SOCIETY OF ACTUARIES
Quantitative Finance and Investment Advanced Exam

Exam QFIADV

MORNING SESSION

Date: Thursday, April 30, 2015
Time: 8:30 a.m. – 11:45 a.m.

INSTRUCTIONS TO CANDIDATES

General Instructions

1. This examination has a total of 100 points. It consists of a morning session (worth 60 points) and an afternoon session (worth 40 points).

   a) The morning session consists of 9 questions numbered 1 through 9.

   b) The afternoon session consists of 6 questions numbered 10 through 15.

   The points for each question are indicated at the beginning of the question.

2. Failure to stop writing after time is called will result in the disqualification of your answers or further disciplinary action.

3. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions on the exam booklet.

Written-Answer Instructions

1. Write your candidate number at the top of each sheet. Your name must not appear.

2. Write on only one side of a sheet. Start each question on a fresh sheet. On each sheet, write the number of the question that you are answering. Do not answer more than one question on a single sheet.

3. The answer should be confined to the question as set.

4. When you are asked to calculate, show all your work including any applicable formulas. When you are asked to recommend, provide proper justification supporting your recommendation.

5. When you finish, insert all your written-answer sheets into the Essay Answer Envelope. Be sure to hand in all your answer sheets because they cannot be accepted later. Seal the envelope and write your candidate number in the space provided on the outside of the envelope. Check the appropriate box to indicate morning or afternoon session for Exam QFIADV.

6. Be sure your written-answer envelope is signed because if it is not, your examination will not be graded.

Tournez le cahier d'examen pour la version française.

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Exam QFIADV

****BEGINNING OF EXAMINATION****
Morning Session

1. (*7 points*) You are the Chief Investment Officer of a large life insurance company which has a substantial amount of residential mortgage loans in its investment portfolio. In view of the recent global credit crisis you have been asked to re-evaluate the default model the company uses for predicting defaults on residential mortgage loans.

(a) (*1 point*) Define "mortgage default" for the purpose of the model.

(b) (*2 points*) Describe four determinants of mortgage defaults that would be essential parameters of the predictive default model and explain difficulties encountered in determining each.

The CEO of your company has commented, *"The recent turmoil in the subprime market has led to changes in mortgage underwriting standards. The profile of subprime borrowers in the future, and the mix of mortgage products, will be quite different from that of the past. This presents a challenge to predictive models."*

(c) (*1 point*) Explain the challenges the CEO is suggesting.

(d) (*1 point*) Suggest possible ways to overcome these challenges.

(e) (*1 point*) Describe two other models that you would combine with the default model for valuation of mortgage credit risk.

(f) (*1 point*) Explain the relationship of prepayment and default rates implied by the cumulative loss percentages in the table below:

Cumulative Losses (%) by Voluntary Prepayment Rate and Default Rates						
Default Rate (%)	***Voluntary Prepayment Rate (%) - assuming a 50% loss severity***					
	10	***20***	***30***	***40***	***50***	***60***
5	15	9	6	5	3	3
10	24	16	11	9	5	5
15	29	21	16	12	8	8
20	33	25	19	15	10	10
25	36	28	22	18	12	12
30	38	30	25	21	14	14

2. (*7 points*) Your team is trying to create a smooth function for the density of the price of NJ index in 1 month. To do so, you use European call option prices observed on the market. You are also given:

- As of today, the index value is 100.
- The risk-free rate $r = 3\%$.

Your co-worker has started working towards calculating the value of the function at different strike prices, and he was told there is a mistake in his preliminary work. He shows you this excerpt of the table.

K	$C(K,T)$	$\partial^2 C(K,T)/\partial K^2$
95	7.02	-0.0425
100	3.67	0.0481
105	1.84	0.0375

For all strikes, $T = 1$ month.

(a) (*1 point*) Identify where the mistake is and explain why the number is wrong.

2. Continued

After doing some more work, your co-worker asks you to help him complete the task. He gives you the following table (assume these numbers are correct):

K	$C(K,T)$	$\partial^2 C(K,T)/\partial K^2$
65	35.5299	0.0003
70	30.5753	0.0011
75	25.6366	0.0020

For all strikes, $T = 1$ month. In addition, you assume that for $K \le 60$, $\frac{\partial^2 C(K,T)}{\partial K^2} = 0$.

You denote the risk-neutral density of the strike price in 1 month by $\phi_T(K)$.

Assume that $\phi_T(K)$ is constructed using linear interpolation.

(b) Calculate the following:

(i) (*1 point*) $\phi_T(65)$, $\phi_T(70)$ and $\phi_T(75)$.

(ii) (*1 point*) The risk-neutral probability that the index price in one month is between 70 and 75.

(iii) (*1 point*) The risk-neutral probability that the index price in one month is below 65.

When your team is done fitting a smooth function for the density of the NJ index in one month, your manager asks you to use it to price an up-and-out put option that expires one month from now and that pays $\max(90 - S_T, 0)$ only if the index price remains below 110 until maturity.

(c) (*1 point*) Explain why the density you retrieved is not sufficient to price the up-and-out put option explained above.

Your manager is now considering using a fully stochastic volatility model or a jump diffusion model to price the one-month up-and-out put.

(d) (*1 point*) Compare and contrast the two models.

(e) (*1 point*) Recommend one of the two models and justify your answer.

3. (*8 points*) You are studying the performance of five stocks (stock S_1, S_2, S_3, S_4, S_5) using Principal Component Analysis. Each of the five stocks is from either the Financial Services industry or from the IT industry. You have gathered historical monthly log returns for these stocks. The log return from each stock is denoted as a random variable $x_i (i = 1...5)$.

(a) (*2 points*) Outline the steps required to identify the *first* and *second* principal components for the random variables x_i.

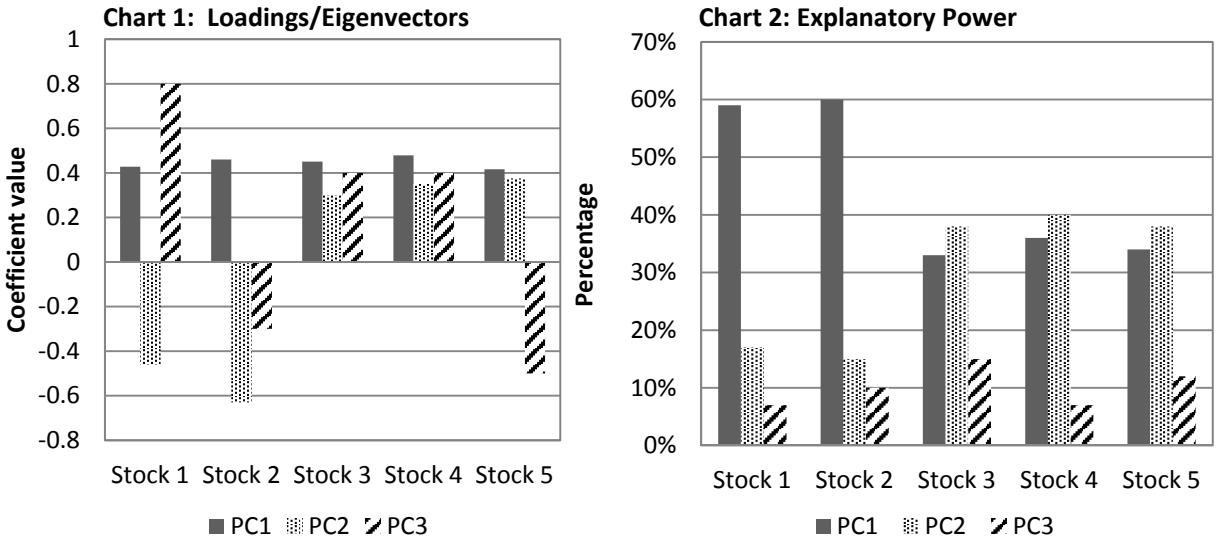
You are given the following principal components that were calculated from the historical log returns on the five stocks. The principal components have not been ranked in any particular order.

Principal Component	Eigenvalue (λ_i)
A	0.7
B	1.0
C	2.6
D	0.2
E	1.8

(b) (*2 points*) Determine which of the above principal components should be used to explain at least 80% of the variance in the original data.

3. Continued

You have selected the three principal components that have the highest explanatory power. The following charts were produced:



(c) (*1 point*) Contrast the nature of the information that can be obtained from a loadings/Eigenvectors chart (such as Chart 1) compared to an explanatory power chart (such as Chart 2).

(d) (*2 points*) Interpret the results for PC1 and PC2 in Chart 1.

You have assumed that overall market returns generally contribute more to the variance of financial services firms than to that of IT companies.

(e) (*1 point*) Determine which stocks in the above charts are more likely to be from the financial services industry and which are more likely to be from IT stocks, given your assumption.

4. (*5 points*) You are a credit portfolio manager for XYZ Investment Services. You are evaluating the following portfolio of corporate bonds:

Issuer	Spread	Spread Duration	Portfolio Weight
AAA	50	11	40%
DOVE	80	8	20%
RISK	250	2	20%
MAX	500	1	20%
Portfolio	186	6.6	100%

The investment policy for this portfolio has a contribution to DTS per issuer limit of 150 and does not allow short selling of assets nor the holding of cash positions.

(a) (*1 point*) Describe how DTS-based issuer caps can result in a "credit torpedo" and suggest how to minimize this risk.

(b) (*1 point*) Determine which bond in the portfolio is the riskiest with respect to excess return volatility. Show your work.

(c) (*1 point*) Explain why the current portfolio does not comply with the investment policy.

(d) (*2 points*) Determine portfolio weights for each bond above that will bring the portfolio into compliance with the investment policy.

5. (*4 points*) You are given the following Prospect theory model:

$$P = w(p)v^{+}(x) + w(1-p)v^{-}(x)$$

$v^{+}(x) = x^{0.8}$: value of the outcome for $x \geq 0$

$v^{-}(x) = 0.5x^{0.4}$: value of the outcome for $x < 0$

$$w(p) = p^2 / \left(p^2 + (1-p)^2\right)^{0.5}$$

(a) (*2 points*) Describe the major features of the Prospect theory model.

(b) (*2 points*) Identify the mistakes in the above Prospect theory model and provide a numeric example of each mistake.

6. (*8 points*) You are a qualified CERA actuary for Iron & Wine Life Co. and have been asked by the Chief Risk Officer (CRO) to provide a risk assessment of the company from a liquidity stress testing standpoint.

(a) (*2 points*) Describe the problems associated with each of the three quantitative methods for scenario-based liquidity stress testing as described in "*Liquidity Risk: Measurement and Management – A Practitioner's Guide to Global Best Practices."*

Iron & Wine Life Co. has interest-sensitive liabilities and its asset portfolio consists of A-rated corporate bonds and AA-rated asset-backed securities (ABS). You have been provided the following information from the Head of the Stress Testing division of the company on two deterministic liquidity stress scenarios:

Scenario 1: A credit rating downgrade on the ABS held on the company's balance sheet 12 months from now that increases the net asset haircuts to 15%. Liability time factor remains at 50%.

Scenario 2: An interest rate increase of 5% occurring 12 months from now that also increases the liability time factor to 80%.

(Values in millions)	One Year
Base Case – asset market value	1,300
Base Case – liability value	1,100
Base Case – haircut	5%
Base Case – liability time factor	50%
Duration of Assets	6

(b) (*3 points*) Assess the liquidity risk for Iron & Wine Life Co. by applying liquidity scenario analysis for each of the following:

(i) Base Case

(ii) Scenario 1

(iii) Scenario 2

(c) (*2 points*) Recommend next steps to the CRO based upon this liquidity scenario analysis.

(d) (*1 point*) Determine the minimum amount of additional cash that would be needed to support the liabilities based upon the results of the scenario analysis.

7. (*5 points*) As a pricing actuary at Guaranteed Life, you are charged with selecting an interest rate model to price interest rate derivatives embedded in the company's product portfolio.

(a) (*1 point*) Identify six practical considerations in choosing an interest rate model.

You are given the Hull-White Extended Vasicek Model as shown below:

$$dr(t) = \left(\theta(t) - ar(t)\right)dt + \sigma(t)\,dW(t)$$

(b) (*1 point*) State the advantages and disadvantages of this model.

You are given the following values of Hull-White Extended Vasicek Model as of time 0 $(t = 0)$:

- $a = 0.1$
- $\sigma(t) = 0.02$ for all t
- The market price of zero-coupon bond that matures at time $t = T$ with \$1 par is:
  $P(0,T) = e^{-0.01\left(T+T^2\right)}$

(c) (*3 points*) Calculate, as of time 0:

(i) The expected value of the short rate at time $t = 1$

(ii) The standard deviation of the short rate at time $t = 1$

(iii) The risk neutral probability that short rate exceeds 5% at time $t = 1$.

8. (*6 points*) Your company would like to consider new investments into direct ownership of commercial real estate investments instead of purchasing public REITs for specific individualized mass affluent clients with a substantial part of their wealth in a closely held company.

Your target is to invest the funds over a long term period but clients are reluctant since the last financial crisis.

To predict future real estate capitalization rates you use the following model and variables on a quarterly time step:

$$\log\left(C_{j,t}\right) = a_0 + a_1\log\left(C_{j,t-1}\right) + a_2\log\left(C_{j,t-4}\right) + a_3\log\left(\text{RRI}_{j,t}\right) + a_4\text{RTB}_t + a_5\text{SPREAD}_t + a_6\text{DEBTFLOW}_t + a_7\text{Q2}_t + a_8\text{Q3}_t + a_9\text{Q4}_t + a_{10}\text{D}_j$$

Variable	Description
$\log\left(\text{RRI}\right)_{j,t}$	Real rent index calculated as ratio of real rent data for a given Metropolitan Statistical Area (MSA) in a given quarter to the historical average of real rent for this MSA.
RTB_t	Real T-bond yield calculated as nominal yield minus inflation rate.
SPREAD_t	Economy wide risk premium over the risk-free rate calculated as the difference between Moody's AAA Corporate Bond Index and 10-year T-bond.
DEBTFLOW_t	Debt flow calculated as a ratio of Total Net Borrowing and Lending from the Federal Reserve's Flow of Funds Database to that quarter's nominal GDP level.

(a) (*2 points*) Describe the anticipated impact for each variable (in the above table) on the capitalization rate and asset value.

(b) (*1 point*) Discuss the critical issue on the purchase price if your Opportunity Cost of Capital (OCC) on your real estate portfolio is much higher than a REIT competitor assuming neither use debt.

(c) (*1 point*) Identify the specific issues needed to satisfy a good due diligence for those clients.

(d) (*2 points*) Explain why the NCREIF benchmark used in the United States evolved into two indices.

9. (*10 points*) You are assessing a number of different fixed income portfolios based on their historical performance data for potential use by your company.

You are given the following information pertaining to Portfolio A and Portfolio B:

- Both are managed against the same benchmark.
- Both are managed in a top-down fashion.
- Interaction return has been folded into the asset allocation return.

Table 1:

Portfolio A

Rating	Weight (%)	Return (%)	Asset Allocation (%)	Security Selection (%)	Benchmark Weight (%)	Benchmark Return (%)
A	30	6	-0.2	1.20	40	2
BB	w1	8	1.05	0.45	w2	7
B	w3	10	-0.30	1	w4	6
Total	100				100	

Table 2:

Portfolio B

Rating	Weight (%)	Return (%)	Asset Allocation (%)	Security Selection (%)	Benchmark Weight (%)	Benchmark Return (%)
A	30	6	a1	s1	40	2
BB	25	8	a2	s2	w2	7
B	45	10	a3	s3	w4	6
Total	100				100	

Table 3:

1-year bond rating transition matrix "M"

Initial Rating	Rating at Year End (%)					
	AA	A	BB	B	C	Default
AA	95	3	2	0	0	0
A	3	94	2	1	0	0
BB	n^2	$2n$	94	2	1	0
B	0	2	5	90	3	0
C	0	0	3	6	90	1
Default	0	0	0	0	0	100

Question 9 continued on next page

9. Continued

(a) (*1.5 points*) Calculate each of the missing values w1, w2, w3, w4 in Table 1.

(b) (*1.5 points*) Calculate the asset allocation return and the security selection return pertaining to each bond rating for Portfolio B.

(c) (*1 point*) Determine the appropriate value of *n* such that the bond rating transition matrix "M" is a stochastic matrix.

(d) (*1 point*) Calculate the cumulative 2nd year default probability for bond currently rated B.

You are given the following investment objectives for your company:

(i) Both asset allocation and security selection of the portfolio are more than 0.5%.

(ii) The cumulative 2nd year default probability for B rated bond is less than 0.05%.

(e) (*2 points*) Assess which of the two portfolios (Portfolio A and Portfolio B) is most appropriate for your company.

You are given the following information on the performance data of a third portfolio, Portfolio C:

- The "Factor Return model" for return splitting has been used.
- The total returns from both the portfolio and from the benchmark have been split into the linear contributions from two factors: (i) shift, and (ii) twist

	Portfolio Weight (%)	Benchmark Weight (%)	Portfolio Shift Return (%)	Benchmark Shift Return (%)	Portfolio Twist Return (%)	Benchmark Twist Return (%)
Governments	20	37	1.5	1	0.025	-0.025
MBS	20	34	0.75	0.75	-0.05	-0.075
ABS	9	2	0.5	1	-0.1	-0.05
CMBS	9	3	4/3	1	0	-0.025
Corporates	42	24	1	1.5	-0.025	0.075

9. Continued

Your company's chief economist has made the following qualitative observations on the performance of Portfolio C:

(i) The portfolio had a neutral overall duration bet.

(ii) The portfolio had a slightly higher exposure to a fall in the longer end of the yield curve.

(f) (*3 points*) Critique your economist's observations.

****END OF EXAMINATION****
Morning Session

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