
- REML is a superior estimation technique compared to the method of moments.
- Hypothesis tests can be used to compare alternative structures.

7. Learning Objectives:

5. The candidate will understand methodologies for determining an underwriting profit margin.

Learning Outcomes:

(5b) Calculate an underwriting profit margin using the capital asset pricing model.

(5c) Calculate an underwriting profit margin using the risk adjusted discount technique.

Sources:

Ratemaking: A Financial Economics Approach – D'Arcy and Dyer

Solution:

(a) Calculate $\dfrac{\partial P}{\partial L}$.

$$P = \frac{0.5L}{0.95} + \frac{0.5L}{0.95^2} + 20 + \frac{(P-20)(0.35)}{1.00} - \frac{L(0.35)}{0.95}$$
$$= \left(\frac{0.5}{0.95} + \frac{0.5}{0.95^2} - \frac{0.35}{0.95}\right)L + 0.35P + 20 - 20(0.35)$$
$$0.65P = 0.7119L + 13$$
$$P = 1.0952L + 20$$
$$\partial P / \partial L = 1.0952.$$

(b) Calculate the premium when L is 80.

Commentary on Question:

Full credit is given if an incorrect formula from part (a) is correctly used here.

From part (a) P = 1.0952(80) + 20 = 107.62.

(c) Describe how the assumptions would be adjusted to reflect increased risk in the amount of losses.

The risk-adjusted rate for losses should be decreased.

(d) Explain how risk is accounted for in the Capital Asset Pricing Model applied to insurance.

Risk is reflected in the underwriting beta. It only reflects risk that is systematic with investment risk.

8. **Learning Objectives:**

4. The candidate will understand how to apply the fundamental techniques of reinsurance pricing.

Learning Outcomes:

(4e) Describe considerations involved in pricing property catastrophe covers.

Sources:

Basics of Reinsurance Pricing – Clark

Solution:

(a) Calculate the nominal rate on line.

The nominal rate on line is 10,000,000/100,000,000 = 10%.

(b) Calculate the underwriting loss (excluding expenses) to Property R Us if a loss fully exhausts the limit.

The underwriting loss is
10,000,000 – 100,000,000 + 50% x (100,000,000 + 1,000,000 – 10,000,000)
= –44,500,000.

(c) Calculate the premium for an equivalent traditional risk cover.

The premium for the equivalent traditional risk cover is 10,000,000 – 90% x (90% x 10,000,000) = 1,900,000.

(d) Calculate the rate on line for an equivalent traditional risk cover.

The rate on line for the equivalent traditional risk cover is 1,900,000/(44,500,000 + 1,900,000) = 4.1%.

8. Continued

(e) Recommend whether or not Property R Us should accept the proposal. If your answer is no, offer a counterproposal. Justify your answer.

Because 4.1% < 1/15, Property R Us should reject the proposal.
A counterproposal should include:

- An increase in premium;
- A decrease in profit commission;
- An increase in margin; and/or
- An increase in the additional premium

so that the rate on line for the equivalent traditional risk cover is greater than 1/15.