

# MLC Spring 2015 Written Answer Questions

## Model Solutions

## MLC Spring 2015 Question 1 Model Solution

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Learning Objectives: 1(d), 2(a), 3(a), 4(a)

Textbook References: 8.6, 8.7

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(a)

$$\begin{aligned}\bar{a}_{x:\overline{10}|}^{0j} &= \int_0^{10} {}_t p_x^{0j} e^{-\delta t} dt \\ \Rightarrow \bar{a}_{x:\overline{10}|}^{00} + \bar{a}_{x:\overline{10}|}^{01} + \bar{a}_{x:\overline{10}|}^{02} &= \int_0^{10} ({}_t p_x^{00} + {}_t p_x^{01} + {}_t p_x^{02}) e^{-\delta t} dt \\ \text{and } {}_t p_x^{00} + {}_t p_x^{01} + {}_t p_x^{02} &= 1 \\ \Rightarrow \bar{a}_{x:\overline{10}|}^{00} + \bar{a}_{x:\overline{10}|}^{01} + \bar{a}_{x:\overline{10}|}^{02} &= \int_0^{10} e^{-\delta t} dt = \bar{a}_{\overline{10}|}\end{aligned}$$

*Comments: Many candidates did well on this part. Some candidates gave a verbal explanation, rather than mathematical proof. Generally, this earned partial credit, but full credit was available if the verbal proof was sufficiently detailed.*

*In the mathematical proof, the candidate was required to indicate clearly that  ${}_t p_x^{00} + {}_t p_x^{01} + {}_t p_x^{02} = 1$  for full credit.*

*In verbal explanations, candidates were required to state that  $\bar{a}_x^{0j}$  is the actuarial value of a benefit of 1 per year paid ‘while’ the person is in state  $j$ , not ‘when’ or ‘if’ the person transitions to state  $j$ .*

*Some candidates wrote out the sum and the result that had to be proved, but gave no details on the intermediate steps. These answers received no credit.*

(b)

$$\text{EPV Premiums: } P\bar{a}_{x:\overline{10}|}^{00} = 4.49P$$

$$\text{EPV Disability Annuity: } 1000\bar{a}_{x:\overline{10}|}^{01} = 1000 \left( \bar{a}_{\overline{10}|} - (\bar{a}_{x:\overline{10}|}^{00} + \bar{a}_{x:\overline{10}|}^{02}) \right)$$

$$\text{where } \bar{a}_{\overline{10}|} = \frac{1 - e^{-10\delta}}{\delta} = 6.321$$

$$\Rightarrow \text{EPV Dis. annuity} = 471.21$$

$$\text{EPV Death Benefit: } 1000 \bar{A}_{x:\overline{10}|}^{02} = 3871.0$$

$$\text{So the Premium is } P = \frac{3871 + 471.21}{4.49} = 967.10$$

*Comments: This part was done well by many candidates. Some candidates did not realize that the result in (a) could be used to find the missing annuity value in (b). Other candidates did notice this, but could not calculate the required annuity-certain,  $\bar{a}_{\overline{10}|}$ .*

(c) The Thiele equation at time  $t$  is

$$\frac{d}{dt} {}_tV^{(0)} = \delta {}_tV + P - \mu_{x+t}^{01} ({}_tV^{(1)} - {}_tV^{(0)}) - \mu_{x+t}^{02} (10,000 - {}_tV^{(0)})$$

so at  $t = 3$  we have

$$\begin{aligned} \frac{d}{dt} {}_tV^{(0)} &= 0.1 \times 1304.54 + 967.1 - 0.04(7530.09 - 1304.54) - 0.06(10000 - 1304.54) \\ &= 130.45 + 967.1 - 249.02 - 521.73 \\ &= 326.80 \end{aligned}$$

*Comments: This part proved more challenging to many candidates. The responses indicated that many candidates are memorizing Thiele's formula rather than understanding the intuition behind it. Common errors included the following:*

*Putting the State 1 annuity rate (1000) in the term which values the instantaneous cost of transition to state 1 (the one with  $\mu_{x+t}^{01}$ ). Thiele's equation allows for instantaneous payments at transfer, such as a death benefit; The annuity cost is captured in  ${}_tV^{(1)}$ .*

*Using wrong signs for the release of  ${}_tV^{(0)}$  terms, or for the  ${}_tV^{(1)}$  term.*

*Subtracting the premium rather than adding it.*

*Candidates making one or two of these common errors would receive partial credit.*

*A few candidates wrote down a generic formula for Thiele's equation, but did not adapt it to this problem, nor indicate any appropriate numerical values. These answers received no credit.*

(d) Let  $P^*$  denote the new premium. The EPV of return of premiums is

$$10P^* e^{-10\delta} {}_{10}p_x^{\overline{00}}$$

$$\begin{aligned} \text{where } {}_{10}p_x^{\overline{00}} &= e^{-\int_0^{10} \mu_{x+t}^{01} + \mu_{x+t}^{02} dt} \\ &= e^{-0.4-1} = 0.24660 \end{aligned}$$

$$\text{So, the EPV is } 10 \times P^* \times 0.24660 \times e^{-1} = 0.90718P^*$$

$$\text{Hence } P^* = \frac{3871 + 471.21}{4.49 - 0.90718} = 1211.95$$

$$\Rightarrow (P^* - P) = 244.87$$

*Comments: This part was done reasonably well by many candidates who attempted it. Most candidates received partial credit, but only a few candidates received full credit.*

*A common error was to omit the discount factor,  $e^{-10\delta}$ . The reason may have been that the question said the premiums were returned without interest. That wording means that the benefit is  $10P$ , i.e. a simple sum, not  $P\bar{s}_{\overline{10}|j}$  which would be the payment made if premiums were returned with interest credited at an interest rate of  $j$  per year.*

*The wording does not mean that the insurer does not earn any interest between inception and time 10, which is what the omission of  $e^{-10\delta}$  implies.*

*Another, related error was to replace  $e^{-10\delta}$  in the return of premium benefit with  $\bar{a}_{\overline{10}|}$ . This would be correct if premiums are returned at time 10, with interest at the same rate as the valuation (i.e.  $j = i = e^\delta - 1$ ). It is incorrect in this case as the returned premiums were not credited with interest.*

*Candidates who carried forward an error, for example in the annuity values, were not penalized again here.*

## MLC Spring 2015 Question 2 Model Solution

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Learning Outcomes: 1(a),1(b), 1(d), 1(e)

Textbook References: 8.9

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- (a) Let  $l_{x+t-}^{(\tau)}$  denote the value immediately before exits at exact age  $x + t$ , and  $l_{x++t}^{(\tau)}$  denote the value immediately after.

At  $t = 0.5$ , just before the decrement 2 exits, we have

$$q_{60}^{(2)} = \frac{d^{(2)}}{l_{60.5-}^{(\tau)}}$$

$$l_{60.5-}^{(\tau)} = 1000 \exp\left(-\int_0^{0.5} 1.2t dt\right) = 1000 \times 0.86071 = 860.71$$

$$\text{So } q_{60}^{(2)} = \frac{60}{860.71} = 0.0697$$

*Comment: Many candidates gave 60/1000 as the answer, i.e. did not adjust the exposure to allow for decrement (1) departures before decrement (2) applied.*

- (b)

$$l_{60.5+}^{(\tau)} = 800.71 \Rightarrow l_{61-}^{(\tau)} = 800.71 e^{-\int_{0.5}^1 1.2t dt} = 800.71(0.63763) = 510.6$$

$$l_{61+}^{(\tau)} = 510.6 - 45 = 465.6$$

$$\begin{aligned} \Rightarrow d_{60}^{(1)} &= l_{60}^{(\tau)} - d_{60}^{(2)} - d_{60}^{(3)} - l_{61+}^{(\tau)} \\ &= 1000 - 60 - 45 - 466 = 429 \end{aligned}$$

*Comments: There are different ways of doing this part; full credit was awarded for any correct method.*

*Quite a few candidates gave the answer as 451, which is obtained by ignoring decrement (2); i.e.*

$$p_{60}^{(1)} = e^{-\int_0^1 1.2t dt} = 0.5488$$

$$1000(1 - 0.5488) = 451.2$$

- (c) (i) If decrement 2 occurs at the start of the year, there are fewer lives exposed to force of decrement 1, so  $q_{60}^{(1)}$  would be smaller.

- (ii) If decrement 2 occurs at the start of the year, there are more lives exposed to force of decrement 2, so  $q_{60}^{(2)}$  would be bigger.
- (iii) Because all the decrement 3 exits happen at the end of the year, we have

$$q_{60}^{(3)} = \frac{d_{60}^{(3)}}{l_{60}^{(\tau)}} \quad q_{60}^{\prime(3)} = \frac{d_{60}^{(3)}}{l_{61-}^{(\tau)}}$$

where  $l_{61-}^{(\tau)}$  is the expected number of in-force immediately before the decrement 3 exits at the year end. Also

$$l_{61-}^{(\tau)} = l_{60}^{(\tau)} p_{60}^{(1)} p_{60}^{\prime(2)}$$

and since the independent rates are unchanged,  $l_{61-}^{(\tau)}$  is unchanged, which means that  $d_{60}^{(3)}$  is unchanged, which means that  $q_{60}^{(3)}$  is unchanged.

*Comment: Quite a few candidates simply gave an answer (i.e. decrease, increase etc) without justification. Partial credit was given if all the answers were correct.*

## MLC Spring 2015 Question 3 Model Solution

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Learning Objectives: 1(a), 2(a), 3(a)

Textbook References: 6.5, 6.6, 6.7

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(a) The EPV of the annuity payments and expenses, given single premium  $G$ , is

$$50100 a_{65} + 3000 + 0.1G \quad \text{where } a_{65} = (\ddot{a}_{65} - 1) = 8.8969$$

$$\Rightarrow G = 1.1(50100(8.8969) + 3000 + 0.1G) = \frac{493608.2}{0.89} = 554,616$$

*Comments: This part was done reasonably well. Many candidates achieved full marks, and most others received significant partial credit.*

*Applying the 1.1 factor created some confusion. Calculating a premium and then multiplying by 1.1 was a common approach, but is incorrect because the commission valued is based on the wrong premium.*

*Some candidates used  $\ddot{a}_{65}$  in place of  $a_{65}$  for the annuity. Others omitted expenses or commission.*

(b)

Let  $K$  denote the curtate future lifetime random variable. The loss at issue random variable is  $L$  where

$$L = 50100(\ddot{a}_{\overline{K+1}|} - 1) + 3000 + 0.1G - G = 50100 \ddot{a}_{\overline{K+1}|} - 546254$$

The probability of profit is

$$\begin{aligned} \Pr[L < 0] &= \Pr[50100 \ddot{a}_{\overline{K+1}|} - 546254 < 0] = \Pr[\ddot{a}_{\overline{K+1}|} < 10.9033] \\ &= \Pr\left[\frac{1 - v^{K+1}}{d_{6\%}} < 10.9033\right] \quad d_{6\%} = 0.056604 \\ &= \Pr[1 - v^{K+1} < 0.61717] \\ &= \Pr[v^{K+1} > 0.38283] \\ &= \Pr\left[K + 1 < \frac{\log(0.38283)}{\log v}\right] = \Pr[K + 1 < 16.5] \\ &= \Pr[K \leq 15] = {}_{16}q_{65} = 1 - \frac{l_{81}}{l_{65}} = 0.5222 \end{aligned}$$

*Comments: Few candidates did this part completely correctly, but many candidates achieved partial credit.*

*Some candidates did not reverse the inequality, ending with an answer of  ${}_{16}p_{65} = 0.4778$ .*

*Some candidates found the correct probability statement for  $K$ , but used  ${}_{15}q_{65} = 0.4804$  instead of  ${}_{16}q_{65} = 0.522$ .*

*Some candidates used  $a_{\overline{K+1}|}$  instead of  $\ddot{a}_{\overline{K+1}|} - 1$ , (or, equivalently,  $a_{\overline{K}|}$ ) in the loss function.*

*Some candidates did not adjust the answer for discrete payment periods – i.e. solved for non-integer mortality period.*

*Some candidates set up the loss random variable using expectations instead of random variables. This is a more serious error and received little credit.*

*Some candidates tried to apply a normal approximation. This is not appropriate for an individual policy, and no credit was given.*

- (c) Now let  $L_j$  denote the loss from the  $j$ th policy, given a premium of  $G^P$ . Let  $K_{x_j}$  denote the curtate future lifetime for the  $j$ th life.

$$L_j = 50100(\ddot{a}_{\overline{K_{x_j}+1}|} - 1) + 3000 - 0.9G^P$$

$$\Rightarrow E[L_j] = 50100(8.8969) + 3000 - 0.9G^P = 448735 - 0.9G^P$$

$$\begin{aligned} \text{and } V[L_j] &= 50100^2 V[\ddot{a}_{\overline{K_{x_j}+1}|}] = 50100^2 V\left[\frac{1 - v^{K_{x_j}+1}}{d}\right] \\ &= \frac{50100^2}{d^2} V[v^{K_{x_j}+1}] = \frac{50100^2}{d^2} ({}^2A_{65} - A_{65}^2) \\ &= \frac{50100^2}{d^2} (0.23603 - 0.43980^2) = 182695^2 \end{aligned}$$

Let  $L$  denote the total loss on 8000 independent and identical contracts. Using the normal approximation, and

$$\begin{aligned} E[L] &= 8000E[L_j] & V[L] &= 8000V[L_j] \\ \Rightarrow \Pr[L \leq 0] &\approx \Phi\left(\frac{-8000(448735 - 0.9G^P)}{\sqrt{8000 \cdot 182695}}\right) \end{aligned}$$

Set the probability to 0.9, noting that  $\Phi(1.282) = 0.90$ ,

$$\frac{\sqrt{8000}(0.9G^P - 448735)}{182695} = 1.282 \Rightarrow G^P = 501504$$

*Comment: Few candidates completed this part correctly. The main problem was the calculation of the variance, but candidates were also challenged by the immediate annuity part.*



- (d) Under the portfolio approach, on average, each policy makes a profit. Although the probability of profit for each policy is less than 0.9, the large number of diversified policies means that, with probability 0.9, the gains on the policies where the profit is positive will be greater than the losses on the other policies.

*Comments: This part was answered well.*

## MLC Fall 2014 Question 4 Model Solution

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Learning Outcomes: 1(a), 2(a), 3(a), 4(a)

Textbook References: 7.3, 7.9, 6.5

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*General comments: Parts (a) and (b) were answered well by most candidates; those who continued did well on parts (c) and (d). Relatively few candidates could explain the reasoning behind the mechanics of modified premium reserves, as required for part (e).*

(a)

$$({}_0V + P)(1 + i) = q_x(180000) + p_x(8147.08)$$

$$\text{where } {}_0V = 0, (1 + i) = 1/0.9, q_x = 0.1, p_x = 0.9$$

$$\Rightarrow P = 22,799$$

*Comment: The recurrence method is the natural way to do this problem. Most candidates recognized this and correctly calculated the premium.*

(b) Let  $S_t$  denote the death benefit in the  $t$ th year.

$$({}_1V + P)(1 + i) = q_{x+1}(S_2) + p_{x+1}(12480.86)$$

$$\text{where } {}_1V = 8147.08, q_{x+1} = 0.2, p_{x+1} = 0.8$$

$$\Rightarrow S_2 = 122,000 \quad (123116 \text{ using } P = 23,000)$$

$$({}_2V + P)(1 + i) = q_{x+2}(S_3)$$

$$\text{where } {}_2V = 12480.86, q_{x+2} = 0.4$$

$$\Rightarrow S_3 = 98,000 \quad (98,600 \text{ using } P = 23,000)$$

*Comment: Most candidates received full credit for this part.*

(c) (i)

The first year net premium is the cost of insurance, i.e.  $P_1$ , where

$$P_1 = 180\,000q_x v = 180\,000(0.1)(0.9) = 16200$$

(ii)

Premiums in year 2 and 3 are level, equal to  $P_2$ , say, where

$$\begin{aligned} P_2 &= \frac{122000 q_{x+1} v + 98\,000 {}_1|q_{x+1} v^2}{1 + p_{x+1} v} \\ &= \frac{122\,000(0.2)(0.9) + 98\,000(0.8)(0.4)(0.9)^2}{1 + 0.8(0.9)} = \frac{47361.6}{1.72} \\ &= 27\,536 \end{aligned}$$

(d) Let  ${}_2V^F$  denote the FPT reserve.

$${}_2V^F = 98000q_{x+2}v - 27536 = 7744.00$$

### Alternative Solution

$$\begin{aligned} {}_2V^F &= \frac{({}_1V^F + P_2)(1 + i) - 122,000 q_{x+1}}{p_{x+1}} = \frac{(0 + 27536)/0.9 - 122,000(0.2)}{0.8} \\ &= 7744.00 \end{aligned}$$

*Comments: Many candidates stopped after part (b). Those who did not typically earned full credit for parts(c) and (d).*

- (e)
- Modified premium reserves use an adjusted net premium schedule. The net premium is not assumed to be level (even if the gross premium is level). Instead, premiums in the first year are assumed to be lower, so that the excess of gross premium over the net premium is implicitly assumed to be available for the acquisition expenses.
  - The method gives lower reserves, more consistent with a gross premium policy value approach, whilst maintaining the net premium reserve principle.

*Comment: For full credit, candidates were required to give a coherent explanation of how and why premiums are modified for modified net premium reserves. Relatively few candidates earned full credit for this part.*

## MLC Fall 2014 Question 5 Model Solution

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Learning Outcomes: 3(b), 4(d)

Textbook References: 7.6

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*General comments: On the whole, this question was done well, with many candidates earning full credit.*

(a)

$${}_{20}V = 10000A_{60} - 120(0.95)\ddot{a}_{60} = 2420.72$$

$$A_{60} = 0.36913 \quad \ddot{a}_{60} = 11.1454$$

$$CV = (0.85)_{20}V = 2057.6$$

*Comments: Some candidates omitted commissions. Others attempted to use a retrospective reserve formula, which is incorrect in this case as the premium is not an equivalence principle premium under the assumptions given.*

(b)

(b)(i) Let  $S^*$  denote the revised death benefit.

$$2057.6 = S^* A_{60:\overline{10}|}^1 + 80 - (0.95)120 \ddot{a}_{60:\overline{10}|}$$

$$A_{60:\overline{10}|}^1 = 0.36913 - (0.45120)(0.51495) = 0.13678 \quad \ddot{a}_{60:\overline{10}|} = 7.2789$$

$$\Rightarrow S^* = 20524$$

(b)(ii) Let  $P^*$  denote the revised premium.

$$2057.6 = 10\,000 A_{60:\overline{10}|} + 80 - (0.95)P^* \ddot{a}_{60:\overline{10}|}$$

$$A_{60:\overline{10}|} = 0.36913 - (0.45120)(0.51495) + 0.45120 = 0.58798 \quad \ddot{a}_{60:\overline{10}|} = 7.2789$$

$$\Rightarrow P^* = 564.31$$

(b)(iii) Let  $P^\ddagger$  denote the revised premium.

$$2057.6 = 9\,000 A_{60} + 80 - (0.95)P^\ddagger \ddot{a}_{60}$$

$$\Rightarrow P^\ddagger = 126.98$$

*Comments: Some candidates used  ${}_{20}V$  as the single premium for the policy alteration, instead of the cash value. This resulted in a relatively small deduction of marks. There was also some confusion about the \$80 charge for the policy change. However, most candidates who attempted this part did well.*

- (c) In (b)(iii) the benefit is lower than the original policy, but the premium is higher, and all other factors are the same as the original policy. A rational policyholder would not make this change

*Comment: Many candidates spotted that this would be a bad deal for the policyholder.*

- (d) In (b)(i) the death benefit is significantly increased, based on the change from whole life to term, with no increase in premium. A policyholder with compromised mortality (eg impaired health) might choose this option as a cheap way to increase the payoff. To allow for adverse selection, the insurer would likely assume higher mortality.

*Comment: Many candidates correctly identified the potential for adverse selection. However, option (ii) (the endowment insurance) was also a fairly popular choice. Candidates were expected to recognize that the risk is much greater from the term insurance because the net amount at risk is substantially higher.*

## MLC Spring 2015 Question 6

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Learning Objectives: 1(d), 3(c), 4(e)

Textbook References: 13.4

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*General Commentary: This question tested material related to Type B universal life insurance policies, specifically calculating account values and performing profit testing for these policies. In general, candidates did well on the more basic portions of the question requiring only straightforward calculations, but poorly on the part requiring an understanding of the ramifications of changing one of the underlying assumptions.*

*This question was omitted by a large number of candidates. Of the candidates attempting it, most candidates received significant credit, though very few earned full marks.*

(a)

$$\begin{aligned} AV_1 &= (2500(0.95) - 0.005(100\,000)v_{6\%} - 30)(1.06) = 1985.70 \\ \Rightarrow CV_1 &= 0.2(AV_1) = 397.1 \end{aligned}$$

*Commentary: This part required candidates to know how to calculate an account value for a Type B universal life policy. Most candidates did very well on this part. The most common error was failing to discount the COI charge. Some candidates did calculations for a Type A universal life, instead of a Type B policy. A small number of candidates incorrectly calculated the surrender charge. Each of these mistakes resulted in small point deductions.*

(b) Using the notation of the textbook:

$$Pr_1 = (P - E)(1 + i^e) - EDB_1 - ESV_1 - EAV_1$$

$$\text{where } P = 2500 \quad E = 120 \quad i^e = 0.11$$

$$EDB_1 = 0.004(100\,000 + AV_1 + 200) = 408.7$$

$$ESV_1 = 0.1(0.996)(CV_1 + 100) = 49.5$$

$$EAV_1 = 0.996(0.9)AV_1 = 1780.0$$

$$\Rightarrow Pr_1 = 403.6$$

*Commentary: This part involved profit testing for a Type B universal life policy. Candidates did moderately well on this part and most knew the general formula for  $Pr_1$ , but there were several common errors. The most common errors related to the probabilities*

in the EAV and ESV terms; for example, many candidates calculated the probability of surviving the year as  $1 - 0.1 - 0.004$  instead of  $(0.996)(0.9)$ . Many candidates failed to include the account value in the EDB term. Some candidates failed to correctly incorporate expenses, sometimes erroneously using contract expenses charged to the policyholder.

(c)

$$NPV(1) = Pr_0 + Pr_1 v_{14\%} = (-200) + 403.6/(1.14) = 154.0$$

*Commentary: This part required candidates to perform a partial NPV calculation. Most candidates did well on this part. The most common mistakes were using the wrong discount rate, omitting the pre-contract expenses, and incorporating survival probabilities into the calculation.*

(d) The NPV is

$$NPV = Pr_0 + Pr_1 v_{14\%} + p_x^{(\tau)} Pr_2 v_{14\%}^2 + {}_2p_x^{(\tau)} Pr_3 v_{14\%}^3 + \dots$$

Let \* denote values under revised assumptions.

$$Pr_t^* = Pr_t \quad \text{for } t = 0, 2, 3, \dots$$

$$EDB_1^* = EDB_1 = 408.7$$

$$ESV_1^* = 0.2(0.996)(497.1) = 99.0$$

$$EAV_1^* = 0.996(0.8)AV_1 = 1582.2$$

$$Pr_1^* = (2500 - 120)(1.11) - 408.7 - 99.0 - 1582.2 = 551.9$$

$${}_k p_x^{(\tau)*} = \frac{0.8}{0.9} {}_k p_x^{(\tau)}$$

$$\Rightarrow NPV^* = (NPV - (Pr_0 + Pr_1 v_{14\%})) \frac{0.8}{0.9} + Pr_0 + Pr_1^* v_{14\%}$$

$$= (2000 - 154) \frac{0.8}{0.9} - 200 + 551.9 v_{14\%} = 1925.0$$

*Commentary: In this part, the surrender assumption was changed, and the candidate was required to recompute the NPV under this new assumption. The new surrender assumption had two implications: (1)  $Pr_1$  changed as a result of changes to ESV and EAV, and (2), the survival probabilities applied to  $Pr_k, k \geq 2$  when calculating NPV, also changed. Most candidates recognized the first implication, but the majority of candidates ignored the second implication, resulting in significant point deductions.*

## MLC Spring 2015 Question 7

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Learning Objectives: 5(a), 5(c)

Textbook References: 10.2, 10.6.1

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*General Comments: The candidates who attempted this question did very well, with most earning full credit for the parts attempted. However, a very large number of candidates omitted the question.*

*The most common error involved using the wrong number of years for final salary calculations.*

- (a)  $RR$  denotes the replacement rate,  $FAS$  denotes final average salary,  $S_x$  denotes salary in the year of age  $x$  to  $x + 1$ . Then

$$RR = \frac{9.5 \times 900 + 15.5 \times FAS \times 0.03}{S_{64}}$$

$$S_{64} = 30\,000(1.02)^{24} = 48\,253$$

$$FAS = 30\,000 \frac{1.02^{22} + 1.02^{23} + 1.02^{24}}{3} = 47\,313$$

$$\Rightarrow RR = \frac{30\,550}{48\,253} = 63.3\%$$

- (b)

$$RR = \frac{900n + (25 - n)47\,313(0.03)}{48\,253} \geq 0.65$$

$$\Rightarrow \frac{35\,485 - 519.40n}{48\,253} \geq 0.65 \Rightarrow n \leq 7.9 \text{ years}$$

- (c) *Note: the question does not specify whether the accrued benefits in XYZ are based on FAS at retirement or FAS at age 55. Either interpretation was acceptable.*

**Version A: using FAS at retirement.**

Total accrued benefit based on first 15 years of employment

$$7 \times 900 + 8(0.03)FAS = 17\,655$$

The required benefit is

$$0.65 \times S_{64} = 31\,364$$



So the annuity payments are 13709 per year, requiring premium  $P$  where

$$P\ddot{a}_{55:\overline{10}|} = {}_{10}E_{55} \times 13709 \times \ddot{a}_{65}$$

$${}_{10}E_{55} = 0.48686 \quad \ddot{a}_{55:\overline{10}|} = 12.2758 - 0.48686(9.8969) = 7.4574 \quad \ddot{a}_{65} = 9.8969$$

$$\Rightarrow P = 8857.8$$

**Version B: using FAS at time 15.**

Total accrued benefit based on first 15 years of employment

$$7 \times 900 + 8(0.03)(S_{52} + S_{53} + S_{54})/3 = 15615$$

The required benefit is

$$0.65 \times S_{64} = 31364$$

So the annuity payments are 15749 per year, requiring premium  $P$  where

$$P\ddot{a}_{55:\overline{10}|} = {}_{10}E_{55} \times 15749 \times \ddot{a}_{65}$$

$${}_{10}E_{55} = 0.48686 \quad \ddot{a}_{55:\overline{10}|} = 12.2758 - 0.48686(9.8969) = 7.4574 \quad \ddot{a}_{65} = 9.8969$$

$$\Rightarrow P = 10175.8$$