GI ADV Model Solutions
Fall 2015

1.

Learning Objectives:
4. The candidate will understand how to apply the fundamental techniques of reinsurance pricing.

Learning Outcomes:
(4c) Calculate the price for a casualty per occurrence excess treaty.

Source:
Basics of Reinsurance Pricing, Clark

Commentary on Question:
In addition to performing standard calculations, candidates were asked about special considerations for umbrella policies. With regard to the calculations, candidates tended to either not use the correct arguments and/or used the distribution function rather than the limited expected value function. Most candidates were able to provide a definition of drop down exposure, had difficulty describing how to deal with it, and had little problem coming up with two cautions.

Solution:
(a) Calculate the expected losses in the layer using an exposure rating approach with an expected loss ratio of 60%, ignoring "drop down" exposure.

The expected payment in the layer from 1,000,000 to L is

$$E\left[x;L\right]=\int_{1{,}000{,}000}^{L}[1-F(x)]dx=\int_{1{,}000{,}000}^{L}\left(\frac{x}{1{,}000{,}000}\right)^{-2}dx=1{,}000{,}000\left(1-\frac{1{,}000{,}000}{L}\right).$$

The expected loss is 0 if the policy limit is less than or equal to 1,000,000. For the three remaining cases the calculations are (all numbers in millions):

$$4\frac{E[x;3]-E[x;2]}{E[x;3]-E[x;1]}=4\frac{2/3-1/2}{2/3-0}=1.0$$

$$2.25\frac{E[x;3]-E[x;2]}{E[x;4]-E[x;1]}=2.25\frac{2/3-1/2}{3/4-0}=0.5$$

$$1.5\frac{E[x;4]-E[x;3]}{E[x;4]-E[x;2]}=1.5\frac{3/4-2/3}{3/4-1/2}=0.5.$$

The total is 2.0. Multiplying by the expected loss ratio of 60% gives the expected losses in the layer of 1.2 (million).

1. Continued

(b) Define “drop down” exposure.

Drop down exposure arises because umbrella policies provide coverage that would otherwise be provided by the underlying policy when the aggregate limit in the underlying policy is exhausted.

(c) Explain how your analysis would have to be modified to take into account “drop down” exposure.

A portion of the exposure would be rated assuming an underlying limit of zero.

(d) Describe two cautions that should be considered when using such factors.

Commentary on Question:
Any two of the following items were sufficient to earn full credit.

- The report lag may vary by company.
- The mix of attachment points and limits may not be clearly broken out.
- Data may or may not be exclusive of asbestos and environmental claims.
- There may be inconsistent handling of workers compensation claims.

2. **Learning Objectives:**

5. The candidate will understand methodologies for determining an underwriting profit margin.

Learning Outcomes:

(5d) Allocate an underwriting profit margin (risk load) among different accounts.

Source:

An Application of Game Theory: Property Catastrophe Risk Load, Mango

Commentary on Question:

Candidates generally did well on this question though few candidates were able to get every step correct in part (a). Candidates who had incorrect values from part (a) were not penalized for properly using them in later parts.

Solution:

(a) Calculate the variance risk load for the portfolio, before and after the addition of the earthquake coverage.

Commentary on Question:

Several candidates employed the formula $L^2 p(1-p)$ *but that formula does not give the correct variance. It is designed for adding independent risks which have a single possible loss L with probability p. This setting had one risk with several possible outcomes. Candidates who used the incorrect formula were not further penalized if the calculated variance was properly used.*

The hurricane mean and variance are, respectively:

$900\left[0.3(0.5)+0.1(5)\right] = 585$ and

$900^2[0.3(0.5)^2+0.1(5)^2]-585^2 = 1,743,525.$

The variance risk load before adding earthquake coverage is 0.00025(1,743,525) = 435.88.

The earthquake mean and variance are, respectively:

$100[0.2(0.5)+0.1(5)] = 60$ and

$100^2[0.2(0.5)^2+0.1(5)^2]-60^2 = 21,900.$

The variance of the combined portfolio is

$1,743,525+21,900+2(-0.1)\sqrt{1,743,525(21,900)} = 1,726,344.$

The variance risk load after adding earthquake coverage is 0.00025(1,726,344) = 431.59.

2. **Continued**

(b) Calculate the renewal risk loads for both hurricane and earthquake coverages using the Marginal Variance method.

The hurricane marginal variance risk load is
0.00025(1,726,344 – 21,900) = 426.11.
The earthquake marginal variance risk load is
0.00025(1,726,344 – 1,743,525) = –4.30.
Note that the same answers can be obtained using an alternative formula that employs the covariance (illustrated here for hurricane)
0.00025[1,743,525 + 2(–19,541)] = 426.11.

(c) Calculate the renewal risk loads for both hurricane and earthquake coverages using the Shapley method.

The covariance is $(-0.1)\sqrt{1,743,525(21,900)} = -19,541.$
The hurricane Shapley risk load is
0.00025(1,743,525 – 19,541) = 431.00.
The earthquake Shapley risk load is
0.00025(21,900 – 19,541) = 0.59.

3. Learning Objectives:

1. The candidate will understand how to use basic loss development models to estimate the standard deviation of an estimator of unpaid claims.

Learning Outcomes:

(1e) Apply a parametric model of loss development.

(1f) Estimate the standard deviation of a parametric estimator of unpaid claims.

Source:

LDF Curve-Fitting and Stochastic Reserving: A Maximum Likelihood Approach, Clark

Commentary on Question:

Candidates did not do particularly well on this question. About half were able to get parts (a) and (b) completely correct. It was common to see only partial credit for the other parts.

Solution:

(a) Calculate the expected amount to be paid in calendar year 2015.

The calculation is:

$$7,000\frac{\left(1-e^{-30/7.94}\right)-\left(1-e^{-18/7.94}\right)}{1-e^{-18/7.94}}+6,000\frac{\left(1-e^{-18/7.94}\right)-\left(1-e^{-6/7.94}\right)}{1-e^{-6/7.94}}=4,773.$$

(b) Calculate the expected amount to be paid in calendar year 2016.

The calculation is:

$$6,000\frac{\left(1-e^{-30/7.94}\right)-\left(1-e^{-18/7.94}\right)}{1-e^{-6/7.94}}=914.$$

(c) Explain the purpose of the scaling factor.

The scaling factor arises from the use of the over-dispersed Poisson distribution. It allows the model for incremental loss emergence to have a variance that is not equal to the mean. The scaling factor is the ratio of the process variance to the mean.

(d) Estimate the process standard deviation of losses paid in calendar year 2015.

The process standard deviation is:

$$\left(4,773\cdot 318\right)^{0.5}=1,232.$$

3. Continued

(e) Estimate the process standard deviation of losses paid in calendar year 2016.

The process standard deviation is:

$$(914 \cdot 318)^{0.5} = 539 .$$

(f) Calculate the discounted loss reserve for all accident years combined using an annual discount rate of 5%.

The discounted loss reserve is:

$$\frac{4{,}773}{(1.05)^{0.5}} + \frac{914}{(1.05)^{1.5}} = 5{,}507 .$$

(g) Estimate the process standard deviation of the discounted loss reserve for all accident years combined.

The estimated process standard deviation is:

$$\left(\frac{4{,}773 \cdot 318}{1.05} + \frac{914 \cdot 318}{1.05^{3}} \right)^{0.5} = 1{,}303 .$$

4. Learning Objectives:

1. The candidate will understand how to use basic loss development models to estimate the standard deviation of an estimator of unpaid claims.

Learning Outcomes:

(1a) Identify the assumptions underlying the chain ladder estimation method.

(1b) Test for the validity of these assumptions.

(1c) Identify alternative models that should be considered depending on the results of the tests.

(1d) Estimate the standard deviation of a chain ladder estimator of unpaid claims.

Sources:

Measuring the Variability of Chain Ladder Reserve Estimates, Mack
Testing the Assumptions of Age-to-Age Factors, Venter

Commentary on Question:

Candidates generally did very well on this question with the exception of Part (f) where Venter's approach was often not clearly stated.

Solution:

(a) Demonstrate that the value of α_4^2 was correctly calculated. (Your calculation need not match to all three decimal places.)

The calculation is:

$$\alpha_4^2 = \frac{1}{7-4-1}\sum_{j=1}^{7-4} C_{j,4}\left(\frac{C_{j,5}}{C_{j,4}} - f_4\right)^2$$

$$= \frac{1}{2}\left[C_{1,4}\left(\frac{C_{1,5}}{C_{1,4}} - f_4\right)^2 + C_{2,4}\left(\frac{C_{2,5}}{C_{2,4}} - f_4\right)^2 + C_{3,4}\left(\frac{C_{3,5}}{C_{3,4}} - f_4\right)^2\right]$$

$$= \frac{1}{2}\left[30,915\left(\frac{31,365}{30,915} - 1.03198\right)^2 + 16,824\left(\frac{16,909}{16,824} - 1.03198\right)^2 + 28,617\left(\frac{30,524}{28,617} - 1.03198\right)^2\right]$$

$$= 27.980.$$

4. **Continued**

(b) Demonstrate that the standard error for accident year 3 was correctly calculated.

The calculation is:

$$C_{3,7}^2 \sum_{k=7+1-3=5}^{7-1=6} \frac{\alpha_k^2}{f_k^2}\left(\frac{1}{C_{3,k}}+\frac{1}{\sum_{j=1}^{7-k} C_{j,k}}\right) = C_{3,7}^2\left[\frac{\alpha_5^2}{f_5^2}\left(\frac{1}{C_{3,5}}+\frac{1}{\sum_{j=1}^{7-5=2} C_{j,5}}\right)+\frac{\alpha_6^2}{f_6^2}\left(\frac{1}{C_{3,6}}+\frac{1}{\sum_{j=1}^{7-6=1} C_{j,6}}\right)\right]$$

$$=31,858^2\left[\frac{0.202}{1.02136^2}\left(\frac{1}{30,524}+\frac{1}{31,365+16,909}\right)+\frac{0.00146}{1.02188^2}\left(\frac{1}{31,176}+\frac{1}{32,082}\right)\right]$$

$$=10,599.$$

The standard error is the square root, 103.

(c) Explain why f_k has only the subscript *k* and not both *i* and *k*.

Mack's assumptions are designed to reproduce the chain ladder results. A key feature of the chain ladder method is that the same age-to-age factor is used for each accident year.

(d) Describe a situation where these ratios may be correlated.

Commentary on Question:
The solution provides one illustration of how the values may be correlated. Other, valid, illustrations were acceptable.

Suppose in one calendar year there is a change in procedures that leads to a speedup of claims paying. This would lead to a larger ratio in one development year followed by a smaller ratio the next year. This would produce a negative correlation.

(e) Explain why the formula used to estimate α_1^2 through α_5^2 cannot be used to estimate α_6^2.

Commentary on Question:
Two explanations are presented. Either one was sufficient for full credit.

4. **Continued**

There is only one observation, hence it is not possible to estimate a variance. Alternatively, an attempt to use the formula leads to

$$\alpha_6^2 = \frac{1}{7-6-1}\sum_{j=1}^{7-6=1} C_{j,6}\left(\frac{C_{j,7}}{C_{j,6}} - f_6\right)^2 = \frac{1}{0}C_{1,6}\left(\frac{C_{1,7}}{C_{1,6}} - f_6\right)^2 .$$

.

Because $f_6 = C_{1,7} / C_{1,6}$, the expression is 0/0 which is undefined.

(f) Explain what Venter means when using the terms "doesn't work" and "try."

Commentary on Question:

The solution uses quotations from the article. Paraphrasing was acceptable for full credit.

The phrase "doesn't work" means that the method "fails the assumptions of least squares optimality." By "try" Venter means to "test the underlying assumptions of [a different model]."

5. **Learning Objectives:**

3. The candidate will understand how to use a credibility model with parameters that shift over time.

Learning Outcomes:

(3a) Identify the components of a credibility model with shifting risk parameters.

(3c) Estimate the parameters of the model.

(3d) Compare various models that might be used.

Source:
Credibility with Shifting Risk Parameters, Klugman

Solution:

(a) Identify the simplifying assumptions reflected in this formula.

Commentary on Question:
Both assumptions had to be stated to receive full credit.

- All groups must have the same structural parameters.
- Correlations between observations depend only on the differences between the time periods.

(b) Explain why it is not possible to estimate δ_{10} and δ_{11} without further assumptions.

These values represent covariances for observations that are ten and eleven years apart. With ten years of data, the maximum separation is nine years. So there is no data on which to base an estimate.

(c) Explain why assuming an MA(2) model overcomes the issue identified in part (b).

For an MA(2) model all observations more than two years apart have a covariance of zero. Hence, these two values are known to be zero and do not need to be estimated.

5. Continued

(d) State an advantage of REML with respect to MLE and a different advantage with respect to MM.

Versus MLE, REML has a smaller bias. Versus MM, REML generally has a smaller mean squared error. An alternative answer is that formulas are more difficult to develop for MM than for REML.

6. **Learning Objectives:**

5. The candidate will understand methodologies for determining an underwriting profit margin.

Learning Outcomes:

(5a) Calculate an underwriting profit margin using the target total rate of return model.

(5b) Calculate an underwriting profit margin using the capital asset pricing model.

(5c) Calculate an underwriting profit margin using the risk adjusted discount technique.

Source:

Ratemaking: A Financial Economics Approach, D'Arcy and Dyer

Commentary on Question:

For part (a) most all candidates were able to complete the calculations for the risk-adjusted discount technique. For the other two methods, in most all cases there were elements of the formula that were not correctly used. For part (b) most candidates were able to state some of the issues while few were able to get all three correct.

Solution:

(a) Demonstrate that the Target Total Rate of Return Model, the Capital Asset Pricing Model, and the Risk Adjusted Discount Technique all produce a premium of 100, if taxes are ignored.

For the Target Total Rate of Return Model, the calculation is:

$$13\% = \left(\frac{P-30+S}{S}\right)1.25\% + \left(\frac{P}{S}\right)UPM$$

$$S = 0.5P, UPM = (P-30-65)/\mathrm{P}$$

$$0.13 = 0.0125\left(3-\frac{60}{P}\right)+2\left(\frac{P-95}{P}\right)$$

$$0.13P = 0.0375P - 0.75 + 2P - 190$$

$$1.9075P = 190.75$$

$$P = 100.$$

For the Capital Asset Pricing Model, the calculation is:

$$UPM = -k(1.25\%) + 0.5(11.75\%)$$

$$\left(\frac{P-95}{P}\right) = -\left(\frac{P-30}{P}\right)0.0125 + 0.05875$$

$$0.95375 = \frac{95.375}{P}$$

$$P = 100.$$

6. **Continued**

For the Risk Adjusted Discount Technique, the calculation is:

$$P = \frac{65}{0.92857} + 30 = 100.$$

(b) Identify one drawback of each of the three methods.

Commentary on Question:
Only one drawback was needed for each method.

Target Total Rate of Return

- Difficult to obtain the required inputs
- Lack of theoretical justification

Capital Asset Pricing Model

- Ignores risk that is unique to insurance that is not systematic with investment risk

Risk Adjusted Discount Technique

- Expenses are not proportional to premiums
- The risk-free rate is not appropriate for the lag in premium collection
- Difficult to determine the risk-adjusted discount rate
- Difficult to allocate equity to policies
- Considers only one policy form

7. **Learning Objectives:**

2. The candidate will understand the considerations in selecting a risk margin for unpaid claims.

Learning Outcomes:

(2a) Describe a risk margin analysis framework.

(2b) Identify the sources of uncertainty underlying an estimate of unpaid claims.

(2c) Describe methods to assess this uncertainty.

Source:
A Framework for Assessing Risk Margins, Marshall, et al.

Solution:

(a) Calculate the internal systemic risk coefficient of variation for outstanding claim liabilities for all lines combined.

Commentary on Question:
The most common error was not to scale the proportions to sum to one.

The variance terms are:

$$(6.5)^2(0.1/0.5)^2 + (4.5)^2(0.1/0.5)^2 + (7.5)^2(0.3/0.5)^2 = 22.75.$$

The covariance terms are:

$$0.5(6.5)(4.5)(0.1/0.5)(0.1/0.5) + 0.25(6.5)(7.5)(0.1/0.5)(0.3/0.5)$$
$$+0.25(4.5)(7.5)(0.1/0.5)(0.3/0.5) = 3.06.$$

The total is 22.75 + 2(3.06) = 28.87.
The coefficient of variation is the square root, 5.37%.

(b) Define external systemic risk.

External systemic risk is risk that is external to the actuarial modeling process.

(c) Describe two sources of external systemic risk.

Commentary on Question:
Any two from the list were sufficient for full credit.

7. **Continued**

- Economic, inflation in particular
- Legislative and political
- Claims management and processing
- Expenses of managing run off
- Natural or man-made events
- Latent claims
- Recovery

(d) Explain why traditional quantitative modeling techniques alone are inadequate to capture external systemic risk.

With respect to external systemic risk, the past may not be predictive of the future.

8. **Learning Objectives:**

4. The candidate will understand how to apply the fundamental techniques of reinsurance pricing.

Learning Outcomes:

(4d) Apply an aggregate distribution model to a reinsurance pricing scenario.

(4e) Describe considerations involved in pricing property catastrophe covers.

Source:

Basics of Reinsurance Pricing, Clark

Solution:

(a) Identify two types of information that would be needed to price the cover using a catastrophe model.

Commentary on Question:

Any two of the following were sufficient to receive full credit.

- A measure of exposure
- Geographic information
- Terms of the insurance policies
- Details of inuring reinsurance

(b) Calculate the probability that annual losses will exceed 20,000,000.

In units of millions,

$P(0) = P(N = 0) = e^{-0.1} = 0.9048$

$P(10) = P(N = 1)P(X_1 = 10) = 0.1e^{-0.1}(0.5) = 0.0452$

$P(20) = P(N = 1)P(X_1 = 20) + P(N = 2)P(X_1 = 10, X_2 = 10)$

$\quad = 0.1e^{-0.1}(0.5) + (0.1^2)(e^{-0.1} / 2)(0.5)^2 = 0.0464$

$P(S > 20) = 1 - 0.9048 - 0.0452 - 0.0464 = 0.0036.$

The recursive formula may also be used to obtain these probabilities.

(c) Propose terms on a traditional basis. Justify your proposal.

The expected loss is 0.1[10,000,000(0.5) + 20,000,000(0.5)] = 1,500,000. Applying a traditional loading of 100/80 yields a premium of 1,875,000. This assumes an unlimited number of free reinstatements.

Alternatively, there could be a charge for reinstatements, which would make the analysis more complicated. This was not required for full credit. Any reasonable loading was acceptable.

8. Continued

(d) Propose terms on a finite basis. Justify your proposal.

Begin by assuming a profit commission of 80% after a 10% margin on Annual Premium (A) and additional premium of 50% of (Loss (L) + Margin – Annual Premium).

Setting the expected premium equal to the expected loss plus the margin yields:

$$A - 0.8[0.9A]\Pr(L=0) + 0.5[E(L \mid L>0) + 0.1A - A]\Pr(L>0) = E(L) + 0.1A$$

$$0.9A[1 - 0.8\Pr(L=0) - 0.5\Pr(L>0)] + 0.5E(L) = E(L)$$

$$A = \frac{0.5E(L)}{0.9\left[1 - 0.8\Pr(L=0) - 0.5\Pr(L>0)\right]} = \frac{0.5(1,500,000)}{0.9\left[1 - 0.8e^{-0.1} - 0.5\left(1 - e^{-0.1}\right)\right]}$$

$$= \frac{750,000}{0.9\left[0.5 - 0.3e^{-0.1}\right]} = 3,646,195.$$

Other terms were also acceptable as long as they were justified by an analysis such as that shown above.