
SOCIETY OF ACTUARIES
Enterprise Risk Management – Investment Extension

Exam ERM-INV

Date: Tuesday, October 27, 2015

Time: 8:30 a.m. – 12:45 p.m.

INSTRUCTIONS TO CANDIDATES

General Instructions

1. This examination has a total of 80 points.

This exam consists of 10 questions, numbered 1 through 10.

The points for each question are indicated at the beginning of the question. Questions 9 and 10 pertain to the extension readings and/or the Case Study, which is enclosed inside the front cover of this exam booklet.
2. Failure to stop writing after time is called will result in the disqualification of your answers or further disciplinary action.
3. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions on the exam booklet.

Written-Answer Instructions

1. Write your candidate number at the top of each sheet. Your name must not appear.
2. Write on only one side of a sheet. Start each question on a fresh sheet. On each sheet, write the number of the question that you are answering. Do not answer more than one question on a single sheet.
3. The answer should be confined to the question as set.
4. When you are asked to calculate, show all your work including any applicable formulas.
5. When you finish, insert all your written-answer sheets into the Essay Answer Envelope. Be sure to hand in all your answer sheets because they cannot be accepted later. Seal the envelope and write your candidate number in the space provided on the outside of the envelope. Check the appropriate box to indicate Exam ERM-INV.
6. Be sure your written-answer envelope is signed because if it is not, your examination will not be graded.

Tournez le cahier d'examen pour la version française.

CASE STUDY INSTRUCTIONS

The case study will be used as a basis for some examination questions. Be sure to answer the question asked by referring to the case study. For example, when asked for advantages of a particular plan design to a company referenced in the case study, your response should be limited to that company. Other advantages should not be listed, as they are extraneous to the question and will result in no additional credit. Further, if they conflict with the applicable advantages, no credit will be given.

****BEGINNING OF EXAMINATION****

1. (6 points) You work for a small multi-line insurance company that sells:

- Individual universal life products (UL)
- Fixed deferred annuities (FA)
- Group long-term disability (GLTD)

To address the sustained low interest rate environment, your company recently lowered crediting rates on the UL and FA products, and it has also received state approvals for higher premium rates for the GLTD.

Your CRO wants to evaluate the effect of a rapid increase in interest rates, both on quantifiable risks and on those that are not easily quantifiable.

- (a) (1 point) Describe ways to evaluate non-quantifiable risks, in general.
- (b) (2 points) Describe the potential risks to your company under the following three risk categories if a rapid increase in interest rates occurs:
- I. Product / Pricing
 - II. Liquidity
 - III. Operational
- (c) (1.5 points) Recommend a specific approach for measuring liquidity risk for each of the company's three products.
- (d) (1.5 points) Explain how you will monitor and report on the operational risks identified in part (b).

2. (12 points) Lily Life is a medium-sized U.S. life insurer that has issued only traditional life and annuity products, including immediate annuities. Lily recently expanded its product offerings and launched a single premium variable annuity product, LVA.

LVA premiums are invested in the actively-managed All-American Total Return (TR) Equity Fund which is benchmarked against the S&P 500 TR Index. LVA provides a return of premium (ROP) guarantee at the end of the first contract year only. Lily has received \$5,800,000 in LVA premium to date.

Lily's risk management policy requires hedging of market risk exposures. For its immediate annuity block, Lily currently has a \$5 million notional interest rate swap position with Magnolia Bank that hedges the interest rate mismatch.

You have been asked to assess potential hedges of the new U.S. equity risk exposure that has arisen from the LVA product. You are reviewing three alternatives:

- I. Enter into an exchange-traded 1-year S&P 500 futures contract
- II. Buy an exchange-traded 1-year S&P 500 put option
- III. Buy a 1-year over-the-counter (OTC) S&P 500 put option offered by Magnolia Bank

To assess the hedging alternatives, you consider a simplified delta hedging strategy using the S&P 500 TR Index as a reasonable proxy for the All-American TR Equity Fund, assuming all policies are sold at $t = 0$. You have decided to use the following formulas to assess the cost of the one-year ROP guarantee and delta:

$$P_0 = -X_0\Phi(-d_1) + Ee^{-r}\Phi(-d_2)$$

Where

$$d_1 = \frac{\ln\left(\frac{X_0}{E}\right) + \left(r + \frac{\sigma_X^2}{2}\right)}{\sigma_X}$$

$$d_2 = d_1 - \sigma_X$$

$$\Delta_p = \Phi(d_1) - 1$$

2. Continued

You gather the following additional information:

- The level of the S&P 500 TR Index at $t = 0$ is 2100
- The volatility of the S&P 500 TR Index is 20% per annum
- $r = 1\%$
- The strike price, E , is set such that the option is at-the-money (ATM)
- Only 95% of the current LVA premium is expected to be in force in 1 year

- (a) (2 points) Calculate delta, that is, the total exposure to the underlying S&P 500 TR Index that needs to be hedged relative to the one-year ROP option written by Lily. Show your work.

Additionally, you determine that:

- The volatility of the S&P 500 futures contract is 16%
- The correlation between the S&P 500 TR Index and the S&P 500 futures contract is 0.80
- $\beta_M = \rho_{F,M} \sigma_M / \sigma_F$
- The size of the S&P 500 futures contract offered by the exchange is $\$250 \times \text{Index level}$

- (b) (2 points) You now consider alternative I to delta-hedge the equity risk exposure from LVA.

- (i) Calculate the optimal hedge ratio to minimize the volatility of the S&P 500 one-year futures hedge.
- (ii) Determine the number of one-year S&P 500 futures contracts that Lily should enter into to cover the delta risk exposure from LVA.

Show your work.

- (c) (2 points) You now consider alternative II to delta-hedge the equity risk exposure from LVA.

- (i) Calculate the theoretical cost of an S&P 500 ATM put option.
- (ii) Calculate the number of put options that Lily should purchase.

Show your work.

Question 2 continued on next page

2. Continued

With respect to alternative III, Magnolia Bank has offered an OTC put option for \$2,300 that will fully hedge the current delta exposure of the LVA put options written. You have gathered the following information about current financial exposures related to Magnolia Bank:

Financial Instrument	Current Market Value
Magnolia Bank Stock held by Lily	\$2,456,378
Magnolia Bank Bonds held by Lily	\$7,901,346
Lily Life Bonds held by Magnolia Bank	\$1,880,245
Lily's value of Interest Rate Swap with Magnolia Bank	-\$50,903

- (d) (1 point) Assume Lily purchases the put option from Magnolia Bank.

Calculate Lily's current counterparty risk exposure to Magnolia Bank under each of (i) and (ii):

- (i) Assume no netting agreement is in place
- (ii) Assume a cross-product netting agreement is in place

Show your work.

- (e) (2 points) Compare and contrast the nature of the counterparty risk between the exchange-traded put option hedge and the OTC put option hedge with Magnolia Bank.
- (f) (3 points)
- (i) Identify the criteria you would use in evaluating a recommendation to pursue one of the three alternative hedging approaches.
 - (ii) Recommend one of the three alternative hedging approaches based on the criteria identified in (i). Justify your response.

3. (9 points) Rose Life uses delta-hedging to manage the risk of the equity guarantees embedded in its variable annuity products.

(a) (1 point) Consider the observation that financial market volatility is predictable.

Describe three implications for risk management assuming this is the case.

(b) (0.5 points) Describe two features of empirical equity returns that are inconsistent with the assumption that returns are normally distributed.

You explore methods to predict equity return volatility that could assist in the management of time-varying risk. You learn of two popular time-series models:

I. Moving Average (MA) models:

$$\sigma_t^2 = \frac{1}{M} \sum_{i=1}^M r_{t-i}^2$$

II. A generalized autoregressive conditional heteroskedastic GARCH(1,1) model:

$$h_t = \alpha_0 + \alpha_1 r_{t-1}^2 + \beta h_{t-1}$$

For convenience, you assess the models using raw returns instead of returns around the mean.

In all calculations of one-month values (volatility, returns, etc.), you assume 20 business days of returns.

To aid in your assessment of the MA model, you have gathered the following daily log-return series on the S&P 500 Index:

Day	$t-22$	$t-21$	$t-20$	$t-19$...	$t-4$	$t-3$	$t-2$	$t-1$
Log return (r)	0.1195%	-1.4275%	0.3937%	-1.7107%	...	-0.8835%	-0.3973%	0.3523%	0.6587%
$100 \times r^2$	0.0143%	2.0378%	0.1550%	2.9265%	...	0.7806%	0.1578%	0.1241%	0.4339%

Question 3 continued on next page

3. Continued

- (c) (1.5 points) Calculate the daily σ_t^2 using the MA model with a 20-day window, given $(100 \times \sigma_{t-2}^2) = 0.8311\%$. Show your work.

You next use the GARCH(1,1) model to forecast the equity volatility in one month. You calculate maximum-likelihood estimates (MLE) for the parameters of the GARCH(1,1) model as follows:

α_0	α_1	β
0.000049%	0.0485	0.9459

You are given the following formula:

$$E_{t-1}(r_{t,T}^2) = \frac{\alpha_0}{1 - (\alpha_1 + \beta)} \left[(n-1) - (\alpha_1 + \beta) \frac{1 - (\alpha_1 + \beta)^{n-1}}{1 - (\alpha_1 + \beta)} \right] + \frac{1 - (\alpha_1 + \beta)^n}{1 - (\alpha_1 + \beta)} h_t$$

Where $E(r_t) = 0$

- (d) (2 points)
- (i) Calculate h , the long-run average, unconditional variance. Show your work.
 - (ii) Calculate the expected 1-month equity return variance at t , assuming the one-day variance h_t is equal to 0.008750%. Show your work.

Using your forecasted volatility under each of the two time-series models above, you derive the current theoretical price of a one-month at-the-money (ATM) S&P 500 Index put option, as follows:

Times-Series Model	Put Option Price
MA	\$31.88
GARCH(1,1)	\$33.49

3. Continued

Additionally, you gather the following actual exchange market prices for various ATM put options on the S&P 500 Index:

Expiration	Put Option Price
1-week	\$29.41
1-month	\$46.53
2-month	\$61.94
3-month	\$86.55

- (e) (1 point) Explain specifically how this market option price data can be used to provide a forecast of the 1-month volatility of the S&P 500 Index returns.
- (f) (3 points) Recommend one of the three approaches below to forecast equity volatility, citing the advantages of your recommended approach and the disadvantages of each of the other two approaches:
- I. The MA model
 - II. GARCH(1,1)
 - III. Use of market option prices

- 4.** (7 points) Begonia Life is an insurance company that offers only Guaranteed Investment Contracts (GICs). Begonia's GIC products have a 5-year term with a fixed guaranteed interest rate based on the yield of the 5-year U.S. Treasury note plus a fixed spread at the time of sale. The two products Begonia offers differ only in terms of surrenderability:
- I. The Basic GIC product is non-surrenderable.
 - II. The GIC Plus product is surrenderable at any time after 2 years for the notional amount plus accrued interest.

Regulatory reserves for both products are determined using interest-only discounting of all future cash flows at a prescribed interest rate determined as of the issue date. Regulatory valuation for assets supporting the GIC products is on a book value basis.

Economic liabilities are determined as the expected present value at risk-free rates of all future cash flows. Persistency and expense assumptions for this calculation are re-assessed annually based on experience studies. Begonia's economic scenario generator (ESG) determines future stochastic interest rate paths based on the existing risk-free yield curve as of the valuation date. Assets are valued on a market value basis.

Begonia is reviewing its ALM strategy and will use the following six steps as its Strategic Asset Allocation (SAA) framework:

1. Investment Objectives and Constraints
2. Asset Universe and Assumptions
3. Liability Cash Flow and Replicating Portfolio
4. Risk Measures
5. Risk-Return Tradeoffs
6. Strategic Asset Allocation and Associated Benchmarks

Parts (a) through (d) relate to the Basic GIC product.

- (a) (1.5 points) Identify three Investment Objectives that would be important for this product. Justify your response.

4. Continued

(b) (1.5 points) Consider the following asset classes:

- I. Cash
- II. Investment grade bonds or Treasuries with maturities less than 5 years
- III. Investment grade bonds or Treasuries with maturities greater than 5 years
- IV. Equities

Explain the role, if any, that each of the asset classes I through IV could play in your overall ALM strategy.

- (c) (1 point) Explain how the size of the fixed spread may influence your Investment Objectives and Constraints in step 1 and your Asset Universe in step 2.
- (d) (1 point) Explain the implication of basing the Risk Measures in step 4 on the economic surplus amount instead of the regulatory surplus.
- (e) (2 points) Explain how the following items will differ between Basic GIC and GIC Plus:
- (i) Effective liability duration
 - (ii) Setting Investment Objectives and Constraints
 - (iii) Setting Asset Universe and Assumptions
 - (iv) Risk Measure based on an economic basis

5. (7 points) You are an actuary for Peony Life. The CEO is delighted that you recently earned your CERA designation. The CEO sends you an email regarding an upcoming S&P rating agency assessment:

“The S&P rating agency incorporated an ERM assessment into their ratings, and we need to develop something fast! I’d like to have you design and create our company’s ERM program. You will report directly to me. I want the ERM program to focus on the underwriting department – our losses always stem from the underwriters! I’m not too worried about the other departments – let’s plan on having them complete an ERM check-list annually. Don’t worry – I won’t ask you to give a presentation to the Board of Directors. The Board isn’t interested in developing an ERM program.

Our RBC ratio is strong, well over 300%, so our risks are clearly financially covered. S&P should give us a “strong” ERM score!

I really need to focus on increasing our sales before year-end – everyone expects big bonuses this year! Let’s get this wrapped up as quickly as possible!”

- (a) (1 point) Describe three significant issues with Peony’s risk management culture.
- (b) (1 point) Outline the basic components of the ERM program you will design for Peony.
- (c) (1 point) After conducting its review of Peony’s new ERM program, S&P gives it a “weak” score.

Explain to the CEO why Peony’s strong RBC ratio may not translate into a strong S&P score.

5. Continued

- (d) (2 points) The CEO is reviewing Peony's RBC level. He asks you to provide information on four competitors. You research the RBC data from these companies and obtain the following:

Company #	Authorized Control Level (\$)	Total Adjusted Capital (\$)	RBC Ratio	NAIC Action Level
1	100	195		
2	99	215		
3	242	505		
4	86	125		

Complete the last two columns of the table above.

- (e) (2 points) Companies 1 and 2 are considered peers of Peony. The CEO has proposed maintaining Peony's RBC ratio equal to the average RBC ratio of these two companies.
- (i) Explain the positives and the negatives of maintaining Peony's RBC ratio at that level.
- (ii) Assume you move forward with the CEO's proposal.

Propose three specific ways to lower Peony's RBC ratio to that level.

6. (10 points) You are an actuary in the ERM area of Tulip Life. You are asked to evaluate the current risk aggregation approach taken by the Economic Capital (EC) modeling team. In particular, you are interested in tail dependency.

Tulip calculates required EC using a Gaussian copula simulation approach. The marginal loss distributions assumed for Equity and Credit risk factors are Normal ($\mu = 0$, $\sigma = 500$) and Normal ($\mu = 0$, $\sigma = 1000$), respectively.

You have also been provided the following correlation matrix:

Risk Factor	Equity Risk	Credit Risk
Equity Risk	1.000	0.250
Credit Risk	0.250	1.000

- (a) (5 points) You first calculate the simulated total loss due to Equity and Credit risk factors by using a Gaussian copula.

You are given the factorization of the correlation matrix, $\Sigma = L \times L^t$, done via the Choleski decomposition, where L is a lower triangular matrix and

$$L = \begin{bmatrix} 1.000 & 0.000 \\ 0.250 & 0.968 \end{bmatrix}$$

You are given two independent uniform pseudorandom numbers:

$$u_1 = 0.950$$

$$u_2 = 0.080$$

- (i) Demonstrate that the correlated uniform values using the Gaussian copula are $v_1 = 0.950$, $v_2 = 0.171$. Show your work.
- (ii) Calculate the simulated total loss due to Equity and Credit risk factors using the correlated uniform values from (i). Show your work.

6. Continued

- (b) (3 points) You now calculate the simulated total loss due to Equity and Credit risk factors by using a t -copula with 2 degrees of freedom.

In addition to u_1 and u_2 , you have simulated a single observation from the Gamma distribution with parameters $\alpha = \nu / 2$ and $\theta = 2$. The resulting simulated value is 0.325.

You are given that the cumulative distribution function of the t -distribution with 2 degrees of freedom is:

$$F_{t,2}(x) = \frac{1}{2} + \frac{x}{2\sqrt{2+x^2}}$$

- (i) Demonstrate that the correlated uniform values using the t -copula are $v_1 = 0.972$, $v_2 = 0.071$. Show your work.
- (ii) Calculate the simulated total loss due to Equity and Credit risk factors using the correlated uniform values from (i). Show your work.
- (c) (2 points) You believe the Gaussian copula is not appropriate when modeling these aggregate losses for Tulip. You have proposed using a t -copula.

Outline the advantages and disadvantages of the proposed t -copula over the standard Gaussian copula in this situation.

7. (5 points) Paphos Bank (PB) is a local, hometown bank focused on servicing the residents of a small Caribbean island. It decided to branch out and acquired a multi-line, U.S.-based insurance company now called Paphos Insurance Company (PIC). PIC currently sells only term life and auto insurance, but PIC also has closed blocks of critical illness and disability liabilities.

You recently obtained your CERA and have just been hired by PB. One of your first assignments is to develop PIC's Risk Appetite Statement (RAS), as this is currently not part of its ERM.

A colleague has suggested that the following four items from PB's RAS would be appropriate for PIC:

- I. Be the top bank on the island with a Moody's rating of at least Aa1.
- II. Have an average customer satisfaction rating of at least 90%.
- III. Annually grow the overall client base by 5% while retaining 95% of present clients.
- IV. Meet all withdrawal requests upon demand for all events.

(a) (3 points)

- (i) Evaluate items I to IV in the bank's RAS with respect to their applicability to PIC and, where necessary, explain how they should be modified for PIC's RAS.
- (ii) Identify two additional items that would be important for PIC to include in its RAS.

7. Continued

- (b) (2 points) Your manager wants you to consider reinsurance on the following two blocks of PIC's business to mitigate the earnings volatility:
- I. Critical Illness Insurance – Policies pay a one-time payment of U.S. \$50,000 if the policyholder is diagnosed with a critical illness. PIC has recently stopped selling this type of insurance, but still has 500 policies that will continue to provide coverage for up to 20 more years.
 - II. 20-year Term Insurance – PIC launched this product at the end of 2014. Annual forecasted sales are as follows:

Year	Forecasted Term Sales (Premiums)
2015	\$10 Million
2016	\$15 Million
2017	\$20 Million
2018	\$22 Million
2019	\$23 Million

The following types of reinsurance are available:

- A. Excess of loss
- B. Catastrophic excess of loss
- C. Automatic treaty proportional quota share
- D. Facultative treaty proportional quota share

Recommend the most appropriate type of reinsurance from the list above for each of blocks I and II, based on the objective of mitigating earnings volatility. Justify your answer.

- 8.** (4 points) Fuchsia is a publicly-traded, market-leading mutual fund company. In order to participate in the expanding Variable Annuity (VA) market, the corporation started selling VAs with guarantees in early 2000. Assets under management now total \$10 billion.

Fuchsia's VA product offers investment performance guarantees with almost no investment restrictions. Accordingly, the positive investment performance during the early contract years yielded strong product earnings, but the 2008 global financial crisis resulted in sharp increases in reserves and capital.

In the wake of the global financial crisis, Fuchsia is looking to divest the VA block. The CEO formed a task force for the project that includes professionals from within Fuchsia as well as external consultants who have significant experience with VAs. Mr. A was appointed the project leader.

Mr. A, currently a vice president, has worked at Fuchsia for over 15 years and has extensive experience selling securities. Mr. A was promised a significant bonus if the deal closes by year-end.

From Fuchsia's perspective:

- (a) (2 points)
- (i) Identify and analyze People Risk associated with the VA product.
 - (ii) Propose means for reducing such risk.
- (b) (2 points)
- (i) Identify and analyze two types of People Risk related to the divestiture project.
 - (ii) Propose means for reducing such risks.

*Questions 9 – 10 pertain to the Case Study and/or extension readings.
Each question should be answered independently.*

9. (10 points) You are performing a risk review of the equity exposure arising from the GMAB of SLIC's VA product. Your analysis will focus on allocations to U.S. Equity mutual funds. The value of the GMAB liability at issue is modeled as a written 10-year at-the-money (ATM) put option on the S&P 500 Index, ignoring fund fees. The theoretical value is:

$$P = S[\Phi(d_1) - 1] - Ke^{-r\tau}[\Phi(d_2) - 1]$$

$$d_1 = \frac{\ln\left(\frac{S}{Ke^{-r\tau}}\right)}{\sigma\sqrt{\tau}} + \sigma\frac{\sqrt{\tau}}{2}$$

$$d_2 = d_1 - \sigma\sqrt{\tau}$$

To derive the VaR(99%) of the GMAB option, you map the GMAB position to five selected risk factors: S , S^2 , r , σ , and t . Changes in the value of the GMAB put option are approximated by taking partial derivatives to these risk factors:

$$dP = \Delta dS + \frac{1}{2} \Gamma dS^2 + \rho dr + \Lambda d\sigma + \theta dt$$

- (a) (1.5 points) Define the following sensitivities of the value of a 10-year S&P 500 put option:
- (i) Delta
 - (ii) Rho
 - (iii) Vega
 - (iv) Gamma
 - (v) Theta

Question 9 continued on next page

9. Continued

- (c) (2 points) Assume the hedging group determines that the implied volatility of the 10-year S&P 500 ATM put option is 25%.

Assess the impact of the increased volatility on SLIC’s risk exposure to the GMAB. Show your work.

Your analysis now focuses on the ATM option risk factor mapping to derive the 99% VaR at the 1-month hedge rebalancing horizon of the GMAB option. You have assumed that the σ risk factor has a one-month volatility of 3%, and that the r risk factor has a one-month volatility of 0.3%. You have derived the 1-month VaR(99%) for each of the risk factors as follows:

		Unit	$K = 2100$	Risk Factor	Worst potential
		P	\$229.48	1-month VaR(99%)	1-month loss
Risk factor	Greek	Change per			
S	Δ	+1 Dollar	-\$0.215	\$282.01	
S^2	Γ	+1 Dollar	\$0.00022	\$79,531	
σ	Λ	+1% per annum	\$19.38	6.98%	
r	ρ	+1% per annum	-\$68.01	0.70%	
t	θ	+1 month	\$0.085	0	

- (d) (1 point) Calculate the worst potential one-month loss at a 99% confidence level for each of the risk factor sensitivities above. Show your work.
- (e) (1 point) Assume that SLIC’s current delta-rho hedging program is 100% effective in neutralizing the exposure to these two Greeks.

Assess how effective the delta-rho hedge is for the total GMAB option 1-month VaR(99%).

Show your work.

- (f) (2 points) Identify and describe three aspects of SLIC’s VA hedging program that should be changed to improve the program’s effectiveness in hedging the GMAB liability risk exposure. Justify your response using your analysis.

**Questions 9 – 10 pertain to the Case Study and/or extension readings.
Each question should be answered independently.**

- 10.** (10 points) You are an investment actuary in SLIC’s ALM department. You have been asked to review the extent to which interest rate risk has been immunized within the Term block. The Term block has exhibited a stable duration of about 7.5 years over the last few years.

In analyzing the current asset and liability cash flows for the block, you find that they cluster around four key term points that can be represented as U.S. Treasury zero-coupon bond maturities. You will use \$100 million of liabilities and \$100 million of assets that are representative of the full Term liability and current asset portfolios. You cash flow map the \$100 million portfolios to the four key zero-coupon bonds as follows:

Term (years)	Yield (%)	Modified Duration	Volatility	Term Liabilities (\$ millions)	Current Assets (\$ millions)
1	0.3	0.997	0.30	7.5	65.0
5	1.6	4.921	0.25	43.0	9.0
10	2.1	9.794	0.27	47.0	6.0
30	2.7	29.211	0.24	2.5	20.0

You then perform a Principal Components Analysis (PCA) of the four key points on the U.S. Treasury zero-coupon curve and produce the following results:

Term (years)	Factor 1	Factor 2	Factor 3	% of Variance explained			Total Variance Explained (%)
	β_1	β_2	β_3	Factor 1	Factor 2	Factor 3	
1	0.46	0.77	-0.43	75.7	22.6	1.5	99.8
5	0.52	0.08	0.67	96.1	0.3	3.5	99.9
10	0.53	-0.22	0.23	97.5	1.8	0.4	99.7
30	0.49	-0.59	-0.56	84.2	13.3	2.4	99.9
Eigenvalue $\sigma^2(z_i)$	3.54	0.38	0.08				

- (a) (1.5 points) You plan to proceed with your analysis using only the first two principal components.

Justify this approach.

10. Continued

You proceed by using a two-factor PCA model whereby:

$$z_i = \beta_{1i}R_1 + \beta_{2i}R_2 + \beta_{3i}R_3 + \beta_{4i}R_4 \quad \text{for } i = 1, 2$$

$$x = D * \sigma(dy)P$$

$$\beta_{iP} = x' \beta_i$$

$$\sigma^2(R_p) = \sum (\beta_{iP})^2 \sigma^2(z_i)$$

The Term liabilities and current asset portfolio have the following dollar exposures:

Term (years)	β_1	β_2	Liability Exposure (\$ million)	Asset Exposure (\$ million)
1	0.46	0.77	0.022	0.194
5	0.52	0.08	0.529	0.111
10	0.53	-0.22	1.243	0.159
30	0.49	-0.59	0.175	1.402
Eigenvalue $\sigma^2(z_i)$	3.54	0.38		

- (b) (2 points) The \$100 million term liabilities have the following exposure to each of the two principal components and have an overall portfolio variance as follows:

	β_1 Exposure	β_2 Exposure	Variance
Liability	1.030	-0.317	3.792

Calculate, for the \$100 million current asset portfolio:

- (i) The exposure to each of the two principal components
- (ii) The variance of the current asset portfolio

Show your work.

You then decide to consider the following alternative \$100 million asset portfolios:

- Portfolio A created by allocating equal weights to asset maturities from 1 year through 20 years
- Portfolio B created by allocating only to medium term maturities

Question 10 continued on next page

10. Continued

(e) (3 points) You gather additional values as follows:

	Term Liabilities	Current Portfolio	Alternative Portfolio A	Alternative Portfolio B
Modified Duration	7.52	7.52	7.52	
Asset Portfolio Variance (\$ millions)	3.79		4.01	3.92
10× Surplus Variance	n/a	1.01		0.02
Surplus VaR(99%) (\$ millions)	n/a	0.74	0.13	

Compare the interest rate risk exposure of the current asset portfolio and the two alternative asset Portfolios A and B relative to the Term liabilities with respect to:

- I. Modified duration
- II. Asset Portfolio Variance
- III. Surplus risk

(f) (2 points) Recommend the best asset portfolio structure to manage the interest rate risk exposure of SLIC's Term block. Support your recommendation based on your analysis.

****END OF EXAMINATION****

USE THIS PAGE FOR YOUR SCRATCH WORK