
SOCIETY OF ACTUARIES
Quantitative Finance and Investment Advanced Exam

Exam QFIADV

AFTERNOON SESSION

Date: Thursday, April 30, 2015

Time: 1:30 p.m. – 3:45 p.m.

INSTRUCTIONS TO CANDIDATES

General Instructions

1. This afternoon session consists of 6 questions numbered 10 through 15 for a total of 40 points. The points for each question are indicated at the beginning of the question. A template is provided to be used in answering question 11.
2. Failure to stop writing after time is called will result in the disqualification of your answers or further disciplinary action.
3. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions on the exam booklet.

Written-Answer Instructions

1. Write your candidate number at the top of each sheet. Your name must not appear.
2. Write on only one side of a sheet. Start each question on a fresh sheet. On each sheet, write the number of the question that you are answering. Do not answer more than one question on a single sheet.
3. The answer should be confined to the question as set.
4. When you are asked to calculate, show all your work including any applicable formulas. When you are asked to recommend, provide proper justification supporting your recommendation.
5. When you finish, insert all your written-answer sheets into the Essay Answer Envelope. Be sure to hand in all your answer sheets because they cannot be accepted later. Seal the envelope and write your candidate number in the space provided on the outside of the envelope. Check the appropriate box to indicate morning or afternoon session for Exam QFIADV.
6. Be sure your written-answer envelope is signed because if it is not, your examination will not be graded.

Tournez le cahier d'examen pour la version française.

****BEGINNING OF EXAMINATION****

Afternoon Session
Beginning with Question 10

10. (6 points) You manage a portfolio of distressed securities of a large private retirement fund.

- (a) (1 point) Identify reasons why this fund would invest in distressed securities.
- (b) (2 points) Describe the principal risks of the investment in distressed securities.

Your team has considerable experience in bankruptcy proceedings, in corporate activism, and has had the most success by becoming the major creditor of distressed companies.

- (c) (1 point) Recommend a distressed security investment strategy that would take advantage of the strengths of your team.

You use the following hedge fund data to benchmark your portfolio:

Fund	Strategy	Past year performance	Assets Under Management (AUM) (\$ millions)
A	Distressed securities	26%	100
B	Distressed securities	- 15%	1,200
C	Equity market neutral	4%	500
D	Distressed securities	12%	2,400

At the end of last year, fund B was terminated and its assets were returned to its investors.

- (d) (2 points)
 - (i) Develop and describe a benchmark that would best measure the diversification potential of your distressed security portfolio.
 - (ii) Calculate the benchmark return for the past year.

Question 11(b) may be answered using the template provided in your exam booklet. Please include this template in your Answer-Essay Envelope upon completion of the exam.

11. (6 points) You are considering one-factor short rate models.

(a) (2 points) Explain each of the following terms:

- (i) Time Homogenous Models
- (ii) Affine Term Structure Models
- (iii) Affine Coefficient Models

You are given the following four short rate models:

Model	Dynamics of short rate $r(t)$
A	$dr(t) = k(\theta - r(t))dt + \sigma dW(t)$
B	$dr(t) = k(\theta - r(t))dt + \sigma\sqrt{r(t)}dW(t)$
C	$dr(t) = kr(t)dt + \sigma r(t)dW(t)$
D	$dr(t) = (\theta(t) - ar(t))dt + \sigma dW(t)$

(b) (1 point) Fill out the table below with “Yes (Y),” “No (N)” or “Not Applicable (N/A)”

Model features or implications	Model			
	A	B	C	D
The model is Time Homogenous				
The model displays an Affine Term Structure				
The model has Affine Coefficients				
Short rate is always positive in the model				
Short rate is normally distributed in the model				
The model has closed-form formula to price zero-coupon bond				
The model has closed-form formula to price European options on zero-coupon bond				

Your company is evaluating interest rate derivatives in its new product development.

(c) (3 points) Recommend one model from Models A through D above.

12. (8 points) You are a consulting actuary that has been asked by the CFO of XYZ Bank to assess the bank's liquidity risk. Below is the current asset and liability profile of the bank.

Asset Class	Current Value	Book Yield
Commercial Mortgages	600	7%
Corporate Bonds	250	5%
AAA-Rated ABS	150	4%

Liability Type	Current Value
Long-Term Debt	350
30-day Repurchase Agreement	300
Core Deposits	250
Non-Core Deposits	50
Equity	50

The commercial mortgages can be sold however they are not accepted as collateral. The corporate bonds and the ABS can be sold and are accepted as collateral with a haircut of 4% for the corporate bonds and a haircut of 2% for the ABS.

- (1 point) Calculate the leverage of XYZ.
- (1 point) Calculate the cash capital of XYZ.
- (2 points) Recommend changes to the existing assets and liabilities that could reduce the liquidity risk for XYZ.

One month later, XYZ needs to roll over the repurchase agreement funding. However, the level of financial stress in the market has greatly increased to the extent that issuing new equity would be very difficult. The following summarizes other changes due to the market conditions:

- In order to fund core deposit withdrawals, 20% of the ABS is sold. No further ABS can be sold.
 - XYZ would now need to pay 12% on any additional long-term debt it decided to issue.
 - The haircuts on corporate bonds have increased to 30% and those on ABS have increased to 25%.
 - Commercial mortgages can also be sold but at a 40% discount.
- (2 points) Calculate the amount of repurchase agreements that can be rolled over.

12. Continued

The CFO determines that XYZ must roll over all of the repurchase agreements but that given the level of financial stress in the market, the value of long-term liquid assets must exceed the funding agreements by 20% within 90 days. The CFO has asked for a proposal to meet the bank's funding needs.

- (e) (2 points) Recommend a proposal for XYZ to replace the funding.

13. (6 points) You are Chief Investment Officer (CIO) at CAP Insurance Company and your Chief Risk Officer (CRO) has concerns of your debt holding in Jane Elk Corporation (JEC). JEC is financed with a very simple capital structure, namely one debt obligation and one type of equity. JEC's debt is a zero-coupon bond with maturity in 5 years, face value of 1,000 and issued at 800.

(a) (3 points)

- (i) Develop a graph to illustrate the value of CAP's position as a function of the value of JEC's assets at the expiration of the debt. Please include data labels.
- (ii) Identify a theoretical position that hedges CAP's position in JEC's debt.
- (iii) Demonstrate the effectiveness of the hedge by showing cash flows.

Two years into the maturity of the JEC debt holding (i.e. 3 years remaining on the zero coupon bond), your CRO recommends that you execute the hedge strategy outlined in part (a). Given Black-Scholes assumptions and the following parameters:

- Price of JEC debt = 816.25
- Price of JEC equity = 333.75
- Volatility of the underlying JEC asset = 30%
- Risk free rate = 2%

- (b) (2 points) Calculate the option cost to implement the hedge strategy from part (a).
- (c) (1 point) Describe three assumptions about JEC's assets and capital structure which are necessary for the application of Merton's Asset Model.

14. (8 points) You are pricing a three-year zero-coupon bond with a face value of 100. The one-year risk-free interest rate is 5%. The recovery rate is 30% of the face value. Cumulative default experience based on the historical data:

T (year)	Cumulative Actual Default Rate
1	1.30%
2	2.61%
3	3.76%

- (a) (1 point) Calculate the forward survival probability $q(3|2)$.

You know that the market quotes the fair default spread s of a credit default swap with annual payments on this 3-year bond.

- (b) (2 points) Calculate s .
- (c) (2 points) Explain why the risk-neutral default probabilities (PD_t^m) are no less than the actual default probabilities (PD_t^{real}), under the Merton-style approach.

Your boss believes the hazard rate is time varying but deterministic and asks you to model the instantaneous default rate, $h(t)$ where:

$$\int_0^t h(s) ds = \phi(t) \cdot t$$

$$\phi(t) = a_0 + (a_1 + a_2) \left(\frac{1 - \exp(-t/a_3)}{t/a_3} \right) - a_2 \exp\left(-\frac{t}{a_3}\right)$$

- (d) (1 point) Derive the survival function $S(t)$, assuming no hump in the term structure of default intensities.

Industrial statistics show a_1 is negative for investment grade and positive for speculative grade.

- (e) (2 points)
- (i) Explain in words what parameter a_1 represents.
- (ii) Propose one explanation as to why investment grade bonds tend to have a slowly upward sloping term structure whereas those of speculative grade bonds tend to be downward.

- 15.** (6 points) Adam is a grade 5 student who just started at NuSchool Elementary School. At his previous school, OldSchool Elementary, he was an exceptional marble player and won lots of marbles. However, the students at OldSchool could not pay Adam his winnings right away which left Adam without any marbles.

To encourage the OldSchool Elementary students to give Adam his marbles, Adam charges 1 marble for every 10 marbles owed for every week until the marbles are repaid in full.

The amount owed to Adam is given in the table below.

Grade	Number of Students Who Owe Adam Marbles	Total Number of Original Marbles Owed to Adam	Total Number of Extra Marbles Paid to Adam Weekly
1	6	400	40
2	4	200	20
3	8	500	50
Total	18	1100	110

Adam was not a people-person and had a difficult time managing and collecting on all the debts owed to him from the OldSchool Elementary students. Additionally, Adam did not like having these debts owed to him and just wanted his marbles.

Donald, a people-person from NuSchool Elementary grade 4 class, decided to help Adam and gave him 1000 marbles in exchange for the debts.

Donald was nervous about having all these debts and wanted to get some of his marbles back. Donald decided to offer some deals to Eric, Fred and George who each had 300 marbles. In exchange for their marbles Donald would give them some of the marbles being paid by the OldSchool students. Eric was a little nervous about the deal; George was very excited about the deal; and Fred was indifferent.

15. Continued

The details of Donald's deals with Eric, Fred and George are given in the table below:

	Amount of Original Marbles Given to Donald Upfront	Amount of Original Marbles Returned to Eric, Fred or George	Calculation Formula of Extra Marbles Paid to Eric, Fred or George	Order in which Marbles are Paid to Eric, Fred or George
Eric	300	300	1 marble for every 30 outstanding per week	First
Fred	300	300	1 marble for every 20 outstanding per week	Second
George	300	300	1 marble for every 5 outstanding per week	Third
Donald's remaining piece		After all original marbles are returned to Eric, Fred and George, the remaining marbles are Donald's	None	Last

If Donald does not recover all 900 marbles plus due extra marbles that he owes Eric, Fred and George by the end of week 3, Bruno, Donald's Dad, has promised to replace any marbles Donald still owes at that point, up to a maximum of 100 marbles.

If OldSchool students move then the marbles that they owe to Donald will be assumed to never be repaid. Eric, Fred and George will not receive extra marbles on these amounts.

Question 15 continued on next page

15. Continued

Donald received the following marbles at the end of each of the next 2 weeks:

End of Week	Number of Extra Marbles Paid	Number of Original Marbles Returned to Donald
1	110	300
2	80	359

- (a) (3 points)
- (i) Calculate the number of marbles paid to Eric, Fred, George and Donald at the end of Week 1 and Week 2.
 - (ii) Calculate the number of marbles still owed to Eric, Fred and George at the end of Week 2.

At the beginning of Week 3, two events happen:

1. First, Donald found out that some of the OldSchool students had moved and they still owed him 201 marbles that he will never get back.
 2. Second, all remaining OldSchool students (who hadn't moved) repay all of their debts.
- (b) (2 points) Calculate the marbles Donald gets to keep from this transaction after Week 3.

Alternatively assume at the beginning of Week 3, the students who moved owed him 301 marbles and all remaining students paid off their debts.

- (c) (1 point) Calculate the marbles that are distributed to or from all relevant parties after Week 3.

****END OF EXAMINATION****
Afternoon Session

USE THIS PAGE FOR YOUR SCRATCH WORK

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