QFI ADV Model Solutions
Spring 2015

1. Learning Objectives:

2. The candidate will understand and be able to apply a variety of credit risk theories and models.

Learning Outcomes:

(2a) Demonstrate an understanding of events and causes of the recent global credit crisis.

(2c) Demonstrate an understanding of credit valuation models.

(2h) Demonstrate an understanding of credit default swaps (CDS) and the bond-CDS basis, including the use of CDS in portfolio and trading contexts.

Sources:

QFIA-100-13: "Modeling of Mortgage Defaults", Jan 22, 2008, pp. 5- 38 (pp. 13-25 background only)

Commentary on Question:

This question tests the candidates understanding of mortgage defaults and its determinants, and an understanding of events and causes of the recent global crisis with its corresponding impact on predictive modeling.

Most candidates performed relatively poorly on this question.

Solution:

(a) Define "mortgage default" for the purpose of the model.

Commentary on Question:

The candidates performed poorly in this section. Many candidates did not describe that the essential condition for a default to occur is that the lender suffers a loss after liquidating the property.

A mortgage default occurs when a loan is paid off and there was a loss. In other words, a default only occurs when a loan is liquidated and at a loss.

1. Continued

(b) Describe four determinants of mortgage defaults that would be essential parameters of the predictive default model and explain difficulties encountered in determining each.

Commentary on Question:
Candidates performed relatively well in this section. Most candidates identified at least one major determinant of mortgage default. However, some candidates did not explain the difficulties that may arise in measuring those determinants.

Any 4 of the following points will have earned the candidates full credit:

Loan-to-Value Ratio and HPA: LTV measures the amount of equity in the home
Current LTV is estimated using the home price appreciation (HPA)
Difficulties: Inflated home appraisals (were widespread in last housing boom)
Silent second mortgage on the property may not be known

Credit Score: an indicator of creditworthiness
Difficulties: credit scores may have lost their predictive power

Loan documentation status: documents (pay stubs, tax return, etc.) for verification of ability to pay
Difficulties: "reduced" or "no" documentation loans concept in last housing boom has been abused

Occupancy status: owner-occupied or investor property … investor properties considered riskier
Difficulties: Challenge to know whether a loan is truly owner-occupied due to fraud

Loan purpose: home purchase or refinancing
Difficulties: for home purchase LTV based on home price while for refinancing it is based on appraised value, but balancing this purchase loans often taken out by first time buyers who present a higher risk

Debt-to-income (DTI) ratio: ratio of monthly mortgage payments & other debt to monthly income of the borrower; the higher the ratio the greater the strain on the borrower's ability to continue payments
Difficulties: Borrowers may not give accurate information about their debt or income, may be hard to verify.

1. Continued

Job Losses: is a major trigger event for defaults
Difficulties: Changes in unemployment rates are used to capture likelihood of a job loss. Complications come from variation in unemployment from region to region

Spread at origination (SATO): the difference between the coupon on the loan and a prevailing mortgage rate is generally an indicator of an above average risk of the borrower. Due to uncertainties surrounding other variables like LTV and DTI, SATO is considered a critical element in predicting defaults

(c) Explain the challenges the CEO is suggesting.

Commentary on Question:
The candidates performed relatively poorly on this section. Many candidates did not explain the link between changing conditions and challenges to the predictive model.

Predictive models tend to use the past as a guide to future behavior.
There is the problem of limited data and old data was of questionable reliability.

Subprime market underwriting standards have gone from being very loose to extremely tight, & there has been a shift from adjustable- to fixed-rate mortgages. Not only underwriting standards changed over time, but there was no uniformity in the market. This is a major challenge to predictive models.

(d) Suggest possible ways to overcome these challenges.

Commentary on Question:
The candidates performed poorly in this section. Most candidates did not describe ways the model can be adjusted to improve its accuracy. Most candidates instead explained general ways of overcoming subprime mortgage issues which was not asked for in this section.

The model would use implied "fraud factors" to compensate for shoddy or even fraudulent underwriting.

It would use dynamic deal specific adjustments to incorporate originator- and servicer-specific effects.

(e) Describe two other models that you would combine with the default model for valuation of mortgage credit risk.

1. Continued

Commentary on Question:
The candidates did relatively well in this section. The candidates generally identified the types of models that can combine with the default model. Some candidates lost points for simply naming the model without any explanation.

To value mortgage credit risk, we need to combine the default model with:

A prepayment model: that projects the overall level of payoffs

A loss severity model: that predicts the loss when a mortgage defaults

(f) Explain the relationship of prepayment and default rates implied by the cumulative loss percentages in the table below:

Commentary on Question:
Most candidates did well in this section. Most candidates were able to explain the link between pre-payment rates, default rates and cumulative losses.

Higher speeds of prepayments reduce the overall balance and leave fewer borrowers who could potentially dfault.

For a given default rate, cumulative losses drop as pre-payment percentages increase. Also, for a given pre-payment percentage, cumulative losses increase as default rates increase.
For example, at 15% default rate and 60% voluntary prepayment rate, the cumulative loss is 8%. If the default rate doubles to 30%, then the cumulative loss increases to 14%. If, alternately, the voluntary prepayment speed is assumed to decline by 50%, losses increase even more, rising to a 16% cumulative loss.

2. Learning Objectives:

1. The candidate will understand the standard yield curve models, including:
   - One and two-factor short rate models
   - LIBOR market models

   The candidate will understand approaches to volatility modeling.

Learning Outcomes:

(1n) Calculate the risk-neutral density given call option prices.

(1o) Identify several stylized empirical facts about smiles in a variety of options markets.

(1p) Describe and contrast several approaches for modeling smiles, including: Stochastic Volatility, local-volatility, jump-diffusions, variance-gamma and mixture models.

Sources:

Volatility Correlation – The Perfect Hedger and the Fox, Rebonato, R., 2nd Edition, Ch 9 (9.5-9.8), p. 252-254.

Volatility Correlation – The Perfect Hedger and the Fox, Rebonato, R., 2nd Edition, Ch 8 (8.1,8.3), p. 237-239, 242-243.

Commentary on Question:

The question aimed to test the candidates' knowledge and comprehension of the different ways to price a short-term option while accounting for the volatility smile.

Overall, the question was answered relatively well by candidates. Most candidates were able to successfully compare and contrast the fully stochastic volatility and jump diffusion model, and to identify the mistake in part a). However, many candidates did not fully explain the mistake identified in part a) nor properly calculated the risk-neutral probabilities in part b).

Solution:

(a) Identify where the mistake is and explain why the number is wrong.

Commentary on Question:

The candidates performed well on this section. Almost all candidates identified the mistake but many failed to fully substantiate the explanation.

The second derivative of the price of the call option with respect to the strike price should always be positive, so $\frac{\partial^2 C(K,T)}{\partial K^2}\Big|_{K=95} = -0.0425$ is wrong.

2. Continued

The second derivative needs to be positive, because up to an accumulation factor (which is always positive), it is the density of the index price at time T. Since this density cannot be negative, the second derivative cannot be negative either.

b) Calculate the following:

(i) (*1 point*) $\phi_T(65)$, $\phi_T(70)$ and $\phi_T(75)$.

(ii) (*1 point*) The risk-neutral probability that the index price in one month is between 70 and 75.

(iii) (*1 point*) The risk-neutral probability that the index price in one month is below 65.

Commentary on Question:
The candidates performed poorly on this section, especially for parts (ii) and (iii). Many candidates failed to retrieve the appropriate formula and to properly perform the calculations. The question should read that $\phi_T(K)$ denotes the density of the index price (not the strike price). Candidates were given appropriate credit whether they noticed this error in the question's language, or if they assumed the density referred to "index" price.

(i) $\phi_T(K) = \frac{\partial^2 C(K,T)}{\partial K^2} e^{rT}$

$\phi_T(65) = 0.0003 \times e^{0.03/12} = 0.000301$

$\phi_T(70) = 0.0011 \times e^{0.03/12} = 0.001103$

$\phi_T(75) = 0.0020 \times e^{0.03/12} = 0.002005$

(ii) $P(K_{i-1} < K < K_i) = \frac{1}{2}\left(\phi(K_{i-1}) + \phi(K_i)\right) \times (K_i - K_{i-1})$

$P(70 < K < 75) = \frac{1}{2}(0.001103 + 0.002005) \times 5 = 0.007769$

(iii) Since $\phi_T(60) = 0$, then the strike at which the density goes to zero is $K_0 = 60$

$P(K < 65) = \frac{1}{2}(\phi(65) + 0) \times (65 - 60) = 0.000753$

(c) Explain why the density you retrieved is not sufficient to price the up-and-out put option explained above.

2. Continued

Commentary on Question:
The candidates performed relatively well on this section. Most candidates were able to provide support as to why the density retrieved is not sufficient to price the option described.

- The payoff of the up-and-out put depends on the maximum attained by the index value between today and 1 month from now (maturity of the option). Therefore, in order to price it, we must have some information on the path taken by the index value during the next month.
- The function obtained using the second derivatives of call option prices with respect to the strike only reflects the unconditional distribution of the index value 1 month from now. Since we only used options expiring in 1 month, the function obtained does not give information about the possible paths of the index value during the next month.

(d) Compare and contrast the two models.

Commentary on Question:
The candidates performed well on this section. Most candidates were able to compare and contrast the two models in regards to their abilities to replicate various types of volatility. However, only few candidates mentioned that the two models can be linked to the Black-Scholes model under certain conditions.

- Both models can be linked to the Black-Scholes model under certain conditions (independence of the Brownian motions in the SV case, log-normal distribution of jumps in JD case).
- Both models allow to model the complete path of the stock price process (compared to the method suggested in c)
- In a stochastic volatility model, the market is typically not complete. It can only be completed by specifying a utility function or a market price of volatility risk. Under jump diffusion models, the market can be completed by adding more assets if the distribution of jump amplitudes is discrete. If the distribution of jump amplitudes is continuous, the market is incomplete.
- Under reasonable parameters, stochastic volatility models can only replicate shallow smiles for short maturities. Since jump diffusion models allow for big price moves in a short period of time (through discontinuities), they can replicate sharper smiles for short maturities, which is typically what is observed.

(e) Recommend one of the two models and justify your answer.

2. Continued

Commentary on Question:
The candidates performed relatively well on this section. Most candidates were able to identify the jump diffusion model as the preferred model and to properly substantiate their claims.

- A jump diffusion model would be more appropriate than a fully stochastic volatility model.
- It has been observed on the market that smiles are sharper for shorter maturities. The model chosen should reflect that.
- Jump diffusion models are better able to replicate sharper smiles at short maturities.

3. Learning Objectives:

4. The candidate will understand important quantitative techniques for analyzing financial time series Performance Measurement and Performance Attribution.

Learning Outcomes:

(4b) Apply various techniques for analyzing factor models including Principal Component Analsis (PCA) and Statistical Factor Analysis.

Sources:

Analysis of Financial Time Series, Tsay, Ruey S., 3rd Edition, 2010. Ch.9 Principal Component Analysis and the Factor Models

Principal Component Analysis on Term Structure of Interest Rates, Antii Malava.

Commentary on Question:

This question asks the candidates to show an understanding of the methodology of PCA in a non-purely technical way.

Overall the candidates performed excellently on this question.

Solution:

(a) Outline the steps required to identify the *first* and *second* principal components for the random variables x_i .

Commentary on Question:

The candidates performed well on this question. Some candidates failed to mention that the first principal component is uncorrelated with the second principal component.

Identify the First Principal Component:

Consider the variables $x_1 \ldots x_5$ as a random vector **x**. Find its variance/covariance matrix **Σ** of **x**. Find the largest eigenvalue λ_1 of the covariance matrix **Σ.** $\boldsymbol{\beta_1}$ is the eigenvector corresponding to the eigenvalue λ_1. The first principal component is $y_1 = \boldsymbol{\beta_1^T}\,\mathbf{x}$

Identify the the Second Principal Component:

Find the linear combination of random variables $x_1, \ldots, x_5$, that is *uncorrelated* with the 1st principal component and contains as much of the variability of the random variables as possible. $Cov(y_1, \boldsymbol{\beta^T x}) = 0$

3. Continued

(b) Determine which of the above principal components should be used to explain at least 80% of the variance in the original data.

Commentary on Question:
The candidates performed excellently on this question. Almost no one missed this.

Find the explanatory degree of each PC using the following formula

$$\frac{\lambda_i}{\sum_{i=1}^{5} \lambda_i}$$

The calculated explanatory degree of each PC is calculated to be the following for the 3 largest eigenvalues:

Principle Component	Eigenvalue (λi)	Explanatory Degree
B	1.0	16%
C	2.6	41%
E	1.8	29%

Rank the PCs in order of their explanatory %: C, E, B, A, D. The sum of the top three 16% + 41% + 29% > 80%. So the PCs B, C, E should be chosen.

(c) Contrast the nature of the information that can be obtained from a loadings/Eigenvectors chart (such as Chart 1) compared to an explanatory power chart (such as Chart 2).

Commentary on Question:
The candidates performed relatively well on this question. Some candidates explained each principal component but did not differentiate the two graphs.

The first graph (loadings/eigenvalues) shows what is the effect of each PCs on the different stocks; The second graph (Breakdown of Explanatory Power) gives the amount of total variation that is explained by each PC.

(d) Interpret the results for PC1 and PC2 in Chart 1.

Commentary on Question:
The candidates performed well on this section. The most common mistake was to not specifically identify the 2nd principal component was likely an industry component since that was provided in the question.

3. Continued

The 1st PC:
The loadings for first component is roughly the same for all five stocks, this suggests that the first component is roughly equally weighted linear combination of the stock returns. The logical interpretation is that it represents the general movement of the stock market and is a market component.

The 2nd PC:
The loadings are different drastically between the stocks, but they are similar for stocks 1 and 2, and similar for stocks 3, 4, 5. Since we know these stocks came from 2 different industries, the second PC may represent an industry component.

(e) Determine which stocks in the above charts are more likely to be from the financial services industry and which are more likely to be from IT stocks, given your assumption.

Commentary on Question:
The candidates performed well on this question. However, many candidates failed to link their conclusions to the assumption.

The loadings for stocks 1 -2 are similar for the 2nd principal component. The loadings for stocks 3 -5 are similar for the 2nd principal component. Stocks 1-2 should belong to the same industry, and stocks 3-5 should belong to the same industry.

The first principal component explains more of the variance for stocks 1-2, and less of the variance for stocks 3-5, and it can be interpreted as the market component. The given assumption is that overall market returns contribute more to the variance of financial services firms than it contributes to IT companies. Hence, stocks 1-2 should belong to financial services industry, stocks 3-5 belong to the IT industry.

4. Learning Objectives:

2. The candidate will understand and be able to apply a variety of credit risk theories and models.

Learning Outcomes:

(2g) Demonstrate and understanding of and be able to apply the concept of Duration Times Spread (DTS).

Sources:

Quantitative Credit Portfolio Management, Ben-Dor, et. al. Ch.1

Commentary on Question:

This question tested the candidates understanding of DTS and included an extension of the principles in the text.

Overall, candidates performed excellently on the question. Most candidates understood how to calculate DTS and apply it to a portfolio management situation.

Solution:

(a) Describe how DTS-based issuer caps can result in a "credit torpedo" and suggest how to minimize this risk.

Commentary on Question:

Candidates performed poorly on this section. Although most recognized that low spread issuers were the source of the risk, very few candidates stated that market weight caps were the solution.

DTS-based issuer caps allow for large positions in low spread issuers resulting in a potential "credit torpedo". The credit torpedo risk can be minimized by using market weight caps in addition to DTS-based caps.

(b) Determine which bond in the portfolio is the riskiest with respect to excess return volatility. Show your work.

Commentary on Question:

Candidates performed relatively well on this section. Candidates recognized the relationship between DTS and excess return volatility. However, many candidates chose the wrong bond. Those that chose the wrong bond received partial credit as long as the rest of the answer was correct.

$$DTS = (Spread\ Duration\ \times Spread)$$

$$DTS_{AAA} = 50 \times 11 = 550$$
$$DTS_{DOVE} = 80 \times 8 = 640$$
$$DTS_{RISK} = 250 \times 2 = 500$$
$$DTS_{MAX} = 500 \times 1 = 500$$

4. Continued

The bond issued by DOVE is the riskiest because it has the highest DTS and therefore the highest excess return volatility. The bond issued by AAA has the highest contribution of excess return volatility to the portfolio but is not the riskiest with respect to excess return volatility.

(c) Explain why the current portfolio does not comply with the investment policy.

Commentary on Question:
Candidates performed excellently on this section. Most candidates received full points.

Using the DTS values calculated in part b multiply each by the portfolio weight to find the contribution to DTS from each issuer:

$$DTS_{AAA}\ Contribution = 550 \times 40\% = 220$$
$$DTS_{DOVE}\ Contribution = 640 \times 20\% = 128$$
$$DTS_{RISK}\ Contribution = 500 \times 20\% = 100$$
$$DTS_{MAX}\ Contribution = 500 \times 20\% = 100$$

Since the $DTS_{AAA}\ Contribution$ exceeds the DTS contribution limit for the portfolio (220 > 150), the portfolio is not in compliance.

(d) Determine portfolio weights for each bond above that will bring the portfolio into compliance with the investment policy.

Commentary on Question:
Candidates performed excellently on this section. If the candidate was able to do part c than likely the candidate was able to figure out how to adjust the portfolio to bring it back into compliance. There were multiples ways to bring the portfolio back into compliance and any answer that satisfied the criteria was accepted.

One approach that works is to find weights that equal 150 for the first bond and then the second bond and so forth until you reach the last bond. The numbers for this approach look like this:

$$New\ Port.Weight_{AAA} = (Contribution\ Limit)/(DTS_{AAA}) = \frac{150}{550} = 27.2\%$$
$$New\ Port.Weight_{DOVE} = (Contribution\ Limit)/(DTS_{DOVE}) = \frac{150}{640} = 23.4\%$$
$$New\ Port.Weight_{RISK} = (Contribution\ Limit)/(DTS_{RISK}) = \frac{150}{500} = 30.0\%$$
$$New\ Port.Weight_{MAX} = 100\% - 27.2\% - 23.4\% - 30.0\% = 19.4\%$$

5. Learning Objectives:

5. The candidate will understand the behavior characteristics of individuals and firms and be able to identify and apply concepts of behavioral finance.

Learning Outcomes:

(5c) Identify and apply the concepts of behavioral finance with respect to individual investors, institutional investors, portfolio managers, fiduciaries and corporate managers.

Sources:

A Survey of Behavioral Finance by Barberis & Thaler

The Impact of Stochastic Volatility on Pricing, Hedging Efficiency of Withdrawal Benefit Guarantees in Variable Annuities

Commentary on Question:

This question tests candidates' understanding of Stochastic Volatility and the application of the concepts of Behavioral Finance with respect to portfolio managers

Solution:

(a) Describe the major features of the Prospect theory model.

Commentary on Question:

The Candidates performed relatively well on this section. Full credit was given for touching on most of the following features.

Important features –

1. utility is defined over gains and losses rather than over final wealth positions, the option payoff exactly satisfies this property
2. The shape of the value function:
   a. concave in the domain of gains and convex in the domain of losses
   b. indicating a greater sensitivity to losses than to gains (loss aversion)
3. Non-linear probability transformation: Small probabilities are over-weighted

(b) Identify the mistakes in the above Prospect theory model and provide a numeric example of each mistake.

Commentary on Question:

The Candidates performed relatively well on this section. Most often full credit was not earned due to incomplete or confusing examples. At least two mistakes with examples were needed in order to receive full credit for this part of the question.

5. Continued

1. The value function is not convex in the domain of loss, it is actually concave

   Example:
   v-(x) = 0.5x^0.4: This is a concave function as the derivative of the function is monotonically decreasing or in layman terms, it has a decreasing slope
   derivative = 0.5 * 0.4 x^(0.4-1)

2. The value function is undefined for x<0 given that the exponent is less than 1.

   Example:
   0.5x^0.4 is not defined for x < 0. E.g. -1^0.4 is not a well defined function.

3. Small probabilities are not over-weighted w(p) < p

   Example:
   W(0.05) = 0.00263 < 0.05 (this is but one example, there can be others as the p=0.05 is not stated in the problem; it is simply a "small probability")

6. Learning Objectives:

3. Candidate will understand the nature, measurement and management of liquidity risk in financial institutions.

Learning Outcomes:

(3f) Apply liquidity scenario analysis with various time horizons.

(3g) Create liquidity risk management plans and procedures, including addressing appropriate product design, investment guidelines, and reporting given a desired liquidity risk level.

Sources:

QFIA-105-13: Report of the Life Liquidity Work Group of the American Academy of Actuaries to the Life Liquidity Risk Working Group of the NAIC

QFIA-106-13: *Liquidity Risk: Measurement and Management – A Practitioner's Guide to Global Best Practices,* Matz, Leonard & Neu, Peter, 2006, Ch. 3

Commentary on Question:

This question tested the Candidate's comprehension of liquidity risk measurement, and the Candidate's ability to analyze and apply a liquidity scenario analysis in practice.

Solution:

(a) Describe the problems associated with each of the three quantitative methods for scenario-based liquidity stress testing as described in "*Liquidity Risk: Measurement and Management – A Practitioner's Guide to Global Best Practices."*

Commentary on Question:

The candidates performed relatively well on this section. Most candidates either knew the three quantitative methods associated with liquidity scenario stress testing, or they confused the question with the three types of frameworks that banks typically used to assess liquidity risk. For those who listed the correct three quantitative methods, many were able to score quite well.

Historical VaR and Extreme Value Analysis

Main problem with both VaR and EV analysis is that the tail in the distribution of liquidity risk only reveals the most severe losses

Historical data do not include the type of extreme events that comprise contingent liquidity risk ("Black Swan" problem)

Deterministic Scenario modeling at multiple stress levels

Deterministic stress testing simulates shocks that never occurred, or did not occur with sufficient frequency or severity in recent historical data

Deterministic modeling provides no information about the probability of an event –just its severity

6. Continued

Monte Carlo modeling to capture stress levels
MC requires a starting state and key parameterization e.g. mean reversion and volatility
Unlikely that observed parameters reflect conditions during extreme liquidity events

(b) Assess the liquidity risk for Iron & Wine Life Co. by applying liquidity scenario analysis for each of the following:

(i) Base Case

(ii) Scenario 1

(iii) Scenario 2

Commentary on Question:
The candidates performed relatively well on this section. Many were able to calculate the required asset haircut and/or value of the liabilities for some (if not all) of the given scenarios, which were fairly straightforward. The key to obtaining full credit was to calculate the liquidity ratios, which was needed to communicate a proper liquidity scenario analysis.

Base Case:	
Haircut	1300 * 5% = 65
MV of Assets Less Haircut	1300 – 65 = 1235
Value of Liabs. * Time Factor	1100 * 50% = 550
Ratio of Net Assets/Liabs.	1235/550 = 2.25
Scenario 1:	
Haircut	1300 * 15% = 195
MV of Assets Less Haircut	1300 – 195 = 1105
Value of Liabs. * Time Factor	1100 * 50% = 550
Ratio of Net Assets/Liabs.	1105/550 = 2.01
Scenario 2:	
Duration Impact	-6 * 5% = -30%
MV of Assets after Haircut	1300*(1-30%)*(1-5%) =
Value of Liabs. * Time Factor	1100 * 80% = 880
Ratio of Net Assets/Liabs.	864.5/880 = 0.9824

6. Continued

(c) Recommend next steps to the CRO based upon this liquidity scenario analysis.

Commentary on Question:
The candidates performed relatively poorly on this section. Rather counter-intuitively, many that did well in part (b) didn't always provide the appropriate recommendations to the CRO. Alternatively, there were a few candidates who didn't perform as well in part (b) and yet managed to provide solid recommendations.

Scenario 2 ratio is below 1.0, is considerably below the others, and is of primary concern to Iron & Wine. Recommend to the CRO to:-

- Mitigate interest-sensitivity of the liabilities
- Consider hedging options to offset the liquidity risk of the current block of liabilities
- Consider restructuring the asset mix in the portfolio
- Line up alternate sources of liquidity such as back-stop liquidity lines

(d) Determine the minimum amount of additional cash that would be needed to support the liabilities based upon the results of the scenario analysis.

Commentary on Question:
The candidates performed relatively poorly on this section. Many did not attempt the question. For those that did, candidates were given credit as long as their answers' were consistent with their findings in part (b), even if the ultimate answer was not correct.

Scenario 2 ratio is most below 1.0, so would need additional cash assets to bring ratio to 1.0
880 – 864.5 = 15.5, so 15.5 cash assets is the minimum amount of assets needed since cash assets would not be expected to have a market value change nor a haircut.

7. Learning Objectives:

1. The candidate will understand the standard yield curve models, including:
   - One and two-factor short rate models
   - LIBOR market models

   The candidate will understand approaches to volatility modeling.

Learning Outcomes:

(1a) Identify and differentiate the features of the classic short rate models including the Vasicek and the Cox-Ingersoll-Ross (CIR) models.

(1c) Explain the dynamics of and motivation for the Hull-White extension of the Vasicek model.

Sources:

Brigo, D and Mecurio F, Interest Rate Models – Theory and Practice, 2nd Edition, Ch.3 (3.1, 3.3)

Commentary on Question:

This question tests candidates' general understanding of interest rate models and particularly, the pro/cons of HW one-factor model and its application.

Solution:

(a) Identify six practical considerations in choosing an interest rate model.

Commentary on Question:

The candidates performed well on this section. The most common reason for not getting full credit was simply to not provide 6 considerations. Generally the considerations provided were correct.

The following are the practical considerations when choosing interest rate model (full credit is earned if candidates listed at least six items):

1. Do the dynamics imply positives rates for each time t?
2. What distribution does the dynamics imply for the short rate, is it for instance, a fat-tailed distribution?
3. Are bond prices or rate curve explicitly computable from the dynamics?
4. Are prices of bond-options and interest rate options (cap, caplet, floor, swaption) explicitly computable?
5. Is the model mean-everting?
6. What is the volatility structure implied by the model?
7. Does the model allow for explicit short rate dynamics under the forward measure?
8. How suited is the model for Monte Carlo simulation?
9. How suited is the model for building recombining lattices?
10. Can the historical estimation techniques be used to estimate model parameters

7. Continued

(b) State the advantages and disadvantages of this model.

Commentary on Question:
The candidates performed excellently on this section. Most candidates provided all the advantages and the disadvantage of negative interest rates. This was generally enough to get full credit.

Advantages:
1. The model can fit the initial term structure
2. The model is analytically tractable
3. The model can analytically price zero-coupon bonds
4. The model can analytically price European options

Disadvantages:
1. Possibility of negative rates
2. It may be dangerous to fit initial term structure of volatilities
3. Does not reflect the correlation of rates on the term structure

(c) Calculate, as of time 0:

(i) The expected value of the short rate at time $t=1$

(ii) The standard deviation of the short rate at time $t=1$

(iii) The risk neutral probability that short rate exceeds 5% at time $t=1$.

Commentary on Question:
The candidates performed relatively well on this section. Part (i) below was missed by most of the candidates. Even when the correct formula was obtained, it was not used properly. Parts (ii) and (iii) were done correctly by most candidates.

As of time 0
(i)

$lnP(0, T) = -0.01(T + T^2)$

The market forward rate = $f^M(0,T) = -\frac{\partial lnP(0,T)}{\partial T}$

$f^M(0,T) = 0.01+0.02\text{T}$

$\alpha(t) = f^M(0,t) + \frac{\sigma^2}{2a^2}(1-e^{-at})^2$

$\alpha(0) = f^M(0,0) = r(0) = 0.01$

$\alpha(1) = 0.03 + \frac{0.02^2}{2\ 0.1^2}(1-e^{-0.1})^2 \approx 0.03$

7. Continued

$$E\{r(t)|\mathcal{F}_s\} = r(s)e^{-a(t-s)} + \alpha(t) - \alpha(s)e^{-a(t-s)}$$
$$E\{r(1)|\mathcal{F}_0\} = r(0)e^{-a(1)} + \alpha(1) - \alpha(0)e^{-a(1)} = \alpha(1)$$
$$E\{r(1)|\mathcal{F}_0\} \approx 0.03$$

(ii)

$$Var\{r(t)|\mathcal{F}_s\} = \frac{\sigma^2}{2a}[1 - e^{-2a(t-s)}]$$
$$Var\{r(1)|\mathcal{F}_0\} = \frac{\sigma^2}{2a}[1 - e^{-2a(1)}]$$
$$Var\{r(1)|\mathcal{F}_0\} = 0.00036$$
$$Stdev\{r(1)|\mathcal{F}_0\} = \sqrt{0.00036} \approx 0.019$$

(iii)

$$\text{Prob}[r(1) > 0.05|\mathcal{F}_0] = 1 - \text{Prob}[r(1) \leq 0.05|\mathcal{F}_0)]$$
$$= 1 - \text{Normal Standard Distribution } \left(\frac{0.05 - E\{r(1)|\mathcal{F}_0\}}{Stdev\{r(1)|\mathcal{F}_0\}}\right)$$
$$\text{Prob}[r(1) > 0.05|\mathcal{F}_0] = 1 - \text{Normal Standard Distribution } \left(\frac{0.05 - 0.03}{0.019}\right)$$
$$\text{Prob}[r(1) > 0.05|\mathcal{F}_0] = 14.9\%$$

8. Learning Objectives:

6. The candidate will understand and be able to describe the variety and assess the role of alternative assets in investment portfolios. The candidate will demonstrate an understanding of the distinguishing investment characteristics and potential contributions to investment portfolios of the following major alternative asset groups:
   - Real Estate
   - Private Equity
   - Commodities
   - Hedge Funds
   - Managed Futures
   - Distressed Securities
   - Farmland and Timber
   -

Learning Outcomes:

(6a) Demonstrate an understanding of the types of investments available in each market, and their most important differences for an investor.

(6b) Demonstrate an understanding of the benchmarks available to evaluate the performance of alternative investment managers and the limitations of the benchmarks.

(6d) Demonstrate an understanding of the due diligence process for alternative investments.

Sources:

QFIA-111-13: Maginn & Tuttle, Managing Investment Portfolios, 3rd Ed. 2007, Ch. 8

QFIA-112-13: Commercial Real Estate Analysis & Investment, Chapter 12

QFIA-113-13: Secular and Cyclic Determinants of Capitalization Rates: The Role of Property Fundamentals, Macroeconomic Factors and "Structural Changes"

Commentary on Question:

This question focuses on understanding of components to consider with real estate investment as: economic components, capitalization rate, the risk on the asset himself, the due diligence process for a specific market of client and the understanding of benchmark.

Solution:

(a) Describe the anticipated impact for each variable (in the above table) on the capitalization rate and asset value.

8. Continued

Commentary on Question:
Candidates performed relatively well on this section. For the Log(RRI) variable a few candidates were able to provide both the forward looking and backward looking explanations. For the other components the candidates did well. Some candidates provided an impact only on the capitalization rate only without mentioning the impact on asset value.

Log(RRI): Can have either positive or negative impact on both capitalization rate and asset value.

If investors are forward looking the market is at the peak of the cycle and downward adjustment is in order so expect lower cash flow in the future then this will have a positive effect on capitalization rates (higher capitalization rates) and negative effect on asset value.
If investors are backward looking, they will project higher rent levels into the future. This will bid up asset values and leading to a negative effect on capitalization rates (lower capitalization rate).

The RRI index has different lags across property types because investors in different property sectors react with different dynamics in response to changes in real estate fundamentals.

RTB:
The risk free rate;
Expected to have a positive effect on the capitalization rates and negative effect on asset value.

Spread Risk premium:
Reflect investors' tolerance for risk. Investors demand compensation for higher risk in the form of lower asset values for the same stream of net operating income. Then expected to have a positive effect on capitalization rate (higher capitalization rate).

Debtflow:
Investors bid up asset values when it is easier to trade them. So greater availability of debt financing provide a positive influence on real estate prices by inflating asset value by same way is associated with lower capitalization rate.

(b) Discuss the critical issue on the purchase price if your Opportunity Cost of Capital (OCC) on your real estate portfolio is much higher than a REIT competitor assuming neither use debt.

8. Continued

Commentary on Question:
Candidates performed poorly on this section. Many candidates explained the basic aspect that a higher OCC results in a lower price but failed to identify to use the OCC of the property. A few candidates explained why OCC of the portfolio is higher than the REIT. Almost no candidates were able to specify the value of the property as the cash flow stream using the OCC of that property.

OCC must reflect the risk in the property. So not a reason to pay a lower price with OCC higher than REIT competitor for the property.

Assuming same cash flow stream for you and the REIT for the property to be purchased then both must have the same value of property using the OCC of that property.

The OCC of your portfolio or the REIT OCC reflects the average risk of all assets in their portfolio which may or may not be the same as the risk of the property to be purchase.

Your higher OCC is likely explained by having more risky assets than the REIT.

(c) Identify the specific issues needed to satisfy a good due diligence for those clients.

Commentary on Question:
Candidates performed relatively poorly on this section. A majority of candidates provided a list of general information (not specific issues) about risk, documentation, organization and people. Very few candidates mentioned tax, concentration, time horizon and liquidity issues.

As part of fiduciary responsibility, communication for determining suitability of this investment need to addresses: taxes, time horizon, liquidity needs, role in the portfolio and risk associated to this change.

Another issue relates to decision of changing strategies within a time horizon and investors reactions to it;
Finally concentrated equity position of the client in a closely held company so impact of concentrated position on client's risk and liquidity position. This will interface with suitability, tax and asset allocation issues.

(d) Explain why the NCREIF benchmark used in the United States evolved into two indices.

8. Continued

Commentary on Question:
Candidates performed poorly on this section. The main errors were: 1) Wrong interpretation by mentioning direct and indirect instead of smooth and unsmooth. 2) Some candidates incorrectly mentioned that the indices were designed one for commercial and the other for residential.
About one third obtained partial credit for a brief description of smoothing effect for the appraisal description.

Almost no candidates specified or provided details relative to unsmooth NCREIF or Transaction-Based index.

The original NCREIF is based on infrequent internal appraisals or estimate valuation of sample properties.
This created less volatile values than market value. So exhibit inertia or smoothing effect, which underestimates volatility then an overstatement of the benefits of real estate in a portfolio.

To correct this bias, a <unsmooth NCREIF> or Transaction-Based Index (TBI) on NCREIF data was developed. This index based on transactions with objective price information of sold properties then gives a more accurate picture of real estate benefit.

9. Learning Objectives:

2. The candidate will understand and be able to apply a variety of credit risk theories and models.

4. The candidate will understand important quantitative techniques for analyzing financial time series Performance Measurement and Performance Attribution

Learning Outcomes:

(2e) Demonstrate an understanding of the term structure of default probability.

(4f) Calculate and interpret performance attribution techniques.

Sources:

QFIA-107-13: *Handbook of Fixed Income Securities,* Fabozzi, F.J., 8th Edition, 2012, Ch. 69.

Introduction To Credit Risk Modeling, Bluhm, Christian, 2nd Edition CH 6

Commentary on Question:

This question tests the following

- *Calculate and interpret performance attribution metrics for a given asset, portfolio.*
- *Demonstrate an understanding of the term structure of default probability.*

Solution:

(a) Calculate each of the missing values w1, w2, w3, w4 in Table 1.

Commentary on Question:

Candidates performed excellently on this section. There were two ways to interpret this question and the majority of candidates, once having chosen one of the correct methodologies, were consistent in their calculations. Areas where candidates did not earn full credit were primarily due to errors in the calculation inputs.

Full-credit Solution submitted by most candidates:
(w1-w2)x7 = 1.05; w1x(8-7) = 0.45 => w2=30%, w1=45%
30% + w1 + w3 = 1 => w3=25%
40% + w2 + w4 = 1 => w4=30%
w1=45%, w2=30%, w3=25%, w4=30%

Alternate Full-credit Solution:
(w1-w2)x8 = 1.05, w2x(8-7) = 0.45 => w2 = 45%, w1= 58.125%
30% + w1 + w3 = 1 => w3 = 11.875%
40% + w2 + w4 = 1 => w4 = 15%
w1=58.125%, w2=45%, w3=11.875%, w4=15%

9. Continued

(b) Calculate the asset allocation return and the security selection return pertaining to each bond rating for Portfolio B.

Commentary on Question:
Candidates performed excellently on this section, with the majority earning full credit. Similarly to part a), there were two possible solutions that earned full credit. Most candidates who did not earn full credit performed calculation errors using incorrect weights or failed to solve for all components.

Full-credit Solution submitted by most candidates:

	Asset	Security
A:	(30%-40%)x2	30%x(6-2)
BB:	(25%-30%)x7	25%x(8-7)
B:	(45%-30%)x6	45%x(10-6)

a1=-0.2%, a2=-0.35%, a3=0.9%, s1=1.2%, s2=0.25%, s3=1.8%

Alternate Full-credit Solution:

	Asset	Security
A:	(30%-40%)x6	40%x(6-2)
BB:	(25%-30%)x8	45%x(8-7)
B:	(45%-30%)x10	15%x(10-6)

a1=0.6%, a2=-0.4%, a3=1.5%, s1=1.6%, s2=0.45%, s3=0.60%

(c) Determine the appropriate value of n such that the bond rating transition matrix "M" is a stochastic matrix.

Commentary on Question:
Candidates performed excellently on this section, with the majority earning full credit. In solving for n, nearly all knew the value of n needed to be positive. A small number of candidates did not earn full credit because they did not attempt the question, or made calculation errors such as omitting the division by 100 in the equation.

Using the BB-rated bond information:
$n^2/100 + 2n/100 + 0.94 + 0.02 + 0.01 = 1$
$n^2/100 + 2n/100 – 0.03 = 0$
Applying the quadratic equation results in n = 1 or n = -3
Since n must be positive, the only acceptable solution is n = 1.

9. Continued

(d) Calculate the cumulative 2nd year default probability for bond currently rated B.

Commentary on Question:
Candidates performed excellently on this section. Most earned full credit and calculated the probability correctly. Less than full credit was earned by candidates who performed calculation errors or performed the correct calculation with incorrect inputs.

Using the B-rated bond information:

The only possibility is the transition from B to C in year 1, then C to default in year 2, i.e., 3% * 1% = 0.03%

(e) Assess which of the two portfolios (Portfolio A and Portfolio B) is most appropriate for your company.

Commentary on Question:
Candidates performed well on this section. The majority of candidates correctly concluded that Portfolio A met both requirements. Less than full credit was earned if the candidate misinterpreted the results following the correct calculations, had calculation errors that led them to incorrect conclusions, or failed to draw any conclusions once the calculations were complete.

	Portfolio A	Portfolio B
Asset	0.2+1.05-0.3=0.55%	0.2-0.35+0.9=0.35%
Security	1.2+0.45+1=2.65%	1.2+0.25+1.8=3.25%

Portfolio A satisfies both conditions but Portfolio B only satisfies the second condition, so Portfolio A should be chosen.

(f) Critique your economist's observations.

Commentary on Question:
Candidates performed relatively well on this section. Many candidates were able to draw the correct conclusions regarding the economist's observations after performing the appropriate calculations. Candidates who earned less than full credit made calculation errors, drew the correct conclusions but did not provide support leading to their conclusions, or failed to opine on whether the economist's observations were correct.

9. **Continued**

Portfolio shift return: (20%x1.5) + (20%x0.75) + (9%x0.5) + (9%x4/3) + (42%x1) = 1.035

Benchmark shift return: (37%x1) + (34%x0.75) + (2%x1) + (3%x1) + (24%x1.5) = 1.035

Portfolio twist return: (20%x0.025) - (20%x0.05) - (9%x0.1) + (9%x0) - (42%x0.025) = -0.0245

Benchmark twist return = (37%x-0.025) - (34%x0.075) - (2%x0.05) - (3%x0.025) - (24%x0.075) = -0.0185

The shift returns are the same (economist is correct with shift) but the twist returns are not (economist is incorrect with twist).

The difference in twist return implies there is less exposure to a fall in the longer end of the yield curve.

10. **Learning Objectives:**

6. The candidate will understand and be able to describe the variety and assess the role of alternative assets in investment portfolios. The candidate will demonstrate an understanding of the distinguishing investment characteristics and potential contributions to investment portfolios of the following major alternative asset groups:
   - Real Estate
   - Private Equity
   - Commodities
   - Hedge Funds
   - Managed Futures
   - Distressed Securities
   - Farmland and Timber

Learning Outcomes:

(6c) Demonstrate an understanding of the investment strategies and portfolio roles that are characteristic of each alternative investment.

(6d) Demonstrate an understanding of the due diligence process for alternative investments.

Sources:

QFIA-111-13: Maginn & Tuttle, Managing Investment Portfolios, 3rd Ed. 2007, Ch. 8 section 8

Commentary on Question:

This question tests the understanding of the use of alternative assets in investment management. In particular, it tests the understanding of distressed securities, the strategies to invest in this asset class and the issues with benchmarking their performance.

A general weakness among the candidates was a lack of proper connection to the firm's experience on distress securities, especially on part (c).

Solution:

(a) Identify reasons why this fund would invest in distressed securities.

Commentary on Question:

The candidates performed well on this section. However, some candidates did not mention the ability to achieve potentially high risk-adjusted return.

- Take advantage of investment opportunities that are not considered by most investors because of investment policy restrictions, regulatory restrictions or lack of analyst coverage.

10. Continued

- Knowledgeable investors have historically been able to achieve high risk adjusted returns in this asset category through security selection and (corporate) activism.

(b) Describe the principal risks of the investment in distressed securities.

Commentary on Question:
The candidates performed relatively poorly on this section. For "liquidity risk", many candidates did not explain why this is a risk when investing in distressed securities. Also, some candidates omitted the "event risk", or failed to explain what event risk is about.

- Liquidity risk: Market liquidity for distressed securities is significantly less than for other securities. Liquidity is based on supply and demand and those can freeze for distressed assets in a bad economic cycle.

- J factor risk (legal, judiciary risk): Court decisions will impact the liquidation or restructuring processes that can affect the value of an investment in distressed securities.

- Event risk: Distressed securities were issued by companies that are now in difficulty and where the risk of company specific event widely changing the situation of the company is higher.

- Market risk: Market movements will also impact the value of distressed securities although it is not as important as other risks.

(c) Recommend a distressed security investment strategy that would take advantage of the strengths of your team.

Commentary on Question:
The candidates performed relatively poorly on this section. Some candidates recommended private equity or active or controlling strategy, but did not connect them with team's experience (corporate activism and bankruptcy proceeding). For example, the phrase "corporate activism" itself is insufficient to explain why a firm may have superior returns.

- The best strategy to invest in distressed securities through a private equity or active structure.

10. Continued

- The experience in corporate activism and bankruptcy proceeding will allow superior returns when exerting control over a distressed company. This is achieved through a private equity type approach where the investors becomes the major creditor and uses this position to influence the board of the distressed company.

(d)

(i) Develop and describe a benchmark that would best measure the diversification potential of your distressed security portfolio.

(ii) Calculate the benchmark return for the past year.

Commentary on Question:
The candidates performed relatively well on this section. Most candidates correctly excluded fund C but often did not explain why it should be excluded. Also about half the candidates incorrectly used a weighted AUM to calculate the benchmark instead of an unweighted AUM.

- The benchmark should exclude fund C because it is not the same strategy.
- The benchmark should include fund B even though it was terminated at the end of the year.
- The benchmark is not weighted on the AUM of the funds because equal weight benchmarks may reflect potential diversification better
- The benchmark is (26% + - 15% + 12%) / 3 = 7.666%

11. Learning Objectives:

1. The candidate will understand the standard yield curve models, including:
   - One and two-factor short rate models
   - LIBOR market models

   The candidate will understand approaches to volatility modeling.

Learning Outcomes:

(1a) Identify and differentiate the features of the classic short rate models including the Vasicek and the Cox-Ingersoll-Ross (CIR) models.

(1b) Understand and explain the terms Time Homogeneous Models, Affine Term Structure Models and Affine Coefficient models and explain their significance in the context of short rate interest models.

(1c) Explain the dynamics of and motivation for the Hull-White extension of the Vasicek model.

Sources:

Brigo, D and Mecurio F, Interest Rate Models – Theory and Practice, 2nd Edition, Ch.3 (3.1-3.3)

Commentary on Question:

This question tests candidates' general understanding of short-rate models covered in the syllabus, and their applications in the context of interest-sensitive product pricing. Most candidates did well on part (a) and (b). For part (c), most candidates made reasonable recommendation of either model B or model D, however, few supported their recommendation in the context of other available model choices (e.g., if Model D is recommended for its various features, why do other models are not as good as Model D).

Solution:

(a) Explain each of the following terms:

(i) Time Homogenous Models

(ii) Affine Term Structure Models

(iii) Affine Coefficient Models

Commentary on Question:

The candidates performed excellently on this section.

(i) "Time Homogenous Models" refer to interest rate models where the short rate dynamics depends **only** on constant coefficients

11. Continued

(ii) "Affine Term Structure Models" refer to interest rate models where the continuously compounded spot rate, *R(t, T)*, is an affine function in the short rate *r(t)*. *Or:* "Affine Term Structure Models" refer to short rate models where the solution to zero-coupon bond price satisfies $P(t,T) = A(t,T)e^{-B(t,T)r(t)}$

(iii) "Affine Coefficient Models" refer to interest rate models in which the coefficient of the drift term and the **squared** coefficient of the stochastic term are affine functions of short rate, respectively

(b) Fill out the table below with "Yes (Y)," "No (N)" or "Not Applicable (N/A)"

Commentary on Question:
The candidates performed well in this section. In order to get credit on a model the candidates needed to get all features correct. Many candidates got most of the features but it was important to get them all correct to demonstrate an understanding of the material.

	Model			
Model features or implications	**A**	**B**	**C**	**D**
The model is Time Homogenous	Y	Y	Y	N
The model displays an Affine Term Structure	Y	Y	N	Y
The model has Affine Coefficients	Y	Y	N	Y
Short rate is always positive in the model	N	Y	Y	N
Short rate is normally distributed in the model	Y	N	N	Y
The model has closed-form formula to price zero-coupon bond	Y	Y	Y	Y
The model has closed-form formula to price European options on zero-coupon bond	Y	Y	N	Y

(c) Recommend one model from Models A through D above.

Commentary on Question:
The candidates performed well in this section. Most candidates made reasonable recommendation of either model B or model D, however, few supported their recommendation in the context of other available model choices (e.g., if Model D is recommended for its various features, why do other models are not as good as Model D).

11. Continued

Based on table in part (b) and the general product development process, either Model B or Model D may be recommended.

If recommend Model D (i.e. the Hull-White model), the supporting rationales are:

1. It has the advantage of being able to fit the initial yield curve, better than Model A and Model B

2. It retains model tractability [or it has closed-form pricing formula for zero-coupon bond and European option on zero-coupon bond]

3. It could theoretically generate negative rates. However, the probability of generating negative rate is lower if the initial rate level is above the very low rate level.

4. It is better than Model C (Dothan Model) because Model C

- cannot fit initial market yield curve very well
- does not have closed-form formula for pricing derivatives
- the pricing formula for zero-coupon bond is complex to evaluate
- suffers "explosion of bank account" problem

If recommend Model B (i.e., the CIR model), the supporting rationales are:

1. It avoids negative rate problem, better than Model A and Model D

2. It retains model tractability [or it has closed form pricing formula for zero-coupon bond and European option on zero-coupon bond].

3. Its' disadvantage is that it cannot fit the initial yield curve very well. This may not be essential given the time period of product development

It is better than Model C (Dothan Model) because Model C

- does not have closed-form formula for pricing derivatives
- the pricing formula for zero-coupon bond is complex to evaluate
- suffers "explosion of bank account" problem

12. Learning Objectives:

3. Candidate will understand the nature, measurement and management of liquidity risk in financial institutions.

Learning Outcomes:

(3a) Understand the concept of liquidity risk and the threat it represents to financial intermediaries and markets.

(3b) Measure and monitor liquidity risk, using various liquidity measurement tools and ratios.

(3g) Create liquidity risk management plans and procedures, including addressing appropriate product design, investment guidelines, and reporting given a desired liquidity risk level.

Sources:

QFIA-117-13: Reflections on Northern Rock: The Bank Run That Heralded the Global Financial Crisis

QFIA-106-13: *Liquidity Risk: Measurement and Management – A Practitioner's Guide to Global Best Practices*, Matz, Leonard & Neu, Peter, 2006, Ch. 2

Commentary on Question:

This question tests the candidates' knowledge of the impact of leverage and the ways to measure and reduce liquidity risk.

Solution:

(a) Calculate the leverage of XYZ.

Commentary on Question:

Candidates did relatively well on this section. Candidates mostly either received full credit or no credit for the answers provided.

Leverage = Assets/Equity
Assets=1000, Equity=50
Leverage=1000/50=20

(b) Calculate the cash capital of XYZ.

Commentary on Question:

Candidates did well on this part. Many candidates received full credit for the answer provided. A common mistake was to include the mortgages in the calculation of collateral value.

12. Continued

Cash capital = Collateral value of unencumbered assets less the sum of short-term funding and non-core funding.
Collateral value of unencumbered assets = the value of the corporate bonds and the ABS after subtracting the haircuts (the mortgage holdings were not to be included in the calculation as they were not accepted as collateral).
= (1-.04)*250 + (1-.02)*150 = 240 + 147 = 387
Short-term funding is the value of the repurchase agreement liability = 300
Non-core deposits = 50
Cash capital = 387 – 300 – 50 = 37

(c) Recommend changes to the existing assets and liabilities that could reduce the liquidity risk for XYZ.

Commentary on Question:
Candidates did relatively well on this part. Most candidates received at least some credit for the answer provided, primarily by at least mentioning that the allocation to commercial mortgages could be reduced.

XYZ could do some or all of the following:

- Reduce the amount of long-term assets supported by short-term financing.
- Increase the amount of long-term financing or reduce the amount of short-term financing.
- Reduce the allocation to commercial mortgages.
- Reduce the amount of leverage by increasing the amount of equity.
- Increase the amount of cash and/or short-term equivalents backing the deposits.

(d) Calculate the amount of repurchase agreements that can be rolled over.

Commentary on Question:
Candidates did relatively poorly on this part. Few candidates understood how to determine the amount of repurchase agreements that could be replaced. Many candidates were able to determine that the amount of ABS was reduced by the sales. A common mistake was to include the commercial mortgages in the calculation of the collateral value.

The amount of repurchase agreements that can be replaced depends upon the value after haircuts of the acceptable collateral which includes only the corporate bonds and the ABS.
The amount of ABS is reduced by the sale, (1-.2)*150 = 120.
Collateral value = (1-.3)*250 + (1-.25)*120 = 175+90 = 265.
Therefore, XYZ can replace 265mm of the repurchase agreements.

12. Continued

(e) Recommend a proposal for XYZ to replace the funding.

Commentary on Question:
Candidates did relatively poorly on this part. A common mistake was to provide a recommendation without any justification. Another common mistake was to suggest securitizing the mortgages, something that Northern Rock was specifically unable to do at a time of heightened market stress.

The value of XYZ's remaining collateral is not sufficient to replace the full $300 million of repurchase agreements, so additional action will be necessary.
The minimum value of the assets to be used as collateral, as requested by the CFO, is $360mm, 120% * $300mm.
This limits the amount of corporate bonds and ABS that could be sold as the total amount of these after the ABS sale is 250mm + 120mm = 370mm.
XYZ could consider selling some of its mortgage holdings to make up for the shortfall since there is time (90 days to the trigger).
Issuing new long-term debt would be at a considerably higher rate (12%) than is being earned on the mortgage holdings.
Issuing new equity is not an option available to XYZ due to the level of stress in the market.

13. Learning Objectives:

2. The candidate will understand and be able to apply a variety of credit risk theories and models.

Learning Outcomes:

(2d) Demonstrate an understanding of Merton asset value models in the context of credit risk.

Sources:

Introduction to Credit Risk Modeling, Bluhm, Christian, 2nd Edition, Chapter 3.

Commentary on Question:

This question tests the understanding of Merton's Asset Value Model with the application to credit risk.

Solution:

(a)

(i) Develop a graph to illustrate the value of CAP's position as a function of the value of JEC's assets at the expiration of the debt. Please include data labels.

(ii) Identify a theoretical position that hedges CAP's position in JEC's debt.

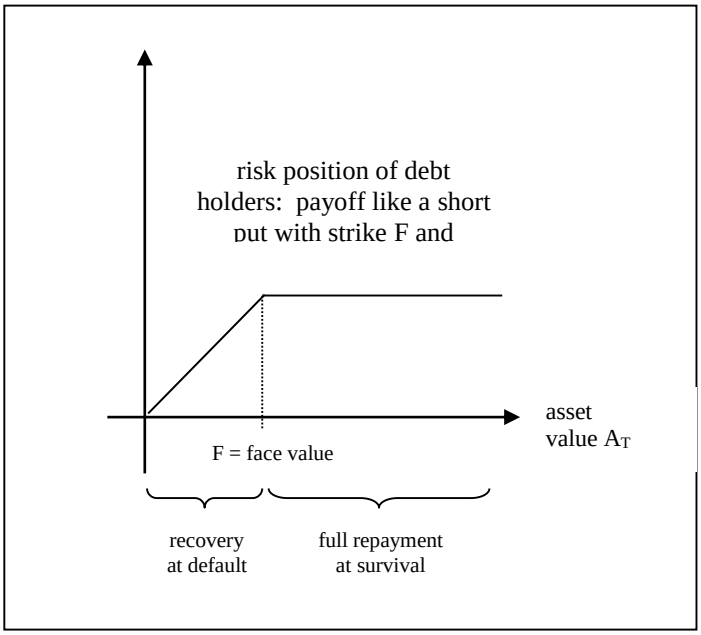
(iii) Demonstrate the effectiveness of the hedge by showing cash flows.

Commentary on Question:

The candidate performed excellently on this section. Maximum points were awarded for an illustrative graph, suggesting long a put option and showing cash flows at all time points. Candidates consistently omitted the initial cash flows (i.e. cost of hedge at time zero).

13. Continued

JEC capital structure is $A_T = E_T + D_T$ thus credit risk arises when $P[A_T < F] > 0$.



To neutralize the credit risk, CAP will purchase a long put with:

- underlying A_T;
- strike price F;
- maturity T.

	asset value	CAP cash flows	CAP payout
t = 0	A0	$-D_0 - P_0$	$-D_0 - P_0$
t = T	$A_T < 1000$	A_T (recovery) $1000 - A_T$ (apply put)	1000
	$A_T \geq 1000$	1000 (receive face value) 0	1000

(b) Calculate the option cost to implement the hedge strategy from part (a).

Commentary on Question:
The candidates performed relatively well on this section. Most candidates were successful at utilizing Black-Scholes formula. The common mistake was pricing the put option for 5 years instead of 3 years.

13. Continued

$P(S, t) = N(-d_2) * Ke^{-r(T-t)} - N(-d_1) * S$, where

- K = face value = 1,000
- S = underlying asset = $D_2 + E_2 = 816.25 + 333.75 = 1150$
- r = 0.02
- T – t = 3 years
- $d_1 = \frac{1}{\sigma\sqrt{T-t}} \left[\ln(\frac{S}{K}) + (r + \frac{\sigma^2}{2})(T - t)\right]$
- $d_2 = \frac{1}{\sigma\sqrt{T-t}} \left[\ln(\frac{S}{K}) + (r - \frac{\sigma^2}{2})(T - t)\right]$
- $\sigma = 0.30$

$$d_1 = \frac{1}{(0.30)\sqrt{3}} \left[\ln(\frac{1150}{1000}) + (0.02 + \frac{(0.30)^2}{2})(3)\right] = 0.644$$

$$d_2 = \frac{1}{(0.30)\sqrt{3}} \left[\ln(\frac{1150}{1000}) + (0.02 - \frac{(0.30)^2}{2})(3)\right] = 0.124$$

$N(d_1) = 0.74 \Rightarrow N(-d_1) = 0.26$
$N(d_2) = 0.55 \Rightarrow N(-d_2) = 0.45$

$$P = (1 - 0.55)(1000)e^{-(0.02)(3)} - (1 - 0.74)(1150) = 125$$

(c) Describe three assumptions about JEC's assets and capital structure which are necessary for the application of Merton's Asset Model.

Commentary on Question:
The candidates performed relatively poorly on this section. Most candidates correctly identified asset volatility. The other assumptions were commonly missed in candidate's papers.

1) Simple capital structure consisting of a zero-coupon bond
2) Asset values are observable
3) Asset volatility if known or can be derived from observed prices
4) Underlying assets of the firm are tradeable
5) Asset prices follow Geometric Brownian motion. (really a Black-Scholes assumption but acceptable for Merton)

14. **Learning Objectives:**

2. The candidate will understand and be able to apply a variety of credit risk theories and models.

Learning Outcomes:

(2e) Demonstrate an understanding of the term structure of default probability.

Sources:

Introduction to Credit Risk Modeling, Bluhm, Christian, 2nd Edition, Ch.6

Commentary on Question:

This question tests an understanding of the term structure of default probability.

Solution:

(a) Calculate the forward survival probability $q(3|2)$.

Commentary on Question:

The candidates did well on this section. This question is primarily derived from prior actuarial exam knowledge.

$q(t|s) = 1 - p(t|s) = P(\tau > t|\tau > s) = S(t)/S(s), \qquad t \geq s \geq 0$

$* \tau$ is the hazard function, which gives the instantaneous probability of default at time t conditional on the survival up to t.

$q(3|2) = 1 - p(3|2) = P(\tau > 3|\tau > 2) = S(3)/S(2)$ = (1-3.76%)/(1-2.61%) = 98.82%

(b) Calculate *s*.

Commentary on Question:

The candidates performed poorly on this section. Some candidates did not recognize that this was a three-year Credit Default Swap (CDS) being calculated and used the one-year formula where spread = PD x (1- LGD). Some candidates used the spread as part of the discount rate (i.e. risk spread) and not the premium amount paid by the pay-leg of the CDS.

The notation shown in the solution is that which is used within the syllabus.

$$0 = \sum_{i=1}^{n} B(0,T_i)s\Delta_i S(T_i) - V(1-REC)\int_0^{T_n} B(0,t)F(dt)$$

$$\sum_{i=1}^{n} B(0,T_i)s\Delta_i S(T_i) = 100 * \left[s * \frac{1-1.3\%}{1+5\%}\right] + \left[s * \frac{1-2.61\%}{(1+5\%)^2}\right] + \left[s * \frac{1-3.76\%}{(1+5\%)^3}\right]$$

$$= s * 265$$

14. Continued

$$V(1-REC)\int_0^{T_n} B(0,t)F(dt)$$
$$= 100 * (1-30\%) * \left[\frac{1.3\%}{1.05} + \frac{(2.61\% - 1.3\%)}{1.05^2} + \frac{(3.76\% - 2.61\%)}{1.05^3}\right] = 2.40$$

$s * 265 = 2.39 \implies s = 0.90$

(c) Explain why the risk-neutral default probabilities $\left(PD_t^{rn}\right)$ are no less than the actual default probabilities $\left(PD_t^{real}\right)$, under the Merton-style approach.

Commentary on Question:
The candidates performed relatively well on this section. Candidates who understood the Merton-style approach did well on this question. The key was relating the PD back to the risk-free rates and then recognizing that investors will invest in risky assets only if they are compensated for that risk.

The actual cumulative default probability from time 0 to time t in a real, risk averse world is given by:

$$PD_t^{real} = N(-\frac{\log\left(\frac{A_0}{C}\right)+\left(\mu-\frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}})$$

In a world where investors are neutral to risk, all assets should yield the same risk-free return r. So, the risk-neutral default probabilities are given as:

$$PD_t^{rn} = N(-\frac{\log\left(\frac{A_0}{C}\right)+\left(r-\frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}})$$

Because investors refuse to hold risky assets with expected return less than the risk-free base rate, μ must be larger than r. It follows that $PD_t^{rn} \geq PD_t^{real}$

(d) Derive the survival function $S(t)$, assuming no hump in the term structure of default intensities.

Commentary on Question:
The candidates did well on this section. The key to this question was to identify that a2 = 0 when there is no hump in the term structure.

Assuming no hump in the term structure of default probability, $a_2 = 0$, so

$$\emptyset(t) = a_0 + a_1\left(\frac{1-\exp(-t/a_3)}{t/a_3}\right)$$

$S(t)=e^{-\int_0^t h(s)ds} = \exp[-(a_0 + a_1\left(\frac{1-\exp(-t/a_3)}{t/a_3}\right))*t]$

14. Continued

(e)

(i) Explain in words what parameter a_1 represents.

(ii) Propose one explanation as to why investment grade bonds tend to have a slowly upward sloping term structure whereas those of speculative grade bonds tend to be downward.

Commentary on Question:
The candidates did relatively well on this section. The candidates that did well on this question gave reasons and examples as to why bonds have particular term structure shape. The candidates that did poorly simply described the formula using words and provided no further insight into the shape of the term structures.

The parameter a_1 represents its current deviation from the mean.

Investment grade bonds are classified that way because they are deemed to have less risk when initially graded. Over time however circumstances will change making it more likely that default could happen. Historical evidence has shown that. Another, rudimentary way to think of it is that a highly rated bond cannot become any less risky (i.e. it has the least risky credit rating) and can only get more risky over time.

On the other hand, speculative grade bonds are classified that way because they are deemed to be quite risky when initially graded. If they can survive the initial risky period the bonds will likely improve in quality over time leading to a slightly downward sloping term default structure. Historical evidence has shown that. Again, a very risky bond can only default, or improve over time.

15. Learning Objectives:

2. The candidate will understand and be able to apply a variety of credit risk theories and models.

Learning Outcomes:

(2m) Understand the rationale, markets and risks of structured finance.

Sources:

QFIA-101-13: *Managing Credit Risk, The Great Challenge for Global Financial Markets*, Caouette, John B. et al., 2nd edition 2008, chapter 24

Commentary on Question:

This question tested the understanding of an ABS investment vehicle in an unfamiliar context.

The intro to this question, all numerical values, and names were identical to the Fall 2014 Question 15, however we asked calculation questions instead. Although the question was long and located at the end of the exam, we felt that this was fair since we had indicated in the Fall 2014 Illustrative solutions that we would likely reuse the question, thus giving the candidates advanced warning to review it.

Overall the candidates did relatively poorly on this question.

Solution:

(a)

(i) Calculate the number of marbles paid to Eric, Fred, George and Donald at the end of Week 1 and Week 2.

(ii) Calculate the number of marbles still owed to Eric, Fred and George at the end of Week 2.

Commentary on Question:

The candidates performed relatively poorly on this section. The key here was to get the order of payment correctly. There were 3 fixed income tranches and one equity tranche. Interest is paid first to all the tranches. Interest is not accumulated or compounded for the subordinate tranches, which was a common error. The equity tranche receives nothing until all the senior and subordinate tranches are paid, which was another common error made. The principal for the fixed income tranches are paid in order of seniority, payment starting for a tranche when the previous tranche is entirely paid out.

Here is a very detailed answer, followed by a more simpler one that was worth full credit:

15. Continued

Week 1:
Total payment from old school : 410 marbles
Interest due on credit tranches:
Eric : = 300 x (1/30) = 10
Fred : = 300 x (1/20) = 15
George: = 300 x (1/5) = 60

Marbles left after payment of interest = 410 - 10 -15 - 60 = 325
payment of principal
Eric : 300 due, 300 availables, paid in total and 25 marbles left
Fred : 300 due, 25 availables after payment to Eric, 275 due next week
George: 300 due, no marbles left to pay
Donald: equity tranche, paid after all credits tranches are paid

total payment: = interest + principal
Eric :10 + 300 = 310
Fred :15 + 25 = 40
George:60 + 0 = 60

Week 2:
Total payment from old school : 439 marbles
Interest due on credit tranches:
Eric := 0 x (1/30) = 0
Fred := 275 x (1/20) = 14
George:= 300 x (1/5) = 60

Marbles left after payment of interest = 439 - 0 -14 - 60 = 365
payment of principal
Eric : 0 due,
Fred : 275 due, 275 availables paid in total and 90 marbles left
George: 300 due, 90 availables after payment to Fred, 210 due next week.
Donald: equity tranche, paid after all credits tranches are paid
total payment: = interest + principal
Eric : 0 + 0 = 0
Fred : 14 + 275 = 289
George: 60 + 90 = 150

Marbles owed after week 2:
= initial balance – principal paid week 1 – principal paid week 2
Eric : 300 – 300 – 0 = 0
Fred : 300 – 25 – 275 = 0
George: 300 – 0 – 90 = 210

15. Continued

Total paid week 1	410	310	40	60	0
Balance end of week 1		0	275	300	
Interest week 2	80	0	14	60	= 6
Principal week 2	359	0	275	90	= -6
Total paid week 1	410	310	40	60	= 0
Balance end of week 2		0	0	210	

(b) Calculate the marbles Donald gets to keep from this transaction after Week 3.

Commentary on Question:
The candidates performed well on this section. Since this section depended on part a) it was graded assuming the answer provided in a) was correct. Most candidates were able to draw a correct conclusion based on their starting point from a).

Part b) and c) tested the candidate's knowledge of the interaction of the fixed income tranches, equity tranche and the guarantor. In this case Donald owns the equity tranche and Bruno is the guarantor.

Principal left at beginning of week 3:

=	Total owed beginning	1100
	principal paid week 1	-300
	principal paid week 2	-359
	amount of loans default	-201
	Principal beginning week 3	240

Interest paid by debtor week 3	= 240*(1/10)	=	24
Marbles received week 3	= 240 + 24	=	264

Required payment to George

=	Principal owed (from a))	210		
+	interest due	42	=	210*(1/5)
		252		

There is enough marbles received to pay George
Donald keep 12 (= 264 – 252) marbles from the equity tranche

15. Continued

(c) Calculate the marbles that are distributed to or from all relevant parties after Week 3.

Commentary on Question:
The candidates performed relatively well on this section. Since this section depended on part a) it was graded assuming the answer provided in a) was correct.

The difference compared to part b) was that the resulting deficit required Bruno, the guarantor, to cover the amount due to George. Candidates often missed the fact that any amount paid by Bruno was after Donald had made all payments to the fixed income tranches. Thus Donald received nothing.

Principal left at beginning of week 3:

=	Total owed beginning	1100
-	principal paid week 1	-300
-	principal paid week 2	-359
-	amount of loans default	-301
	Principal beginning week 3	140

Interest paid by debtor week 3 = 140*(1/10) = 14
Marbles received week 3 = 140 + 14 = 154

Required payment to George (same as in b) above)

=	Principal owed (from a))	210	
+	interest due	42	= 210*(1/5)
		252	

- There are no enough marbles received to pay George
- Bruno, as the guarantor, will pay 98 marbles to Donald
- Those 98 marbles will be used to pay George what he is owed, so George will receive his 252 marbles
- Donald keeps nothing