1. From the ILT, we have:

$$ {}_{25}p_{10} = \frac{\ell_{35}}{\ell_{10}} = \frac{9420657}{9705588} = 0.97064 $$
$$ {}_{25}p_{35} = \frac{\ell_{60}}{\ell_{35}} = \frac{8188074}{9420657} = 0.86916 $$

The expected number of survivors from the sons is 1941.285 with variance 56.991.

The expected number of survivors from fathers is 1738.323 with variance 227.4393.

The total expected number of survivors is therefore 3679.61.

The standard deviation of the total expected number of survivors is therefore $\sqrt{(56.991 + 227.4393)} = \sqrt{(284.43)} = 16.865$

Both np>10 and nq>0, so that normal approximation to the binomial is appropriate.

The 99th percentile equals $3679.61 + 2.33 * 16.865 = 3718.9$

2. The number of left-handed members after each year *k* is:

$$ \begin{aligned} L_k &= L_{k-1} \times 0.75 + 35 \times 0.75 \\ &= 75 \times 0.75^k + 35 \times \left(0.75 + 0.75^2 + ...0.75^{k-1}\right) \end{aligned} $$

Similarly, the number of right-handed members after each year *k* is:

$$ R_k = R_{k-1} \times 0.50 + 15 \times 0.50 $$
$$ = 25 \times 0.50^k + 15 \times \left(0.50 + 0.50^2 + ...0.50^{k-1}\right) $$

At the end of year 5, the number of left-handed members is expected to be 89.57520, and the number of right-handed members is expected to be 14.84375.

The proportion of left-handed members at the end of year 5 is therefore

$$ \frac{89.57520}{89.57520 + 14.84375} = 0.85784 $$

3. The probability that Johnny will not have any accidents in the next year is:

$$p_{(1)}^{\overline{00}} = e^{-\int_0^1 \left(\mu_s^{01}+\mu_s^{02}\right)ds}$$, where

$$\mu_t^{01} + \mu_t^{02} = 3.718\mu_t^{01} = 3.718\left(0.03+0.06\times 2^t\right)$$

So that

$$\int_0^1 3.718\left(0.03+0.06\times e^{t(ln2)}\right)dt = 3.718\left(0.03\right) + \frac{3.718\left(0.06\right)}{ln\left(2\right)} = 0.4333764$$

and

$$p_{(1)}^{\overline{00}} = e^{-0.4333764} = 0.6483164$$

The probability Johnny will have at least one accident is therefore

$1-0.6483164 = 0.3516836$.

4. $Z_3 = 2Z_1 + Z_2$ so that $\text{Var}\left(Z_3\right) = 4\text{Var}\left(Z_1\right) + \text{Var}\left(Z_2\right) + 4Cov\left(Z_1, Z_2\right)$

where $Cov\left(Z_1, Z_2\right) = \underbrace{E\left[Z_1 Z_2\right]}_{=0} - E\left[Z_1\right]E\left[Z_2\right] = -(1.65)(10.75)$

$$\begin{aligned}\text{Var}\left(Z_3\right) &= 4(46.75) + 50.78 - 4(1.65)(10.75) \\ &= 166.83\end{aligned}$$

5.

$$\begin{aligned}a_{50:60:\overline{20|}} &= a_{50:60} - v^{20}\ {}_{20}p_{50:60}\ a_{70:80} \\ &= a_{50:60} - v^{20}\ {}_{20}p_{50}\ {}_{20}p_{60}\ a_{70:80} \\ &= \left(\ddot{a}_{50:60} - 1\right) - {}_{20}E_{50}\frac{\ell_{80}}{\ell_{60}}\left(\ddot{a}_{70:80} - 1\right) \\ &= 9.1944 - 0.23047\left(\frac{3914365}{8188074}\right)\left(4.0014\right) \\ &= 8.7535\end{aligned}$$

6.

$$\begin{aligned} APV &= 30,000A_{50:50} + 70,000A_{\overline{50:50}} \\ &= 30,000A_{50:50} + 70,000\left(A_{50} + A_{50} - A_{50:50}\right) \\ &= 70,000(2)A_{50} - 40,000A_{50:50} \\ &= 140,000(0.24905) - 40,000(0.34049) \\ &= 21,247.40 \end{aligned}$$

7. Per equivalence Principle:

$$G\ddot{a}_{35} = 100,000A_{35} + 0.4G + 150 + 0.1G\ddot{a}_{35} + 50\ddot{a}_{35}$$

$$1770\ddot{a}_{35} = 100,000\left(1 - d\ddot{a}_{35}\right) + 0.4(1770) + 150 + 0.1(1770)\ddot{a}_{35} + 50\ddot{a}_{35}$$

$$1770\ddot{a}_{35} = 100,000 + 708 + 150 + \ddot{a}_{35}\left(177 + 50 - 100,000\left(\frac{0.035}{1.035}\right)\right)$$

Solving for $\ddot{a}_{35}$, we have

$$\ddot{a} = \frac{100,858}{1770 + 3154.64} = \frac{100,858}{4924.64} = 20.48$$

8. The loss at issue is given by:

$$\begin{aligned} L_0 &= 100v^{K+1} + 0.05G + 0.05G\,\ddot{a}_{\overline{K+1|}} - G\,\ddot{a}_{\overline{K+1|}} \\ &= 100v^{K+1} + 0.05G - 0.95G\left(\frac{1 - v^{K+1}}{d}\right) \\ &= \left(100 + \frac{0.95G}{d}\right)v^{K+1} + 0.05G - 0.95\frac{G}{d} \end{aligned}$$

Thus, the variance is

$$\begin{aligned} Var\left(L_0\right) &= \left[100 + \frac{0.95(2.338)}{0.04/1.04}\right]^2\left({}^2A_x - \left(A_x\right)^2\right) \\ &= \left[100 + \frac{0.95(2.338)}{0.04/1.04}\right]^2\left(0.17 - \left(1 - \frac{0.04}{1.04}(16.50)\right)^2\right) \\ &= 908.1414 \end{aligned}$$

9. $${}_{2|2}A_{65} = \underbrace{v^3}_{\text{payment year 3}} \underbrace{{}_2p_{[65]}}_{\text{Lives 2 years}} \times \underbrace{q_{[65]+2}}_{\text{Die year 3}}$$

$$+ \underbrace{v^4}_{\text{payment year 4}} \underbrace{{}_3p_{[65]}}_{\text{Lives 3 years}} \times \underbrace{q_{65+3}}_{\text{Die year 4}}$$

$$= \left(\frac{1}{1.04}\right)^3 \times (0.92)\times(0.9)\times(0.12)$$

$$+ \left(\frac{1}{1.04}\right)^4 \times (0.92)(0.9)(0.88)(0.14)$$

$$= 0.088 + 0.087 = 0.176$$

The actuarial present value of this insurance is therefore $2000 \times 0.176 = 352$.

10. $$\bar{A}_{35} = \left(1 - e^{-35(\mu+\delta)}\right) \times \left(\frac{\mu}{\mu+\delta}\right) + e^{-35(\mu+\delta)} \bar{A}_{70} = 0.063421 + 0.146257 = 0.209679$$

$$\bar{a}_{35} = \frac{1-\bar{A}_{35}}{\delta} = \frac{1-0.209679}{0.05} = 15.80642$$

$$\bar{P}_{35} = \frac{\bar{A}_{35}}{\bar{a}_{35}} = \frac{0.209679}{15.80642} = 0.0132654$$

The annual net premium for this policy is therefore $100,000 \times 0.0132654 = 1,326.54$

11. Assuming UDD

Let P = monthly net premium

$$\text{EPV(premiums)} = 12P\ddot{a}_x^{(12)} \cong 12P\left[\alpha(12)\ddot{a}_x - \beta(12)\right]$$
$$= 12P\left[1.00028(9.19) - 0.46812\right]$$
$$= 104.6934P$$
$$\text{EPV(benefits)} = 100,000\bar{A}_x$$
$$= 100,000\frac{i}{\delta}A_x = 100,000\frac{i}{\delta}(1 - d\ddot{a}_x)$$
$$= 100,000\frac{0.06}{\log(1.06)}\left(1 - \frac{0.06}{1.06}(9.19)\right)$$
$$= 49406.59$$
$$P = \frac{49406.59}{104.6934} = 471.9168$$

12. The expected end of the year profit B $= 722 = ({}_{20}V + G - WG - 60)(1.08)$

$$-(0.004736)(150000)$$
$$-(1 - 0.004736)({}_{21}V)$$

Plug in given values, we have

$$722 = \left(24496 + (1 - W)\right)\left((1212) - 60\right)(1.08) - 0.004736(150000) - 0.995264(26261)$$
$$722 = 852.8121 - 1308.96W$$

Solving W, $W = \dfrac{852.8121 - 722}{1308.96} = 10\%$

13. $i^{(12)} = 0.06 \Rightarrow 0.005$ per month

$$AV_{11} = CSV + SC = 10,000 + 3,000 = 13,000$$
$$COI_{12} = 100,000(0.002)/1.005 = 199.00$$
$$AV_{12} = \left[13,000 + 1,000(1-0.25) - 6 - 199.00\right](1.005) = 13,612.75$$
$$COI_{13} = 100,000(0.003)/1.005 = 298.51$$
$$AV_{13} = \left(13,612.73 + 1,000(1-W) - 6 - 298.51\right)(1.005) =$$
$$CSV_{13} = 13,329.51$$
$$CSV_{13} = AV_{13} - SC_{13} = AV_{13} - 1000 \Rightarrow AV_{13} = 13,330 + 1000 = 14,330$$
$$14,330 = \left(13,612.73 + 1000(1-W) - 6 - 298.51\right)(1.005)$$
$$\frac{14,330}{1.005} - 13,612.73 + 6 + 298.51 = (1-W)1,000$$
$$\Rightarrow W = 0.0495$$

14. Given ${}_4V = 27.77$, we have:

Expected total reserve =
$$980\left[\left(19.9 + 8.8\right)\left(1.05\right) - \left(500 - 27.77\right)\left(0.005\right)\right]$$
$$= 27,218.37$$

Actual total reserve =
$$980\left[\left(19.9 + 8.8\right)\left(1 + j\right) - \left(500 - 27.77\right)\left(\frac{7}{980}\right)\right]$$
$$= 28,126\,j + 24,820.39$$

Since there is no gain or loss, expected = actual, so:
$$j = \frac{27,218.27 - 24,820.39}{28,126} = 8.52585\%$$

15. If G denotes the gross premium, then

$$G = \frac{1000 * A_{35} + 30\ddot{a}_{35} + 270}{0.96 * \ddot{a}_{35} - 0.26} = \frac{1000(0.12872) + 30(15.3926) + 270}{0.96(15.3926) - 0.26} = 59.28$$

So that,

$$x = 1000 * A_{36} + (30 - 0.96 * G)\ddot{a}_{36}$$
$$= 1000(0.13470) + (30 - 0.96 \times 59.28) \times (15.2870) = -276.65$$

Note that $y = 0$ as per definition of FPT reserve.

16. $$\pi = \frac{1000 \quad {}_{10|}\ddot{a}_{55}}{\ddot{a}_{55:\overline{10}|} - (IA)^{1}_{55:\overline{10}|}} = \frac{1000(0.48686)(9.8969)}{\left[12.2758 - (0.48686)(9.8989)\right] - 0.51209} = 693.76$$

$${}_{9}V = 1000 \quad {}_{1|}\ddot{a}_{64} + 10\pi A^{1}_{64:\overline{1}|} - \pi\ddot{a}_{64:\overline{1}|}$$

$$= 1000\frac{1}{1.06}\left(\frac{7,533,964}{7,683,979}\right)9.8969 + 10(693.76)\frac{1}{1.06}(0.01952) - 693.76$$

$$= 9154.42 + 127.76 - 693.76 = 8588$$

17. $$\text{AV}_{\text{end}} = \left(\text{AV}_{\text{start}} + G \times (1 - f) - e - \text{COI}\right) \times \left(1 + i^{c}\right)$$

where $\text{COI} = \left[\left(\text{Specified amount}\right) / \left(1 + i^{c}\right)\right] \times \text{COI}$

(COI depends only on the specified amount, not the AV, since the death benefit is Specified Amount + AV)

$$\text{COI}_1 = 100,000 \times 0.01 / 1.04 = 961.54$$
$$\text{COI}_2 = 100,000 \times 0.02 / 1.04 = 1,923.08$$

$$\text{AV}_1 = \left(0 + G \times (1 - 0.3) - 100 - 961.54\right) \times 1.04 = 0.728G - 1,104$$
$$\text{AV}_2 = \left(0.728G - 1,104 + G \times (1 - 0.1) - 50 - 1,923.08\right) \times 1.04 = 1.693G - 3,200.16$$

$$1.693G - 3,200.16 = 10,000$$
$$G = \left(10,000 + 3200.16\right) / 1.693 = 7,796.35$$

18.

Retirement Age	63	64	65
Years of Service (K)	33	34	35
v^{K-20}	0.414964	0.387817	0.367446
Probability of Retirement	0.4	(0.6)(0.2)	(1.0)(0.6)(0.8)
Benefit	(33)(12)(25)(12.0)	(34)(12)(25)(11.5)	(35)(12)(25)(11.0)
Benefit Reduction Factor	0.856	0.928	1.0
Present Value of Retirement Benefit	16,879.56	5,065.87	20,094.01

The actuarial present value of the retirement benefit is therefore:

$$16,879.56 + 5,065.87 + 20,094.01 = 42,039.44$$

19. Let *S* be Colton's starting salary.

$$\overbrace{\frac{S}{30}\left[1+1.025+1.025^{2}+\ldots+1.025^{29}\right]}^{\text{career average salary}} \times 0.02*30 = \overbrace{\frac{S}{5}\left[1.025^{29}+1.025^{28}+\ldots+1.025^{25}\right]}^{\text{find average salary}} y*30$$

$$0.02\frac{1.025^{30}-1}{0.025} = 6y\left(1.025^{25}\right)\frac{1.025^{5}-1}{0.025}$$

$$y = \frac{0.02}{6}1.025^{-25}\frac{1.025^{30}-1}{1.025^{5}-1} = 0.1501727 = 1.5\%$$

20. For Plan I, the accumulated contributions are:

$$(0.15)S_{35}(1.03)^{30}+(0.15)S_{35}(1.03)(1.03)^{29}$$
$$+(0.15)S_{35}(1.03)^{2}(1.03)^{28}+\ldots$$
$$=(0.15)S_{35}(1.03)^{30}(30)=10.923S_{35}$$
$$=(12B)\ddot{a}_{65}^{(12)}$$
$$\Rightarrow B=\frac{10.923S_{35}}{12(9.44)}=0.096S_{35}$$

For Plan II, we have:

$$\frac{1}{2}\left(S_{35}(1.03)^{28}+S_{35}(1.03)^{29}\right)=2.322S_{35}$$
$$B=\frac{1}{12}\left(30\times 0.015\right)S_{35}\left(2.322\right)=0.087S_{35}$$
$$\Rightarrow \frac{0.096}{0.087}=1.107$$