
SOCIETY OF ACTUARIES
Advanced Topics in General Insurance

Exam GIADV

Date: Friday, May 1, 2015
Time: 2:00 p.m. – 4:15 p.m.

INSTRUCTIONS TO CANDIDATES

General Instructions

1. This examination has a total of 40 points.

This exam consists of 8 questions, numbered 1 through 8.

The points for each question are indicated at the beginning of the question.
2. Failure to stop writing after time is called will result in the disqualification of your answers or further disciplinary action.
3. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions on the exam booklet.

Written-Answer Instructions

1. Write your candidate number at the top of each sheet. Your name must not appear.
2. Write on only one side of a sheet. Start each question on a fresh sheet. On each sheet, write the number of the question that you are answering. Do not answer more than one question on a single sheet.
3. The answer should be confined to the question as set.
4. When you are asked to calculate, show all your work including any applicable formulas.
5. When you finish, insert all your written-answer sheets into the Essay Answer Envelope. Be sure to hand in all your answer sheets because they cannot be accepted later. Seal the envelope and write your candidate number in the space provided on the outside of the envelope. Check the appropriate box to indicate Exam GIADV.
6. Be sure your written-answer envelope is signed because if it is not, your examination will not be graded.

Tournez le cahier d'examen pour la version française.

****BEGINNING OF EXAMINATION****

- 1.** (6 points) Casualty R Us Reinsurance Company has been presented with an opportunity to write a casualty per occurrence excess treaty for accident year 2016 covering the layer 500,000 excess of 500,000.

You are given the following information:

- The following loss experience has been provided, evaluated as of 12/31/2014:

Accident Date	Untrended Loss	Untrended ALAE
7/1/2012	400,000	500,000
7/1/2012	750,000	100,000
7/1/2013	450,000	0
7/1/2013	900,000	1,000,000
7/1/2014	500,000	100,000
7/1/2014	1,000,000	0

- All losses of at least 400,000 are shown.
- All policy limits throughout the experience period are 1,000,000 and are expected to remain at this level through 2016.
- On level subject premium is 10,000,000 for each year from 2012-2014.
- ALAE is allocated to layer in proportion to losses.
- Loss and ALAE trend are each 5% per year.
- The following accident year development factors are applicable to both loss and ALAE in the layer 500,000 excess of 500,000:

12-Ultimate	3.00
24-Ultimate	1.50
36-Ultimate	1.10

- (a) (4 points) Estimate the experience rating loss and ALAE cost as a percentage of the subject premium.
- (b) (1.5 points) State an alternative model to experience rating, and identify any additional information you would need to apply this alternative.
- (c) (0.5 points) Recommend the model to which you would assign greater credibility in this case. Justify your recommendation.

2. (5 points) You are calculating a risk margin for claim liabilities using the methodology set out in “A Framework for Assessing Risk Margins.” The following information is provided:

Line of Business	Claim Liabilities	Coefficients of Variation (CoV)		
		Independent Risk	Internal Systemic Risk	External Systemic Risk
Motor	300	5.5%	8.0%	5.0%
Liability	700	6.5%	15.0%	7.0%
Total	1,000	4.8%	11.9%	5.5%

- Claims are assumed to be normally distributed.
 - Risk margins are set at the 90% adequacy level.
 - The z-value of the 90th percentile of the normal distribution is 1.282.
 - No correlation is assumed between independent risk, internal systemic risk and external systemic risk.
- (a) (0.5 points) Define internal systemic risk.
- (b) (1 point) Calculate the required risk margin.
- (c) (0.5 points) Identify two factors that may be causing the differences in CoV between these lines of business.
- (d) (0.5 points) Explain how the two factors identified in part (c) may be causing the differences in CoV between these lines of business.

Your estimates of claim liabilities are currently based on the development method, using data aggregated by line of business.

- (e) (2.5 points) Explain how each of the following changes would be likely to affect your assessment of internal systemic risk:
- (i) Including the Bornhuetter Ferguson method
 - (ii) Including frequency-severity methods
 - (iii) A significant increase in claims processing times
 - (iv) Estimating claim liabilities at the subline level
 - (v) Future unexpected legislative change

3. (5 points) You are given the following triangle of cumulative paid losses:

Accident Year	Months of Development		
	12	24	36
2012	4,000	7,000	8,000
2013	5,000	7,000	
2014	6,000		

On level premium for each of 2012 and 2013 is 12,000. On level premium for 2014 is not available.

You apply Clark's stochastic reserving model using the Cape Cod method for accident years 2012 and 2013 and the LDF method for accident year 2014. You use an exponential distribution with cumulative distribution function, $G(x) = 1 - e^{-x/\theta}$.

The maximum likelihood estimates of ELR , ULT_{2014} , and θ are 66.77%, 11,347, and 7.975, respectively.

- (1.5 points) Provide the fitted triangle of cumulative paid losses.
- (1.5 points) Calculate the reserve for all accident years combined.
- (0.5 points) Identify the number of degrees of freedom associated with the estimate of the scale factor, σ^2 .

The covariance matrix of the estimates of ELR , ULT_{2014} , and θ , respectively, is:

$$\begin{pmatrix} 0.00676 & 30.0 & 0.0314 \\ 30.0 & 6,529,363 & 2,071 \\ 0.0314 & 2,071 & 2.17 \end{pmatrix}.$$

- (0.5 points) Calculate the correlation between the estimate of ELR and the estimate of ULT_{2014} .
- (1 point) Provide an expression for the estimate of the parameter covariance between the accident year 2013 reserve and the accident year 2014 reserve using matrix notation. (Do not compute the result.)

4. (7 points) You are interested in determining the variability of unpaid claim estimates. The triangle of data you are working with is presented below. The shaded cells have been completed using the standard chain ladder method. The missing columns are not needed to respond to the items. It is assumed that all claims are fully developed after 12 years.

Mack's method of estimating reserve variability has been applied to this triangle. The key results are provided in the table.

	Development Year						
AY	1	2	3	10	11	12	Standard error
1	17,652	41,350	50,387	82,540	82,090	82,826	0
2	11,532	15,432	17,590	26,642	26,715	26,955	143
3	9,074	20,036	25,951	41,123	40,981	41,348	337
4	8,655	4,996	5,904	9,273	9,241	9,324	227
5	5,451	6,987	9,388	14,812	14,760	14,893	374
6	4,778	4,413	5,446	9,285	9,253	9,336	389
7	6,758	6,281	7,646	12,088	12,046	12,154	783
8	5,041	2,001	2,937	4,948	4,931	4,975	509
9	5,536	3,196	4,540	7,362	7,337	7,403	659
10	5,937	6,109	7,621	12,218	12,176	12,285	1,187
11	4,403	4,635	5,748	9,215	9,183	9,266	1,341
12	8,928	12,151	15,069	24,158	24,075	24,291	12,790
f_k	1.36100	1.24015	1.26622	0.99655	1.00897		
α_k^2	4,020.395	59.847	51.295	1.352	0.575		

- (a) (1.5 points) Demonstrate that the value of α_{10}^2 was correctly calculated. (Your calculation need not match to all three decimal places.)
- (b) (1.5 points) Demonstrate that the standard error for accident year 3 was correctly calculated.

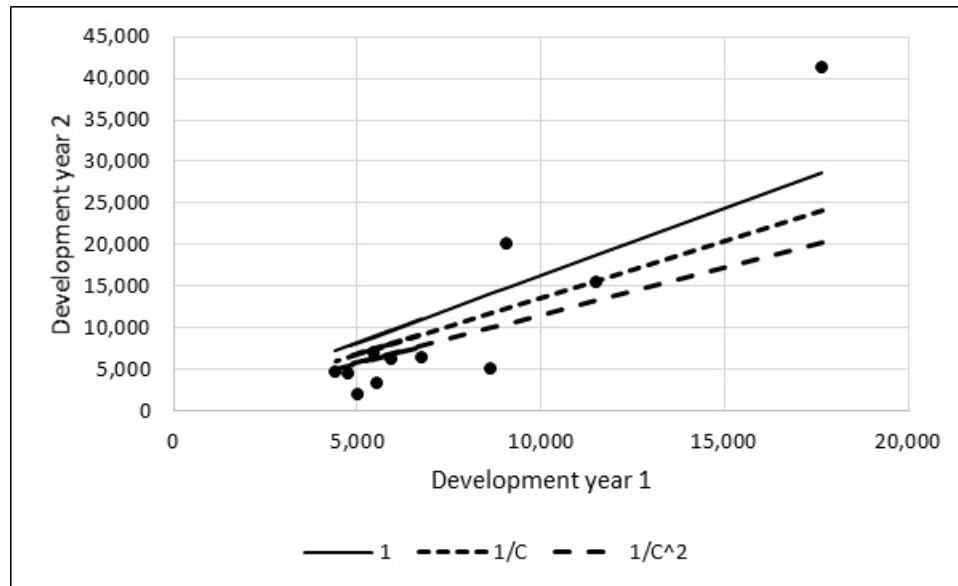
Let $C_{i,k}$ be the cumulative paid claims for accident year i and development year k . The chain ladder method estimates $C_{i,k+1}$ as $f_k \times C_{i,k}$. Mack notes that this can be viewed as a regression model where the intercept term is forced to be zero. Mack further notes that weighted least squares could be used to derive an estimate of f_k . The weight $1/C_{i,k}$ leads to the standard chain ladder estimate. The following table displays estimates of f_1 using three different weights.

4. Continued

Weight	1	$1/C_{i,1}$	$1/C_{i,1}^2$
Estimate of f_1	1.627	1.361	1.151

- (c) (1 point) Explain why the weight $1/C_{i,k}$ is consistent with the variance assumption Mack uses to obtain his standard error estimate.
- (d) (1 point) State the formula for the age-to-age factor f_1 that results from one of the other two weights. Verify that the calculated number (1.627 or 1.151) is correct using that formula.

Mack further suggests that standard regression plots can be used to determine which, if any, of the three weights produces a reasonable model. In the following plot the points are the values of $(C_{i,1}, C_{i,2})$ and the lines are of the form $y = f_1 \times x$, where the value of f_1 is determined using each of the three weights.



- (e) (1 point) Determine, from this graph, which, if any, of the three models is reasonable. Support your answer.

Venter suggests that various models can be compared by adjusting the sum of squared errors by the number of estimated parameters and/or the sample size.

- (f) (0.5 points) Describe one such adjustment.
- (g) (0.5 points) Explain why this method is inappropriate for comparing the three weighted estimates.

5. (4 points) ABC Reinsurance Company has two accounts, X and Y. The following information is provided:

Account	X	Y	X + Y
Expected Losses	100,000	50,000	150,000
Variance of Losses	25,000,000	16,000,000	64,000,000

- ABC uses the Marginal Surplus method to calculate risk loads.
 - The required return on marginal surplus is 10%.
 - The z-score multiplier is 2.2.
- (a) (1 point) Calculate the renewal risk loads for accounts X and Y.
- (b) (0.5 points) Demonstrate that the Marginal Surplus method is not renewal additive.
- ABC is considering using the Marginal Variance method to calculate risk loads with a risk load multiplier of 0.000025.
- (c) (0.5 points) Calculate the renewal risk loads for accounts X and Y.
- (d) (0.5 points) Demonstrate that the Marginal Variance method is not renewal additive.
- (e) (1 point) Calculate the renewal risk loads for accounts X and Y using the Shapley method.
- (f) (0.5 points) Demonstrate that the Shapley method is renewal additive.

6. (5 points) Crow-Hawk Insurance Company insures fifty different groups with exposures that vary both over time and among groups. An examination of the sample autocorrelations of the past fifteen years for each group indicates that for the most part the lag one autocorrelation is positive and significantly larger than zero while all other autocorrelations are essentially zero.

Crow-Hawk has been using Bühlmann-Straub empirical Bayes credibility to estimate future pure premiums for these groups.

Warren, Crow-Hawk's chief actuary, is concerned that the current methodology does not adequately capture the autocorrelation structure and has proposed the following enhancement.

- For each group, forecast the sixteenth observation by fitting an MA(1) model to the fifteen past observations.
- Employ the Bühlmann-Straub approach by using the average of the fifty forecasts as the overall mean, using the average of the estimated variances of these forecasts as the expected process variance, and inserting the fifty forecasts and the expected process variance into the Bühlmann-Straub empirical Bayes formula for the variance of the hypothetical means.
- Use the resulting credibility factors to determine the fifty revised forecasts of the sixteenth observation.

You have read the paper by Klugman that describes an alternative method of incorporating a correlation structure into credibility estimates.

- (a) (2.5 points) Describe the approach taken by Klugman.
- (b) (2.5 points) Explain why this approach is superior to the one proposed by Warren.

7. (4 points) You are setting the premium for a one-year policy using the Risk Adjusted Discount Technique and the following assumptions:

- The premium, P , will be collected when the policy is effective.
- Expenses of 20 will be paid when the policy is effective.
- Losses are expected to be L , half of which will be paid at policy expiration and half of which will be paid one year following policy expiration.
- The tax rate on all income is 35% and taxes will be paid at policy expiration. Reserves are not discounted for tax purposes.
- The risk-free rate is 0%.
- The risk-adjusted rate for losses is -5%.

(a) (1.5 points) Calculate $\frac{\partial P}{\partial L}$.

(b) (1 point) Calculate the premium when L is 80.

(c) (0.5 points) Describe how the assumptions would be adjusted to reflect increased risk in the amount of losses.

(d) (1 point) Explain how risk is accounted for in the Capital Asset Pricing Model applied to insurance.

8. (4 points) A reinsurance broker has proposed that Property R Us Reinsurance Company provide a finite risk cover without reinstatements to All Cat Insurance Company with the following terms:

- Annual Premium: 10,000,000
- Occurrence Limit: 100,000,000
- Profit Commission: 90% after 10% margin on Annual Premium
- Additional Premium: 50% of (Loss + Margin – Annual Premium)

- (a) (0.5 points) Calculate the nominal rate on line.
- (b) (0.5 points) Calculate the underwriting loss (excluding expenses) to Property R Us if a loss fully exhausts the limit.
- (c) (0.5 points) Calculate the premium for an equivalent traditional risk cover.
- (d) (0.5 points) Calculate the rate on line for an equivalent traditional risk cover.

A catastrophe model indicates that a loss will fully exhaust the limit once every 15 years and that the probability of a partial loss is negligible.

- (e) (2 points) Recommend whether or not Property R Us should accept the proposal. If your answer is no, offer a counterproposal. Justify your answer.

****END OF EXAMINATION****

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