MLC Spring 2015 Multiple Choice Solutions

1. $E(N) = 1000\left({}_{30}p_{35} + {}_{30}p_{45}\right) = 1000\left(\frac{7,533,964}{9,420,657} + \frac{5,396,081}{9,164,051}\right) = 1388.56$

$Var(N) = 1000\,{}_{30}p_{35}\left(1 - {}_{30}p_{35}\right) + 1000\,{}_{30}p_{45}\left(1 - {}_{40}p_{45}\right) = 402.27$

Since $1388.56 + 1.645\sqrt{402.27} = 1421.55$

$N = 1422$

2. The desired probability is:

$$
\begin{aligned}
&= \int_0^{14} \exp\left\{-\int_0^u \left(\mu_{01} + \mu_{02}\right) ds\right\} \cdot \mu_{01} \cdot \exp\left\{-\int_0^1 \mu_{12} ds\right\} du \\
&= \int_0^{14} e^{-0.05u} \cdot (0.02) \cdot e^{-0.11} du \\
&= (0.02) \cdot e^{-0.11} \int_0^{14} e^{-0.05u} du \\
&= \frac{0.02}{0.05} \cdot e^{-0.11}\left(1 - e^{-0.7}\right) \\
&= 0.18
\end{aligned}
$$

3. $${}_{10}p^{00}_{30:30} = \exp\left[-\int_0^{10}(\mu^{01}_{30+t:30+t} + \mu^{02}_{30+t:30+t})dt\right]$$

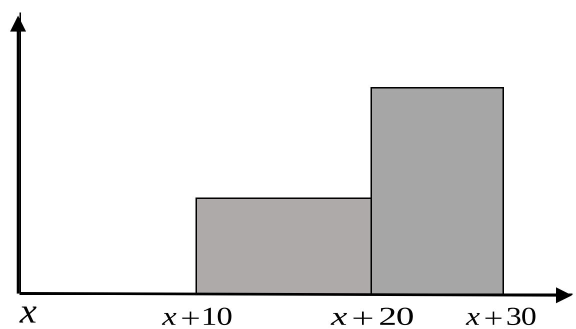
$$= \exp\left[-\int_0^{10}(0.006 + 0.014 + 0.0007 \times 1.075^{30+t})dt\right]$$

$$= \exp[-0.2]\exp\left[-0.0007\int_0^{10} 1.075^{30+t}\,dt\right]$$

$$= \exp[-0.2]\exp\left[-0.0007\frac{1.075^{30+t}}{\ln 1.075}\Bigg|_0^{10}\right] = \exp[-0.2]\exp\left[-0.0007\left(\frac{1.075^{40} - 1.075^{30}}{\ln 1.075}\right)\right]$$

$$= 0.748$$

4. Drawing the benefit payment pattern:



$$E[Z] = {}_{10}E_x \bullet \bar{A}_{x+10} + {}_{20}E_x \bullet \bar{A}_{x+20} - 2\ {}_{30}E_x \bullet \bar{A}_{x+30}$$

5. $Var\left(Z_2\right) = \left(1000\right)^2 \left[{}^2A_{x:\overline{n}|} - \left(A_{x:\overline{n}|}\right)^2 \right] = 15,000$

$$= \left(1000\right)^2 \left({}^2A^{1}_{x:\overline{n}|} + {}^2A_{x:\overline{n}|}^{\;\;1} \right) - \left(1000\right)^2 \left[A^{1}_{x:\overline{n}|} + A_{x\overline{n}|}^{\;\;1} \right]^2$$

$$= \left(1000\right)^2\, {}^2A^{1}_{x:\overline{n}|} + \left(1000\right)^2\, {}^2A_{x:\overline{n}|}^{\;\;1} - \left(1000\right)^2 \left(A^{1}_{x:\overline{n}|} \right)^2 - \left(1000\right)^2 \left(A_{x\overline{n}|}^{\;\;1} \right)^2$$
$$- 2(1000)^2 \left(A^{1}_{x:\overline{n}|} \right) \left(A_{x\overline{n}|}^{\;\;1} \right)$$

$$= (1000)^2 \left[{}^2A^{1}_{x:\overline{n}|} - \left(A^{1}_{x:\overline{n}|} \right)^2 \right] + \left(1000^2\, {}^2A_{x:\overline{n}|}^{\;\;1} \right) - \left(1000A_{x:\overline{n}|}^{\;\;1} \right)^2$$
$$- \left(1000A^{1}_{x:\overline{n}|} \right)^2 - (2)\left(1000A^{1}_{x:\overline{n}|} \right)\left(1000A_{x\,\overline{n}|}^{\;\;1} \right)$$

$$= V\left(Z_1\right) + (1000)\left(1000\, {}^2A_{x:\overline{n}|}^{\;\;1} \right) - \left(1000A_{x\,\overline{n}|}^{\;\;1} \right)^2 - \left(1000A^{1}_{x:\overline{n}|} \right)^2$$
$$-(2)\left(1000A^{1}_{x:\overline{n}|} \right)\left(1000A_{x\,\overline{n}|}^{\;\;1} \right)$$

$15,000 = Var\left(Z_1\right) + (1000)(136) - (209)^2 - 2(528)(209)$

Therefore, $Var\left(Z_1\right) = 15,000 - 136,000 + 43,681 + 220,704 = 143,385.$

6. The probabilities are:

Sick $t = 1 \Rightarrow 0.025$
Sick $t = 2 \Rightarrow (0.95)(0.025) + (0.025)(0.6) = 0.03875$
Sick $t = 3 \Rightarrow (0.95)(0.95)(0.025) + (0.95)(0.025)(0.6) + (0.025)(0.6)(0.6)$
$+(0.025)(0.3)(0.025) = 0.046$

$EPV = 20,000\left(0.025v + 0.03875v^2 + 0.046v^3 \right) = 1934$

7.

$$\ddot{a}^{(2)}_{35:\overline{30|}} \approx \ddot{a}_{35:\overline{30|}} - \frac{(m-1)}{2m}\left(1 - v^{30}\,{}_{30}p_{35}\right)$$

$$\ddot{a}_{35:\overline{30|}} = \frac{1 - A_{35:\overline{30|}}}{d} = \frac{1 - A^{1}_{35:\overline{30|}} - {}_{30}E_{35}}{d}$$

$$= \frac{1 - \left(A_{35} - {}_{30}E_{35} \bullet A_{65}\right) - {}_{30}E_{35}}{d}$$

Since ${}_{30}E_{35} = v^{30}\,{}_{30}p_{35} = 0.2722,$ then

$$\ddot{a}_{35:\overline{30|}} = \frac{1 - \left(A_{35} - v^{30}\,{}_{30}p_{35} \bullet A_{65}\right) - v^{30}\,{}_{30}p_{35}}{d}$$

$$= \frac{1 - (0.188 - (0.2722)(0.498)) - 0.2722}{(0.04/1.04)}$$

$$= 17.5592$$

$$\ddot{a}^{(2)}_{35:\overline{30|}} \approx 17.5592 - \frac{1}{4}(1 - 0.2722) = 17.38$$

$$1000\,\ddot{a}^{(2)}_{35:\overline{30|}} \approx 1000 \times 17.38 = 17{,}380$$

8. In general, the loss at issue random variable can be expressed as:

$$L = \bar{Z}_x - P \bullet \bar{Y}_x = \bar{Z}_x - P \bullet \left(\frac{1 - \bar{Z}x}{\delta}\right) = \bar{Z}_x \bullet \left(1 + \frac{P}{\delta}\right) - \frac{P}{\delta}$$

Using actuarial equivalence to determine the premium rate:

$$P = \frac{\bar{A}_x}{\bar{a}_x} = \frac{0.3}{(1 - 0.3)/0.07} = 0.03$$

$$Var(L) = \left(1 + \frac{P}{\delta}\right)^2 \bullet Var\left(\bar{Z}_x\right) = \left(1 + \frac{0.03}{0.07}\right)^2 \bullet Var\left(\bar{Z}_x\right) = 0.18$$

$$Var\left(\bar{Z}_x\right) = \frac{0.18}{\left(1 + \frac{0.03}{0.07}\right)^2} = 0.088$$

$$Var(L^*) = \left(1 + \frac{P^*}{\delta}\right)^2 \bullet Var\left(\bar{Z}_x\right) = \left(1 + \frac{0.06}{0.07}\right)^2 (0.088) = 0.304$$

9. Need $\text{EPV(Ben} + \text{Exp)} - \text{EPV(Prem)} = -800$

$$
\begin{aligned}
\text{EPV(Prem)} &= G\ddot{a}_{55:\overline{10|}} = G\left(\ddot{a}_{55} - {}_{10}E_{55}\,\ddot{a}_{65}\right) \\
&= G(12.2758 - 0.48686(9.8969)) \\
&= 7.4574G
\end{aligned}
$$

$$
\begin{aligned}
\text{EPV(Ben} + \text{Exp)} &= 12{,}000\ {}_{10|}\ddot{a}^{(12)}_{55} + 300\ddot{a}_{55} \\
&= 12{,}000\ {}_{10}E_{55}\ddot{a}^{(12)}_{65} + 300\ddot{a}_{55} \\
&= 12{,}000\ {}_{10}E_{55}\left(\ddot{a}_{65} - \frac{m-1}{2m}\right) + 300\ddot{a}_{55} \\
&= 12{,}000(0.48686)\left(9.8969 - 11/24\right) + 300(12.2758) \\
&= 58{,}825.8668
\end{aligned}
$$

Therefore, $58{,}825.8668 - 7.4571G = -800$

$G = 7995 \approx 8000$

10. EPV (Premiums) = $Pa_{90} = Pvp_{90}\ddot{a}_{91} = P(1.06^{-1})(0.811227)(3.4611)$

$\text{EPV(Benefits)} = 1000A_{90} = 1000(0.79346) = 793.46$

Therefore,

$$
P = \frac{793.46}{\left((1.06)^{-1}(0.811227)(3.4611)\right)} = 299.25
$$

11. EPV(Premiums) = EPV(Benefits)

$$\text{EPV(Premiums)} = 3P\,\bar{a}_x - 2P\,{}_{20}E_x\,\bar{a}_{x+20}$$
$$= 3P\left(1\big/\mu+\delta\right) - 2P\left(e^{-20(\mu+\delta)}\right)\left(1\big/\mu+\delta\right)$$
$$= 3P\left(1\big/0.09\right) - 2Pe^{-1.8} - 1\big/0.09$$
$$= 29.66P$$

$$\text{EPV(Benefits)} = 1{,}000{,}000\bar{A}_x - 500{,}000\,{}_{20}E_x\,\bar{A}_{x+20}$$
$$= 1{,}000{,}000\left(\mu\big/\mu+\delta\right) - 500{,}000e^{-20(\mu+\delta)}\quad \mu\big/\mu+\delta$$
$$= 1{,}000{,}000\left(0.03\big/0.07\right) - 500{,}000e^{-1.8}\quad 0.03\big/0.09$$
$$= 305{,}783.5$$

$$29.66P = 305{,}783.5$$
$$P = \frac{305{,}783.5}{29.66}$$
$$P = 10{,}309.62 \approx 10{,}300$$

12.

$$G\ddot{a}_{40:\overline{5}|} = 1000A_{40} + 0.15G + 0.05G\ddot{a}_{40:\overline{5}|} + 5 + 5\ddot{a}_{40:\overline{5}|}$$

$$\ddot{a}_{40:\overline{5}|} = \ddot{a}_{40} - {}_5E_{40}\bullet\ddot{a}_{45} = 14.8166 - \frac{735.29}{1000}(14.1121) = 4.44$$

$$G = \frac{161.32 + 5 + 5(4.44)}{-0.15 + 0.95(4.44)} = 46.34$$

13.

$$\text{APV(Premiums)} = \text{APV(Benefits)}$$

$$\text{APV(Benefits)} = 60{,}000\ddot{a}_{45|45} + 3P\ddot{a}_{45|45}$$

$$\text{where } \ddot{a}_{45|45} = \ddot{a}_{45} - \ddot{a}_{45:45}$$

$$= 14.1121 - 12.6994$$

$$= 1.4127$$

$$\text{APV(Premiums)} = P\ddot{a}_{45:45}$$

$$P(12.6994) = 60{,}000(1.4127) + P(4.2381)$$

$$P = 10{,}018$$

14.

$${}_1V_x = A_{x+1} - P_x\,\ddot{a}_{x+1} = 1 - d\ddot{a}_{x+1} - P_x\,\ddot{a}_{x+1}$$

$$= 1 - \underbrace{\left(P_x + d\right)}\ddot{a}_{x+1} = 1 - \ddot{a}_{x+1}\big/\ddot{a}_x$$

$$\Rightarrow \ddot{a}_x\left(1 - {}_1V_x\right) = \ddot{a}_{x+1}$$

Since $\ddot{a}_x = 1 + vp_x\ddot{a}_{x+1}$ substituting we get

$$\ddot{a}_x\left(1 - {}_1V_x\right) = \frac{\ddot{a}_x - 1}{vp_x} \Rightarrow \ddot{a}_x\left(1 - {}_1V_x\right)vp_x = \ddot{a}_x - 1$$

Solving for $\ddot{a}_x$, we get $\ddot{a}_x = \dfrac{1}{1 - \left(1 - {}_1V_x\right)vp_x} = \dfrac{1}{1 - (1 - 0.012)\left(\dfrac{1}{1.04}\right)\left(1 - 0.009\right)}$

$$= 17.07942$$

15. EPV of Premium $= 250\left(1+vp_{50}\right)$

EPV of Profit $= -165+100v+125v^2 p_{50}$

Profit Margin $= \dfrac{-165+100v+125v^2 p_{50}}{250\left(1+vp_{50}\right)} = 0.06$

Solving for p_{50}, we get:

$$p_{50} = \frac{-165+100v-0.06(250)}{0.06(250)v-125v^2} = \frac{-89.09091}{-89.66942}$$
$$= 0.9935484$$
$$\left(\text{where } v = 1.10^{-1}\right)$$

16. ADB = additional death benefit

In each case we have

$$AV_6 = (20,000+1,500-145)(1.06) - \text{COI}(1.06)$$
$$= 22,636.3 - 1.25q_{55}(\text{ADB})\left(\frac{1.06}{1.04}\right)$$
$$= 22,636.3 - 0.011415(\text{ADB})$$

With the corridor factor, $\text{ADB} = 0.8AV_6$, so that

$$AV_6 = \left(\frac{22,636.3}{1+0.8\times 0.011415}\right) = 22,431.40$$

17. Let B be the amount of death benefit.

$$\text{EPV(Premiums)} = 500\ddot{a}_{61} = 500 * 10.9041 = 5{,}452.05$$

$$\text{EPV(Benefits)} = \text{B } A_{61} = 0.38279 \text{ B}$$

$$\text{EPV(Expenses)} = (0.12 \times 500) + \left(0.03 \times 500\ddot{a}_{61}\right) = 0.12 \times 500 + 0.03 \times 5{,}452.05 = 223.56$$

$$\text{EPV(Premiums)} = \text{EPV(Benefits)} + \text{EPV(Expenses)}$$

$$5452.05 = 0.38279\text{B} + 223.5615$$

$$5228.49 = 0.38279\text{B}$$

$$\text{B} = 13{,}659$$

18. Let G be the annual gross premium.

Using the equivalence principle, $0.90G\ddot{a}_{40} - 0.40G = 100{,}000A_{40} + 300$

So $G = \dfrac{100{,}000(0.16132) + 300}{0.90(14.8166) - 0.40} = 1{,}270.36$

The gross premium reserve after the first year and immediately after the second premium and associated expenses are paid is

$$100{,}000A_{41} - 0.90G\left(\ddot{a}_{41} - 1\right)$$

$$= 16{,}869 - 0.90(1270.36)(13.6864)$$

$$= 1{,}221$$

19. Final average salary before retirement $= 40,000\left(\dfrac{1.035^{32}+1.035^{33}+1.035^{34}}{3}\right)$

$= 124,526.80$

Retirement Pension $= 35\times 0.016\times 124,526.80$

$= 69,735.01$

Salary in final year $= 40,000\times 1.035^{34}$

$= 128,834.41$

Replacement Ratio $= \dfrac{69,735.01}{128,834.41} = 54.13\%$

20. Fred gets: $120,000\times 0.8\times 0.02\times 35 = 67,200$

Glenn gets: $\left(120,000+5(4800)\right)\times 0.02\times 40 = 115,200$

Fred gets his for 5 years more, so he is 336,000 ahead of Glenn.

Once Glenn starts drawing he gets 48,000 per year. It takes him $336,000/48,000 = 7$ years to catch up to Fred.