

QFI Core Model Solutions

Fall 2015

1. Learning Objectives:

1. The candidate will understand the fundamentals of stochastic calculus as they apply to option pricing.

Learning Outcomes:

- (1a) Understand and apply concepts of probability and statistics important in mathematical finance.
- (1i) Define and apply the concepts of martingale, market price of risk and measures in single and multiple state variable contexts.
- (1j) Understand and apply Girsanov theorem in changing measures.

Sources:

Neftci, Chapter 5, 6, 15 (has some similarity to Q2 exercises, page 154)

Commentary on Question:

Commentary listed underneath question component.

Solution:

- (a) Calculate:

- (i) $E[S_5 | I_1]$

- (ii) $E[S_5 | I_5]$

Commentary on Question:

Part (a) tests candidates' knowledge of measures in single and multiple state variable contexts

- (i)

$$[S_5 | I_1] = s + \sum_{k=1}^5 E[\Delta_k | I_1] = s + \Delta_1 = S_1$$

where we note that, for instance, $E[\Delta_2 | I_1] = .1 * .75 + (-.3) * .25 = 0$.

1. Continued

(ii)

$$E[S_5 | I_5] = s + \sum_{k=1}^5 E[\Delta_k | I_5] = s + \sum_{k=1}^5 \Delta_k = S_5$$

- (b) Show that the process $M_t = E[S_{t+j} | I_t]$ is a martingale for a fixed and known $j > 0$.

Commentary on Question:

Part (b) tests definition of a martingale.

First show that $\{S_t\}$ is a martingale with respect to a probability measure P and information set $\{I_t\}$ for $t = 1, 2, 3, \dots$ where $I_1 \subseteq I_2 \subseteq I_3 \dots$

By definition we need to prove the following:

- (i) $\{S_t\}$ is I_t -adapted, i.e. given I_t , S_t is known.
- (ii) Unconditional forecasts are finite, i.e., $E|S_t| < \infty$
- (iii) The best forecast of unobserved future values is the last observation, i.e., $E[S_{t+k} | I_t] = S_t$ for any $k > 0$.

Items (i) and (ii) are obvious. For item (iii), we have

$$E[S_{t+k} | I_t] = S_t + \sum_{i=1}^k E[\Delta_{t+i} | I_t] = S_t + \sum_{i=1}^k E[\Delta_{t+i}] = S_t$$

(since Δ_{t+i} are i.i.d. with mean 0 each)

This proves that $\{S_t\}$ is a martingale.

Now since $M_t = E[S_{t+j} | I_t] = S_t$ from (iii) above, we conclude that $\{M_t\}$ is a martingale because $\{S_t\}$ has just been proved to be a martingale.

Alternative solution:

To prove by definition that $\{M_t\}$ is a martingale, similarly as for $\{S_t\}$.

For (iii) as above we calculate:

$$\begin{aligned} E[M_{t+k} | I_t] &= E[E[S_{t+k+j} | I_{t+k}] | I_t] \\ &= E[S_{t+k} | I_t] \\ &= M_t \end{aligned}$$

1. Continued

- (c) Explain why financial assets need not follow a martingale and how to convert them to be a martingale.
- (1) Martingales are random variables whose future movements are completely unpredictable.
 - (2) Financial asset pricing trend is not completely unpredictable. So financial asset price needs not follow a martingale.
 - (3) There are two approaches to convert financial asset prices to be martingales.
 - The first one is so called Doob-Meyer decomposition. This involves making the deviations about the trend in financial asset prices completely unpredictable.
 - The second one is based on Girsanov Theorem. It is to transform the probability distribution of original financial asset price under measure \mathbb{P} to an equivalent martingale measure \mathbb{Q} .

2. Learning Objectives:

1. The candidate will understand the fundamentals of stochastic calculus as they apply to option pricing.

Learning Outcomes:

- (1d) Understand and apply Ito's Lemma.
- (1i) Define and apply the concepts of martingale, market price of risk and measures in single and multiple state variable contexts.
- (1j) Understand and apply Girsanov theorem in changing measures.

Sources:

Neftci, Chapter 6, 8, 10 and 15 (has some similarity to Q2 exercise of Ch. 15)

Commentary on Question:

The question was structured so that candidates were not unfairly punished if they were unable to answer earlier parts of the question, nonetheless, quite a few candidates did not demonstrate a basic understanding of Girsanov theorem and hence failed to achieve a high grade.

This question focused on applying the Ito's Lemma, the characteristics of martingale and Girsanov theorem application. Candidates generally demonstrated a good understanding of Ito's Lemma, fewer candidates were able to correctly prove martingale characteristics, whilst only a handful of candidates demonstrated clear understanding of Girsanov theorem and its application.

Solution:

- (a) Using the Ito's Lemma, show that $d\xi_t = -\alpha\xi_t dW_t$ under the measure \mathbb{P} .

Commentary on Question:

This part was relatively straightforward and most candidates scored the full point, with a few skipping the derivation of each differential calculation and hence not scoring perfectly.

$$\begin{aligned}d\xi_t &= \frac{\partial f}{\partial t} dt + \frac{\partial f}{\partial W_t} dW_t + \frac{1}{2} \frac{\partial^2 f}{\partial^2 W_t} dt \\ &= -\frac{1}{2} \alpha^2 \xi_t dt - \alpha \xi_t dW_t + \frac{1}{2} \alpha^2 \xi_t dt \\ &= -\alpha \xi_t dW_t\end{aligned}$$

2. Continued

- (b) Show that ξ_t is a martingale process under the measure \mathbb{P} based on the definition of a martingale.

Commentary on Question:

This part should have been relatively straightforward, however, many candidates missed the full credit due to failing to mention that $E(|\xi_t|)$ was finite

By integrating part (a) we get

$$\xi_{t+h} = \xi_t - \alpha \int_t^{t+h} \xi_s dW_s$$

It follows that

$$\mathbb{E}^{\mathbb{P}}[\xi_{t+h} | I_t] = \xi_t - \alpha \mathbb{E}^{\mathbb{P}} \left[\int_t^{t+h} \xi_s dW_s \mid I_t \right] = \xi_t$$

Therefore ξ_t is a martingale.

In order to get a full credit, candidate needs to mention $E(|\xi_t|)$ is finite. The integrability of the absolute value of the process follows immediately from: (i) the process is clearly non-negative, and (ii) \mathbb{P} is a probability measure on the filtration I_T

An alternative proof of $\mathbb{E}^{\mathbb{P}}[\xi_{t+h} | I_t] = \xi_t$ exists although no candidate was successful in completing it:

For $s > 0$ consider the conditional expectation

$$\begin{aligned} \mathbb{E}^{\mathbb{P}}[\xi_{t+s} | I_t] &= \mathbb{E}^{\mathbb{P}} \left[e^{-\alpha W_{t+s} - \frac{1}{2}\alpha^2(t+s)} \mid I_t \right] \\ &= e^{-\alpha W_t - \frac{1}{2}\alpha^2(t+s)} \mathbb{E}^{\mathbb{P}}[e^{-\alpha \Delta W_t} \mid I_t] \\ &= e^{-\alpha W_t - \frac{1}{2}\alpha^2(t+s)} e^{\frac{1}{2}\alpha^2 s} = e^{-\alpha W_t - \frac{1}{2}\alpha^2 t} = \xi_t \end{aligned}$$

Here we note that $\Delta W_t = W_{t+s} - W_t$ has a $N(0, s)$ distribution so that using the moment generating function of a normal distribution, we have $\mathbb{E}^{\mathbb{P}}[e^{-\alpha \Delta W_t} \mid I_t] = e^{\frac{1}{2}\alpha^2 s}$

2. Continued

- (c) Calculate the expectation $\mathbb{E}^{\mathbb{Q}}[e^{uB_t}]$ for any constant u .

Commentary on Question:

A lot of candidates went through a complicated yet not 100% accurate calculation to derive the risk-neutral expectation, and confused it with the moment generating function calculation. Some candidates obtained the final answer without proper explanation of the derivation and hence only scored partial credits.

First we note that

$$\mathbb{E}^{\mathbb{Q}}[e^{u(W_t+\alpha t)}] = e^{uat} \mathbb{E}^{\mathbb{Q}}[e^{uW_t}] = e^{uat} \mathbb{E}^{\mathbb{P}}[e^{uW_t}\xi_T].$$

Using law of iterated expectations, we get the last quantity equal to

$$\begin{aligned} e^{uat} \mathbb{E}^{\mathbb{P}}[\mathbb{E}^{\mathbb{P}}[e^{uW_t}\xi_T] | I_t] &= e^{uat} \mathbb{E}^{\mathbb{P}}[e^{uW_t}\mathbb{E}^{\mathbb{P}}[\xi_T | I_t]] \\ &= e^{uat} \mathbb{E}^{\mathbb{P}}[e^{uW_t}\xi_t] \\ &= e^{uat} \mathbb{E}^{\mathbb{P}}\left[e^{(u-\alpha)W_t - \frac{1}{2}\alpha^2 t}\right]. \end{aligned}$$

Finally, we note that W_t has a $N(0, t)$ distribution in \mathbb{P} so that, we have

$$\begin{aligned} \mathbb{E}^{\mathbb{Q}}[e^{u(W_t+\alpha t)}] &= e^{uat} e^{-\frac{1}{2}\alpha^2 t} \mathbb{E}^{\mathbb{P}}[e^{(u-\alpha)W_t}] \\ &= e^{uat} e^{-\frac{1}{2}\alpha^2 t} e^{\frac{1}{2}(u-\alpha)^2 t} \\ &= e^{\frac{1}{2}u^2 t} \end{aligned}$$

- (d) Show that for all $s, t \geq 0$ the increments $B_{t+s} - B_s$ have a normal distribution $N(0, t)$ with mean zero and variance t under the measure \mathbb{Q} .

Commentary on Question:

The answer of this part could follow through part c), or the expectation and variance of the process could be derived directly.

Since $W_{t+s} - W_s$ has the same distribution as $W_t - W_0$ we find that $B_{t+s} - B_s = W_{t+s} + \alpha(t+s) - (W_s + \alpha s) = W_{t+s} - W_s + \alpha t$ has the same distribution as $W_t - W_0 + \alpha t = B_t$.

From part (c) $\mathbb{E}^{\mathbb{Q}}[e^{uB_t}] = e^{\frac{1}{2}u^2 t}$ that is, B_t has the same moment generating function as $N(0, t)$. Thus $B_{t+s} - B_s$ has a normal distribution $N(0, t)$ under the measure \mathbb{Q} .

- (e) Show that B_t is a Wiener process under the measure \mathbb{Q} .

2. Continued

Commentary on Question:

Most candidates failed to answer this part and only a few scored full marks.

This part could be answered using either a simpler or a more tedious derivation, candidates scored high in this part all applied the simpler method. Unlike the more tedious derivation, the simple method does not need to use the intermediate results of (c) and (d), both of which follow fairly easily from the final result (e). Candidates who used the simpler method also received full credits for parts (c) and (d) if they demonstrated how those follow from (e).

Simpler method: Using Girsanov Theorem

Girsanov Theorem states that $B_t = \alpha t + W_t$ is a Wiener process with respect to the measure \mathbb{Q}_1 given by

$\mathbb{Q}_1(A) = \int_A \xi_T^1 d\mathbb{P}$ where $\xi_t^1 = \exp\left(\int_0^t (-\alpha) dW_t - \frac{1}{2} \int_0^t (-\alpha)^2 du\right)$ provided that ξ_t^1 is a martingale with respect to \mathbb{P}

Upon simplifying $\xi_t^1 = \xi_t$ and it implies $\mathbb{Q}_1 = \mathbb{Q}$.

From part (a) ξ_t is a martingale with respect to \mathbb{P} , hence the condition in Girsanov theorem is satisfied and $W_t + \alpha t$ is a Wiener process with the measure \mathbb{Q} .

More tedious method: By definition of Wiener process

- (1) It's clear that B_t is square integrable with $B_0 = 0$ and its trajectories are continuous in t .
- (2) $\mathbb{E}^{\mathbb{Q}}[(B_{t+h} - B_t)^2] = h$ for all $h, t \geq 0$ from Part (d).
- (3) It remains to show that B_t is a martingale $\mathbb{E}^{\mathbb{Q}}[B_{t+h} | I_t] = B_t$. This may be done as below.

First

$$\mathbb{E}^{\mathbb{Q}}[B_{t+h} | I_t] = \frac{\mathbb{E}^{\mathbb{P}}[B_{t+h} \xi_T | I_t]}{\mathbb{E}^{\mathbb{P}}[\xi_T | I_t]}.$$

Second, $\mathbb{E}^{\mathbb{P}}[\xi_T | I_t] = \xi_t$ from part (b).

Finally, $\mathbb{E}^{\mathbb{P}}[B_{t+h} \xi_T | I_t] = \mathbb{E}^{\mathbb{P}}[B_{t+h} \xi_{t+h} | I_t] = B_t \xi_t$ where the first equality is from law of iterated expectations and the second one holds because $B_t \xi_t$ is a martingale under the measure \mathbb{P} as shown below:

$$\begin{aligned} d((W_t + \alpha t)\xi_t) &= \xi_t d(W_t + \alpha t) + (W_t + \alpha t)d\xi_t + \frac{\partial(W_t + \alpha t)}{\partial W_t} \frac{\partial \xi}{\partial W_t} dt \\ &= \xi_t (dW_t + \alpha dt) + (W_t + \alpha t)(-\alpha \xi_t dW_t) + (-\alpha \xi_t) dt \\ &= (1 - \alpha(W_t + \alpha t)) \xi_t dW_t \end{aligned}$$

3. Learning Objectives:

1. The candidate will understand the fundamentals of stochastic calculus as they apply to option pricing.

Learning Outcomes:

- (1a) Understand and apply concepts of probability and statistics important in mathematical finance.
- (1e) Understand and apply Jensen's Inequality.
- (1i) Define and apply the concepts of martingale, market price of risk and measures in single and multiple state variable contexts.
- (1j) Understand and apply Girsanov theorem in changing measures.

Sources:

An Introduction to the Mathematics of Financial Derivatives, Neftci, Salih, 3rd Edition.
Ch. 6

An Introduction to the Mathematics of Financial Derivatives, Neftci, Salih, 3rd Edition.
Ch. 8

An Introduction to the Mathematics of Financial Derivatives, Neftci, Salih, 3rd Edition.
Ch. 14

Frequently Asked Questions in Quantitative Finance, Wilmott, Paul, 2nd Edition Ch 2:
Q23 (p. 103-105)

Commentary on Question:

This question focuses on the concept of Martingale, including application of Jensen's Inequality. Some candidates who understand the concept of Martingale did very well while some other candidates who are not as familiar with the concept struggled to get through c) to e).

Solution:

- (a) Assess whether Statement I is correct.

Commentary on Question:

Candidates did relatively poorly in part a) compare to other parts; one very common error is concluding $S(t)^n$ is a sub-martingale in general, this is not true when n is smaller or equal to 1. Candidates could still get full points without using Jensen's inequality as long as they were able to support their answers with proofs.

3. Continued

Define $\varphi(x) = x^n$

Then

$$E[S^n(t+1)|H_t] = E[\varphi(S(t+1))|H_t]$$

Using Jensen's inequality,

$$E[\varphi(S(t+1))|H_t] \geq \varphi(E[S(t+1)|H_t]) = \varphi(S(t)) = S(t)^n$$

for $n > 1$

$$E[\varphi(S(t+1))|H_t] \leq \varphi(E[S(t+1)|H_t]) = \varphi(S(t)) = S(t)^n$$

for $n < 1$

$S(t)^n$ is a sub-martingale when $n > 1$. It is a martingale when $n = 1$ and a super-martingale when $n < 1$. Therefore statement I is not correct.

- (b) Assess whether Statement II is correct.

Commentary on Question:

Candidates did fairly well in this question. Candidates who set up the correct formula but arrived at the wrong conclusion due to computational errors received partial points.

$$\begin{aligned} E[W_{t+1}^2 | I_t] &= E[\{W_t + (W_{t+1} - W_t)\}^2 | I_t] \\ &= E[(W_t^2 + 2W_t(W_{t+1} - W_t) + (W_{t+1} - W_t)^2) | I_t] \\ &= W_t^2 + 2W_t E[(W_{t+1} - W_t)] + E[(W_{t+1} - W_t)^2] \\ &= W_t^2 + 2W_t \times 0 + 1 \\ E[W_{t+1}^2 | I_t] &= W_t^2 + 1 \end{aligned}$$

The above proves that statement II is correct.

- (c) Derive an expression for $E[E[W_3^2 | I_2] | I_1]$ in terms of W_1 .

Commentary on Question:

Most candidates did well in this part. Some candidates relied on the proof from b); they would receive full score if they actually proved b), but they lost points if they concluded b) was incorrect but assumed b) was correct in c), because they were relying on something they considered incorrect.

$$\begin{aligned} E[E[W_3^2 | I_2] | I_1] &= E[W_2^2 + 1 | I_1] \\ E[E[W_3^2 | I_2] | I_1] &= E[W_1^2 + 1 | I_1] + 1 \\ E[E[W_3^2 | I_2] | I_1] &= W_1^2 + 1 + 1 \\ E[E[W_3^2 | I_2] | I_1] &= W_1^2 + 2 \end{aligned}$$

3. Continued

- (d) Construct a non-constant quadratic function of W_t and t that is a martingale.

Commentary on Question:

Most candidates who did well in previous parts did well in this part. The question asked for a quadratic function, so candidates who provided a non-quadratic function did not receive points.

From part (b)

$$E[W_{t+1}^2 | (I_t)] = W_t^2 + 1$$

$$E[W_{t+1}^2 - (t + 1) | (I_t)] = W_t^2 - t$$

Alternative solution:

$$d[W_t^2] = 2W_t dW_t + \frac{1}{2} 2 dt$$

$$W_t^2 - t = \int_0^t 2W_s dW_s$$

Which is a martingale since the stochastic integral is a martingale.

So $f[W_t] = W_t^2 - t$ is a martingale

- (e) Assess whether Statement III is correct.

Commentary on Question:

Candidates did relatively poor in this part compare to other parts. Many candidates loss points because they only provided verbal explanation but failed to prove M_n Martingale by showing $E[M_{n+1} | F_n] = M_n$

$$M_{n+1} = (A_n + 1)/(A_n + 1 + B_n) \text{ with probability } A_n/(A_n + B_n)$$

$$M_{n+1} = A_n/(A_n + 1 + B_n) \text{ with probability } B_n/(A_n + B_n)$$

$$E[M_{n+1} | F_n] = (A_n + 1)/(A_n + 1 + B_n) * A_n/(A_n + B_n) + A_n/(A_n + 1 + B_n) * B_n/(A_n + B_n)$$

$$E[M_{n+1} | F_n] = A_n/(A_n + B_n) = M_n$$

The above proved that M_n is a Martingale and therefore statement III is correct.

4. Learning Objectives:

1. The candidate will understand the fundamentals of stochastic calculus as they apply to option pricing.
2. The candidate will understand how to apply the fundamental theory underlying the standard models for pricing financial derivatives. The candidate will understand the implications for option pricing when markets do not satisfy the common assumptions used in option pricing theory such as market completeness, bounded variation, perfect liquidity, etc. The Candidate will understand how to evaluate situations associated with derivatives and hedging activities.

Learning Outcomes:

- (1f) Demonstrate understanding of option pricing techniques and theory for equity and interest rate derivatives.
- (1k) Understand the Black Scholes Merton PDE (partial differential equation).
- (2d) Understand the different approaches to hedging.
- (2e) Understand how to delta hedge and the interplay between hedging assumptions and hedging outcomes.

Sources:

Intro Quantitative Finance Wilmott, 2nd Ed. Ch 8-10,26
Nefci Ch.3, 13

Commentary on Question:

This question tests the candidates understanding for BS PDE. Most candidates did well on the question. There are multiple ways to derive the solutions for the sections.

Solution:

- (a) Calculate the amount to be borrowed or lent for the Delta-neutral strategy at its inception.

Commentary on Question:

Some candidates got the calculation right but didn't correctly identify the action which is to borrow cash.

Delta neutral portfolio for the call option sold
= $\Delta \cdot S - \text{Cash} = 0.6 \cdot 1000 - \text{Cash} = \text{Option value} = 100$
The cash to borrow = 500.

4. Continued

- (b) Derive the following Black-Scholes PDE using the Delta-neutral riskless hedge:

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS \frac{\partial V}{\partial S} - rV = 0$$

Commentary on Question:

Most candidates did well on this part.

The delta neutral portfolio is constructed as:

$$\Pi = V(S, t) - \Delta S$$

The increment of the portfolio over time is

$$d\Pi = dV - \Delta dS$$

and it grows at risk free rate

$$= r\Pi dt.$$

Applying Ito's Lemma for option having underlying asset following geometric Brownian motion

$$dV = \frac{\partial V}{\partial t} dt + \frac{\partial V}{\partial S} dS + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} dt$$

By the definition of delta

$$\Delta = \frac{\partial V}{\partial S} \quad \text{and}$$

$$\begin{aligned} d\Pi &= \left(\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} \right) dt \\ \left(\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} \right) dt &= r \left(V - S \frac{\partial V}{\partial S} \right) dt. \\ \frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS \frac{\partial V}{\partial S} - rV &= 0. \end{aligned}$$

- (c) Calculate the Theta of the option from the PDE using the option value, Delta, and Gamma.

From the Ito's Lemma and geometric Brownian motion,

$$\begin{aligned} \frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS \frac{\partial V}{\partial S} - rV &= 0 \\ \Delta &= \frac{\partial V}{\partial S} \\ \Gamma &= \frac{\partial^2 V}{\partial S^2} \\ \Theta &= \frac{\partial V}{\partial t} \end{aligned}$$

4. Continued

$$\Theta = rV - rS\Delta - \frac{1}{2}\sigma^2 S^2 \Gamma = r(V - S\Delta) - \frac{1}{2}\sigma^2 S^2 \Gamma.$$

$$\text{Theta} = 0.02 * 100 - 0.02 * 1000 * 0.6 - 0.5 * 0.2^2 * 1000^2 * 0.002 = -50.$$

- (d) Explain the benefits of making the dynamic hedge Gamma-neutral.
- Measure the change of Delta by the change of underlying assets
 - By taking Gamma position, less frequent trading for Delta change can be achieved.
 - Gamma implies the cost of re-hedge.
 - Measuring convexity of the option.
- (e) Calculate the P&L of the Delta-neutral hedging for the next time period assuming a finite time step of $dt = 0.01$.

Because the call option is sold, the P&L sign of the formula should be changed to interpret. P&L

$$= \frac{1}{2} (\sigma^2 - \tilde{\sigma}^2) S^2 \Gamma^i dt.$$

$$= -0.5 * (0.22^2 - 0.2^2) * 1000^2 * 0.002 * 0.01 = -0.084.$$

Thus loss occurred because the option was sold for less than the actual price.

5. Learning Objectives:

3. The candidate will understand the basic concepts underlying interest rate option pricing models

Learning Outcomes:

- (3a) Demonstrate understanding of interest rate models.
- (3c) Understand the HJM model and the HJM no-arbitrage condition.

Sources:

Quantitative Finance, Wilmott, Paul, 2nd Edition Ch. 17, 19

An Introduction to the Mathematics of Financial Derivatives, Neftci, Salih, 3rd Edition. Ch. 3, 19

Commentary on Question:

This question tested a candidate's understanding of the HJM model. Most candidates did not do well, particularly in the latter parts.

Solution:

- (a) Show that the absence of the arbitrage condition of the HJM model is satisfied in this equation.

$m(t, T)$ = drift of the risk neutral forward rate curve

volatility = $v(t, s) = .01e^{-(s-t)}$

$$\begin{aligned} m(t, T) &= .0001(e^{-(T-t)} - e^{-2(T-t)}) = .01e^{-(T-t)} \int_t^T .01e^{-(s-t)} ds \\ &= v(t, T) \int_t^T v(t, s) ds \end{aligned}$$

So, the HJM condition is satisfied.

- (b) Derive $F(t, T)$ by solving the above equation.

$$\begin{aligned} F(t, T) &= F(0, T) + \int_0^t dF(s, T) \\ F(t, T) &= F(0, T) + .0001 \int_0^t (e^{-(T-s)} - e^{-2(T-s)}) ds + .01 \int_0^t e^{-(T-s)} dW_s \\ F(t, T) &= F(0, T) + .0001 \left(e^{-(T-t)} - e^{-T} - \frac{e^{-2(T-t)}}{2} + \frac{e^{-2T}}{2} \right) \\ &\quad + .01 \int_0^t e^{-(T-s)} dW_s \end{aligned}$$

5. Continued

- (d) Derive an expression for dr_t in terms of $F(0,t)$, $F_t(0,t)$, t , dt , and dW_t .

Commentary on Question:

Most candidates did not answer this question well. Note that the product rule needs to be applied when differentiating the answer in part c.

$$\begin{aligned}
 r_t &= F(0,t) + 0.0001\left(\frac{1}{2} - e^{-t} + \frac{1}{2}e^{-2t}\right) + 0.01e^{-t} \int_0^t e^s dW_s \\
 dr_t &= \left[F_t(0,t) + 0.0001(e^{-t} - e^{-2t}) \right] dt + 0.01 \left[-e^{-t} \left(\int_0^t e^s dW_s \right) dt + dW_t \right] \\
 &= \left[F_t(0,t) + 0.0001(e^{-t} - e^{-2t}) - 0.01e^{-t} \int_0^t e^s dW_s \right] dt + 0.01dW_t \\
 -0.01e^{-t} \int_0^t e^s dW_s &= F(0,t) + 0.0001\left(\frac{1}{2} - e^{-t} + \frac{1}{2}e^{-2t}\right) - r_t \\
 dr_t &= \left[F_t(0,t) + 0.0001(e^{-t} - e^{-2t}) + F(0,t) + 0.0001\left(\frac{1}{2} - e^{-t} + \frac{1}{2}e^{-2t}\right) - r_t \right] dt + 0.01dW_t \\
 &= \left[F_t(0,t) + F(0,t) + 0.0001\left(\frac{1}{2} - \frac{1}{2}e^{-2t}\right) - r_t \right] dt + 0.01dW_t
 \end{aligned}$$

- (e) List the challenges of implementing the HJM model.

The process followed by the short rate is in general non-Markov, meaning that the interest rate process is path-dependent. Monte Carlo simulation has to be used to simulate the process for the short rate in the HJM model

It is difficult to use a tree to represent the term structure movements because the tree is usually non-recombining

The model is expressed in terms of the instantaneous forward rates $F(t, T)$ and these are not directly observable in the market

It is difficult to calibrate the model to prices of actively traded instruments.

6. Learning Objectives:

2. The candidate will understand how to apply the fundamental theory underlying the standard models for pricing financial derivatives. The candidate will understand the implications for option pricing when markets do not satisfy the common assumptions used in option pricing theory such as market completeness, bounded variation, perfect liquidity, etc. The Candidate will understand how to evaluate situations associated with derivatives and hedging activities.
4. The candidate will understand the concept of volatility and some basic models of it.

Learning Outcomes:

- (2a) Identify limitations of the Black-Scholes pricing formula
- (2b) Compare and contrast the various kinds of volatility, (e.g., actual, realized, implied, forward, etc.).
- (2c) Compare and contrast various approaches for setting volatility assumptions in hedging.
- (4a) Compare and contrast the various kinds of volatility, (e.g., actual, realized, implied, forward, etc.).

Sources:

QFIC-103-13

Chapter 9, Risk Management and Financial Institutions, Hull, 2nd Edition

Commentary on Question:

Commentary listed underneath question component.

Solution:

- (a) Calculate an estimate of the annualized volatility (σ) and its standard error.

Commentary on Question:

This question draws on an example presented in Hull's book. Candidates were expected to calculate the annualized standard deviation and the standard error based on given information. Candidates were awarded partial credit for using formula with slight mistakes. Candidates generally did not do well in this part. Most candidates could not recall the correct formulas and/or did not apply them correctly. Many candidates answered the first part of the question (annualized volatility) but omitted the second part (standard error).

6. Continued

$$n = 30$$

There are 31 trading days therefore there are 30 data points for change in prices.

$$s = \sqrt{\frac{0.005235}{30 - 1} - \frac{-0.142511^2}{30(30 - 1)}} = 0.013427$$

$$\begin{aligned} \text{Annualized volatility} \\ &= \text{Annualized } S \\ &= 0.013427\sqrt{255} = 0.2144 \end{aligned}$$

$$\text{Standard error} = \frac{0.2144}{\sqrt{2 \times 30}} = 0.0277$$

- (b) Recommend two trading strategies for Bank ABC based on its belief.

Commentary on Question:

Candidates generally did well in this part. Full credit was given for providing any two strategies that might profit from realized volatility being greater than the current market implied volatility.

Two strategies that Bank ABC can take are:

1. Buy out-of-the-money put options or sell out-of-the-money call options. As volatility increases, out-of-the-money option prices will increase more than in-the-money option prices.
 2. Buy a straddle – buy a call and a put at the same strike price. As volatility increases, prices for both calls and puts increase.
 3. Any other strategies that might profit from realized volatility being greater than the current market implied volatility.
- (c) Describe reasons for this relationship of implied volatilities.

Commentary on Question:

Most candidates received partial credit for this part, but some received full credit.

Options with strikes far away from current price tend to have higher volatility, which is known as a volatility smile.

6. Continued

Black Scholes model assumptions are violated in the market and the option prices are not purely based on BS framework. The assumptions violated include:

- Volatility is constant
- There are no jumps in stock prices
- There are no rehedging or rebalancing costs
- There are no transaction costs

Skew indicates that the returns from equity indices are not exactly lognormal, with a larger tail on the downside or a risk of jumps. The equity market has the tendency to fall with faster speed than it rallies.

Option prices and volatility are also affected by supply and demand in the market. In particular, investors use out-of-the-money options as insurance against market crashes and are willing to pay a premium on them.

7. Learning Objectives:

1. The candidate will understand the fundamentals of stochastic calculus as they apply to option pricing.

Learning Outcomes:

- (1a) Understand and apply concepts of probability and statistics important in mathematical finance.
- (1b) Understand the importance of the no-arbitrage condition in asset pricing.
- (1f) Demonstrate understanding of option pricing techniques and theory for equity and interest rate derivatives.
- (1h) Demonstrate understanding of the differences and implications of real-world versus risk-neutral probability measures.
- (1i) Define and apply the concepts of martingale, market price of risk and measures in single and multiple state variable contexts.
- (1j) Understand and apply Girsanov theorem in changing measures.

Sources:

An Introduction to the Mathematics of Financial Derivatives, Neftci, Salih, 3rd Edition, Chapter 2, 6, 17-19

Commentary on Question:

This question tested candidates' understanding of the interest rate derivative pricing (interest swap in this case).

To receive maximum points, candidates needed to show understanding of interest swap pay-off, the difference of measures in pricing (RN versus Forward), be familiar with the concept of a martingale and apply it in changing measures.

Overall candidates did poorly on the entire question. Most of the candidates can describe the different measures to obtain partial points in c) and provide high level description to obtain partial points in d). Candidates who understood the change of measures did well in a) and b). None of the candidates could fully answer question e)..

Solution:

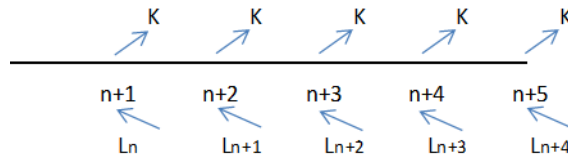
- (a) Derive an expression for the fair strike K^* of the 0×5 swap at time 0 in terms of the expected value of the LIBOR rates using the standard risk-neutral measure.

7. Continued

Commentary on Question:

Around half of the candidates could at least provide payoff of the interest swap and set equation to 0 to obtain partial scores. Only a few candidates knew that discount rate is based on LIBOR and under RN measure it cannot be further simplified.

The payoff for the interest rate swap is:



where the LIBOR rate L_t is known at time t and paid at time $t+1$
the Value of the swap is then

$$\begin{aligned} V_0 &= E_0^Q \left[\sum_{n=1}^5 \frac{1}{\prod_{i=1}^n (1 + L_{i-1})} (L_{n-1} - K) \right] = \\ &= \sum_{n=1}^5 E_0^Q \left[\frac{1}{\prod_{i=1}^n (1 + L_{i-1})} (L_{n-1} - K) \right] = \\ &= \sum_{n=1}^5 E_0^Q \left[\frac{L_{n-1}}{\prod_{i=1}^n (1 + L_{i-1})} \right] - K \sum_{n=1}^5 E_0^Q \left[\frac{1}{\prod_{i=1}^n (1 + L_{i-1})} \right] \end{aligned}$$

Note that the discount rate is based on LIBOR.

The fair strike value is that $V = 0$ (interest swap value = 0):

$$K^* = \frac{\sum_{n=1}^5 E_0^Q \left[\frac{L_{n-1}}{\prod_{i=1}^n (1 + L_{i-1})} \right]}{\sum_{n=1}^5 E_0^Q \left[\frac{1}{\prod_{i=1}^n (1 + L_{i-1})} \right]}$$

where under Risk-Neutral measure, the discount factor cannot be taken out of the expectation sign and $E_0^Q[L_n] \neq F_n$

- (b) Derive an expression for the fair strike K^* of the 0×5 swap at time 0 in terms of bond prices and FRAs using the n -period forward measures ($n = 1, 2, 3, 4, 5$).

Commentary on Question:

Candidates who did well in a) also did well in this question.

7. Continued

Per (i), the value for the swap under the risk-neutral measure is:

$$V_0 = \sum_{n=1}^5 E_0^Q \left[\frac{1}{\prod_{i=1}^n (1 + L_{i-1})} (L_{n-1} - K) \right]$$

under the Forward measure, the value of the swap change to:

$$V_0 = \sum_{n=1}^5 E_0^{F_{n-1}} \left[(L_{n-1} - K) \times \frac{p(0, n)}{p(n, n)} \right]$$

where $p(0, n)/p(n, n)$ is the same as $p(0, n)$ (since by definition $p(n, n)=1$) and $p(0, n)$ is market price known at time 0, therefore it can be taken out of the expectation sign:

$$V_0 = \sum_{n=1}^5 p(0, n) \times E_0^{F_{n-1}} (L_{n-1} - K)$$

In additional, under forward measure

$$E_0^{F_n} [L_n] = F_n$$

where the forward rate F_t is the fair strike fixed at time t and pays in arrears at time $t+1$

Hence, the fair strike value for the interest swap is when the swap value is 0:

$$V_0 = 0 = \sum_{n=1}^5 p(0, n) \times (F_{n-1} - K^*)$$

To solve the equation,

$$K^* = \frac{\sum_{n=1}^5 p(0, n) \times F_{n-1}}{\sum_{n=1}^5 p(0, n)}$$

(c)

- (i) Describe the differences between the two approaches in (a) and (b).
- (ii) Specify, for each approach, the asset used for normalization for asset pricing purposes. Assume that interest rates are deterministic.

7. Continued

Commentary on Question:

Most candidates could write down something from the text book to obtain partial mark, even though candidates might not fully understand the material to apply it mathematically.

Asset used for normalization (numeraire) in the first case is the savings account; for the second it is the ZCB.

The first approach does not allow a simple expression since the numerator for K^* has expectation, which involves 2 possibly correlated entities. The second approach simplifies this expectation as the discounting is done outside.

The second approach under forward measure is an unbiased estimate and the discount factor is a martingale and therefore can be taken out of the expectation.

- (d) Show how the 0×5 interest rate swap struck at K can be valued using the prices of a default-free fixed-coupon bond and a floating-coupon bond (coupons paid annually).

Commentary on Question:

Most of the candidates only provided a high level description but did not provide breakdown view of the payoffs into a set of ZCBs, nor included the face amount in the breakdown.

As question i) showed, the swap pay off can be replicated in terms of ZCBs (receive 5 ZCBs that mature with face value equal to the LIBORs, and pay 5 ZCBs that with flat face value K)

The value of the swap is then:

$$\begin{aligned}V_0 &= E_0^Q \left[\sum_{i=1}^5 (L_{n-1} - K) \times p(i) \right] = \\&= E_0^Q \left[p(5) - p(5) + \sum_{i=1}^5 (L_{n-1} \times p(i) - K \times p(i)) \right] = \\&= E_0^Q \left[p(5) + \sum_{i=1}^5 (L_{n-1} \times p(i)) - p(5) - \sum_{i=1}^5 (K \times p(i)) \right] = \\&= E_0^Q \left[p(5) + \sum_{i=1}^5 (L_{n-1} \times p(i)) \right] - E_0^Q \left[p(5) + \sum_{i=1}^5 (K \times p(i)) \right]\end{aligned}$$

where $p(5)$ is the value of a ZCB that matures at time 5 with face value equal to 1. The first expression is the price of the floating - coupon bond (principal + coupons)

The second expression is the price of the fixed - coupon bond (principal + coupons)

7. Continued

- (e) Show that the Black-76 method can be used to value the swaption with strike K if one were to assume that the fair strike of the $t \times 3$ constant-maturity swap follows a log-normal process.

Hint: Normalize asset prices by $A_t(2,3)$.

Commentary on Question:

Candidates did poorly in this question, none of the candidates obtained full mark.

Let the fair strike of the 2x3 swap at time t ($0 \leq t \leq 2$) be N_t^* , then the fair strike of the 2x3 swap at time 2 is N_2^*

Let $A_0(2,3)$ today's value for this instrument, then $A_2(2,3)$ = the value at time 2 for this instrument.

$$A_2(2,3) = E_2^Q \left[\sum_{n=3}^5 \frac{1}{\prod_{i=3}^n (1+L_{i-1})} \right]$$

Under the Risk-neutral measure, the value of the swap at time 0 is:

$$V_0 = E_0^Q \left[\frac{1}{\prod_{i=1}^2 (1+L_{i-1})} \times (N_2^* - K)^+ \times E_2^Q \left[\sum_{n=3}^5 \frac{1}{\prod_{i=3}^n (1+L_{i-1})} \right] \right]$$

We simplify the calculation by change of measure to normalize the asset using the $A(2,3)$ annuity.

Since the market is complete, by theory, there exists a measure $A(p,m)$ in which all assets normalized by $A(p,m)$ is a martingale:

$$V_0 = A_0(2,3) \times E_0^{A(2,3)} \left[\frac{(N_2^* - K)^+ \times E_2^Q \left[\sum_{n=3}^5 \frac{1}{\prod_{i=3}^n (1+L_{i-1})} \right]}{A_2(2,3)} \right]$$

$$V_0 = A_0(2,3) \times E_0^A [(N_2^* - K)^+]$$

Where $A_0(2,3)$ is known and $E_0^A [(N_2^* - K)^+]$ can be valued using Black formula.

8. Learning Objectives:

4. The candidate will understand the concept of volatility and some basic models of it.

Learning Outcomes:

- (4b) Understand and apply various techniques for analyzing conditional heteroscedastic models including ARCH and GARCH.

Sources:

Analysis of Financial Time Series, Tsay, 3rd Edition - Ch 3 Conditional Heteroscedastic Models (3.1–3.8)

Commentary on Question:

Commentary listed underneath question component.

Solution:

- (a) Critique the use of an ARCH(2) model to predict future XYZ stock returns, based on the above information.

Commentary on Question:

The candidates did very badly in this question. Most of them did not explain the situation of the stock movement and just listed ARCH characteristics. Only partial points were given. Some of them used the data in part (b) (Ljung-Box test) to answer this part. No points were given in that case.

Since the stock price of XYZ experienced periods of substantial increasing and decreasing shocks. XYZ history: volatility clustering is seen – high volatility for certain times and low for others, as well as extended period where there was not much fluctuations in prices,

Two major issues:

1. ARCH (2) assumes that positive and negative shocks have the same effects on volatility, however, stock prices volatility tend to decline as the stock price goes up and increase as the stock price falls.
 2. ARCH (2) tends to overestimate the volatility because the fitted model assumes an ever-increasing estimated volatility from the model.
- (b) Define Null Hypotheses for the Ljung-Box test and conclude whether ARCH(2) is appropriate at the 5% significance level using the test results of Table 1 and Table 2.

8. Continued

Commentary on Question:

No candidate got full credit. Some candidates could not answer the null hypothesis correctly. Though most answered ARCH(2) is appropriate, some of them did not mention the significance level (whether 1% or 5%). Only partial credits would be given.

The Null hypotheses of the Ljung-Box test in this case is:
(null: first m lags of ACF is 0, Tsay pp 114)

The standardized residuals of the ARCH (2) model are independent (are not serially correlated)

The squared standardized residuals of the ARCH (2) model are independent (are not serially correlated).

The p-value for α_2 is greater than 0.05 hence fails to reject α_2 is significantly different from 0, and

The p-value for $\tilde{\alpha}_t^2$ is less than 0.05 hence reject that the null (i.e. squared standardized residuals are serially correlated)

Hence based on the information above, the ARCH (2) model is not appropriate in this case as squared standardized residuals show serial correlation at 5% significance level

Alternatively:

The p-value for α_2 is greater than 0.01 hence fails to reject α_2 is significantly different from 0, and

The p-value for $\tilde{\alpha}_t^2$ is less than 0.01 hence fails to reject the standardized

Hence based on the information above, the ARCH (2) model is appropriate at 1% significance level.

- (c) Critique your colleague's suggestion.

Commentary on Question:

Most candidates answered EGARCH was better, but they only gave 1 good characteristic of EGARCH. Only a few mentioned there was a leverage effect. So only partial credits were given.

EGARCH or GARCH model would be a better choice as it exhibits almost all the major characteristics of asset returns, namely volatility clustering, volatility jumps are rare and volatility does not diverge to infinity, it varies within some fixed range.

8. Continued

With EGARCH, the leverage effect is better reflected. (allow asymmetric effects between positive and negative returns.

- (d) Assess the correctness of the above three statements based on Table 3 as applicable.

Commentary on Question:

(1) Most answered the statement is correct.

(2) Most mentioned kurtosis and that the tail distribution of α_t is fatter than a normal distribution, and hence it is more likely to generate outliers..

(3) Almost all candidate mentioned the statement was not correct but they did not give any reasons.

Statement 1: This statement is correct, and $\text{Var}(\alpha_t) = \alpha_0/(1 - \alpha_1)$

Statement 2: This statement is incorrect, since the unconditional kurtosis of α_t is greater than 3, which suggests that the tail distribution of α_t is fatter than a normal distribution, and hence it is more likely to generate outliers.

Statement 3: This statement is incorrect. For ARCH (1) model, there are some restrictions on the value of α_1 , which is to have a finite fourth moment, it must satisfy: $0 \leq \alpha_1^2 < 1/3$.

In this case, $\alpha_1^2 = 0.4624 > 1/3$, which suggests that the fitted model would not give finite 4th moments.

9. Learning Objectives:

5. The candidate will understand and identify the variety of fixed instruments available for portfolio management. This section deals with fixed income securities. As the name implies the cash flow is often predictable, however there are various risks that affect cash flows of these instruments. In general the candidates should be able to identify the cash flow pattern and the factors affecting cash flow for commonly available fixed income securities. Candidates should also be comfortable using various interest rate risk quantification measures in the valuation and managing of investment portfolios.

Learning Outcomes:

- (5c) Demonstrate understanding of the different characteristics of securities issued by government agencies.
- (5e) Describe the cash flow of various corporate bonds considering underlying risks such as interest rate, credit and event risk.
- (5g) Demonstrate understanding of cash flow pattern and underlying drivers and risks of mortgage-backed securities and collateralized mortgage obligations.

Sources:

An Overview of Mortgages and Mortgage Market, Fabozzi Handbook, Ch. 24

Agency Mortgage Backed Securities, Fabozzi Handbook, Ch. 25

Agency Collateralized Mortgage Obligations, Fabozzi Handbook, Ch. 26

Corporate Bonds, Fabozzi Handbook, Ch. 12

Managing Investment Portfolios: A Dynamic Process, Maginn & Tuttle, 3rd Edition Ch. 6

Commentary on Question:

This question requires application of MBS and Treasuries knowledge in real world situations. Most candidates did fairly well except for part (c). Partial credits were given for reasonable explanation.

Solution:

- (a) Explain how convexity impacts the difference of yields between agency MBS and U.S. Treasuries with the same duration.

Agency MBS exhibits negative convexity, while U.S. Treasuries exhibit positive convexity. When interest rates drop, agency MBS receives more prepayment and faces reinvestment risk at a lower interest rate level. To compensate for this prepayment risk, agency MBS demands a higher yield.

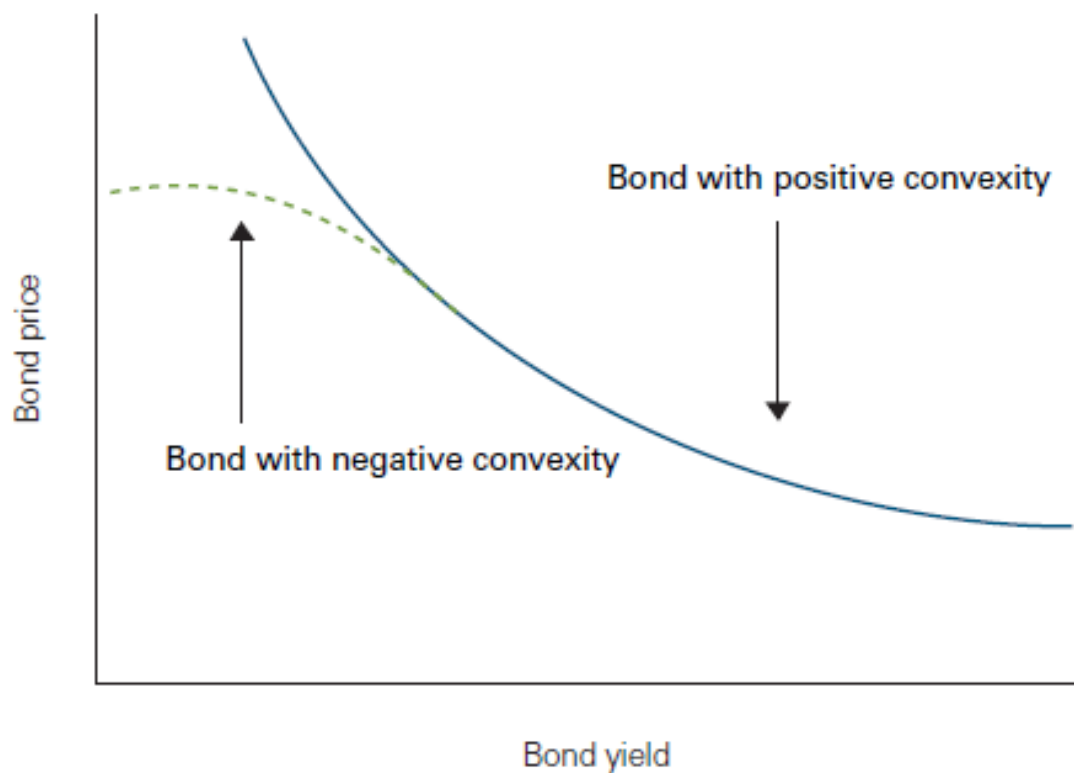
9. Continued

- (b) Explain why GNMA Bond's duration (dotted line) has been more volatile than U.S. Treasury's (solid line).

When interest rates decreased from October 2007 to late 2008, prepayment activity increased and shortened the GNMA portfolio duration. When interest rates slowly rose from their low values in late 2008, prepayment activity slowed and the duration of the GNMA portfolio increased.

During this entire period, because the U.S. 3-5 year Treasury bond index is not exposed to prepayment activity, its duration was fairly stable.

- (c) Draw a chart showing the shape of the price-yield relationship curve for each of the indices. State clearly titles of both axes of the chart.



Dotted Line – GNMA
Solid Line – U.S. Treasury

9. Continued

- (d) Explain the shape of the price-yield relationship curve in part (c).

Commentary on Question:

A general comment on a downward sloping graph was not enough for full marks on this test question. An explanation of the shape in terms of why it displays the shape it does was more important.

Both curves exhibit a typical negative price-yield relationship, where price drops as yield rises. U.S. Treasuries exhibit positive convexity where a small decrease in yield results in a higher price change than what would be experienced from an equal size increase in yields.

At low yields, GNMA bonds experience negative convexity. When yields decrease, there is a greater risk of prepayments because borrowers can refinance at lower interest rates. At some higher yields, the GNMA bond behaves like a typical bond with positive convexity as there is little to no incentive to refinance.

- (e) Design a hedge strategy to maintain a stable duration of a bond portfolio which is invested in GNMA bonds.

Commentary on Question:

Other hedging proposals, besides those described below, could be valid.

To hedge the fluctuation of the duration of the bond portfolio that tracks the U.S. GNMA Bond Index, you can choose to use Treasury futures or interest rate swaps.

If the duration of the bond portfolio decreases below the target level due to a decline of interest rates, BUY Treasury futures, or enter to RECEIVE FIXED interest rate swaps to extend the duration.

If the duration of the bond portfolio rises above the target level due to the rise of interest rates, SELL Treasury futures, or enter to PAY FIXED interest rate swaps to shorten the duration.

To maintain the target level, dynamic rebalancing is needed.

10. Learning Objectives:

5. The candidate will understand and identify the variety of fixed instruments available for portfolio management. This section deals with fixed income securities. As the name implies the cash flow is often predictable, however there are various risks that affect cash flows of these instruments. In general the candidates should be able to identify the cash flow pattern and the factors affecting cash flow for commonly available fixed income securities. Candidates should also be comfortable using various interest rate risk quantification measures in the valuation and managing of investment portfolios.

Learning Outcomes:

- (5e) Describe the cash flow of various corporate bonds considering underlying risks such as interest rate, credit and event risk.
- (5g) Demonstrate understanding of cash flow pattern and underlying drivers and risks of mortgage-backed securities and collateralized mortgage obligations.

Sources:

An Overview of Mortgages and Mortgage Market, Fabozzi Handbook, Ch. 24

Agency Mortgage Backed Securities, Fabozzi Handbook, Ch. 25

Agency Collateralized Mortgage Obligations, Fabozzi Handbook, Ch. 26

Commentary on Question:

Overall and on a relative basis, it was an easy question. Partial credits for similar or reasonable explanation were given.

Solution:

- (a) Analyze the impact of a falling interest rate environment on the IO tranches and PO tranches.

A PO strip represents all the principal payments and an IO strip represents all the interest payments. The interest rate sensitivity of IO strips and PO strips is determined on the rate of prepayment, which in turn primarily depends on the interest rate.

Falling interest rate environment:

PO strips:

- 1) When the interest rate drops, the prepayment accelerates.
- 2) Since POs are priced at deep discount, the value of POs increases as the prepayment accelerates.

10. Continued

- 3) Combining the effects of a lower interest rate (discounting effect) and a high prepayment speed (prepayment effect), the value of POs is more sensitive to the interest rate than the mortgage pass-through.

IO Strips:

- 1) When interest rate drops, the discounting effect causes the value of interest payments to rise.
 - 2) Meanwhile, declining rate increases the prepayment rate, cutting short the expected stream of future interest payments, and thus reducing the value of the IO strip.
 - 3) If prepayment effect is large enough, it can outweigh the discounting effect, causing the value of IO strip to fall.
- (b) Explain how IO tranches can be used to hedge interest rate risk of a portfolio that contains ordinary fixed income securities.

IO strips typically have negative duration.

When interest rate rises, the increase in the value of IO strips can balance the decrease in the value of other ordinary fixed income securities in the portfolio.

When interest rate falls, the decrease in the value of IO strips can balance the increase in the value of other ordinary fixed income securities in the portfolio.

The right proportion of IO strips in the portfolio can insulate the value of the combined portfolio from interest rate changes.

- (c) Explain how PO tranches can be used to hedge interest rate risk of fixed income liabilities.

Because the value of the fixed income securities typically move inversely with interest rates, the debt burden of companies that have issued fixed income securities also move inversely with interest rates.

PO strips typically have long, positive duration.

If the company holds PO as hedge assets in the right proportion, then the changes in the debt burden induced by the changes of interest rates can be offset (or mitigated) to some extent by the changes of the values of the PO strip assets.

11. Learning Objectives:

5. The candidate will understand and identify the variety of fixed instruments available for portfolio management. This section deals with fixed income securities. As the name implies the cash flow is often predictable, however there are various risks that affect cash flows of these instruments. In general the candidates should be able to identify the cash flow pattern and the factors affecting cash flow for commonly available fixed income securities. Candidates should also be comfortable using various interest rate risk quantification measures in the valuation and managing of investment portfolios.

Learning Outcomes:

- (5h) Construct and manage portfolios of fixed income securities using the following broad categories.
- (i) Managing funds against a target return
 - (ii) Managing funds against liabilities.

Sources:

Managing Investment Portfolios: A Dynamic Process, Maginn & Tuttle, 3rd Edition

- Ch. 6, Fixed Income Portfolio Management

Commentary on Question:

Overall most candidates did fairly well on this question.

Solution:

- (a) Define tracking risk.

Commentary on Question:

Most candidates received full credit for this question.

Tracking risk is defined as the standard deviation of the portfolio's active return, where the active return for each period is defined as the difference between the portfolio's return and the benchmark index's return.

Tracking risk or tracking error is a measure of the variability with which a portfolio's return tracks the return of a benchmark index.

Active return = Portfolio's return – Benchmark index's return.

Tracking risk = standard deviation of the active return

- (b) List and describe risk factors that should be managed to minimize tracking risk.

Commentary on Question:

Candidates were given full credit for listing and explaining the risk factors.

11. Continued

Portfolio duration

If the difference between the portfolio duration (i.e. parallel shifts in interest rates) and benchmark duration increases, the tracking risk will increase.

Key rate duration and present value distribution of cash flows

These two strategies capture nonparallel shifts in the yield curve.

Mismatches in the key rate duration will increase tracking risk. In addition, if the portfolio distribution does not match the benchmark, it increases tracking risk.

Sector and quality percent

If the portfolio overweighs AAA securities compared with the benchmark, the tracking risk will increase.

Sector duration contribution

If financial sector duration is 5.0 for the portfolio and 6.5 for the benchmark, the tracking risk might increase.

Quality spread duration contribution.

If the changes in spread duration between qualities of bonds are different between the portfolio and the benchmark, the tracking risk increases.

Sector/Coupon/maturity cell weights

If the call exposure of the index is matched by matching sector, coupon and maturity weights of the callable sectors, the tracking risk will be reduced.

Issuer exposure

If the index is replicated with too few securities, event risk will increase and tracking risk will increase.

- (c) Suggest three index enhancement strategies.

Commentary on Question:

Candidates were given full credit for suggesting and explaining three index enhancement strategies. No additional credit was given if a candidate provided more than three enhancement.

Lower cost enhancements

The manager can increase the portfolio's net return by simply maintaining tight control on trading costs and management fees.

11. Continued

Issue selection enhancements

The manager may identify and select securities that are undervalued using his own analysis; select issues that may will soon to be upgraded and avoid issues that are on the verge of being downgraded.

Yield curve positioning

Some maturities along the yield curve tend to remain consistently overvalued or undervalued. By overweighting the undervalued areas of the curve and underweighting the overvalued areas, the manager may be able to enhance the portfolio's return.

Sector and quality positioning

- a) Maintaining a yield tilt toward short duration corporates, which has shown by experience to have the best yield spread per unit of duration.
- b) Periodic over- or underweighting of sectors (e.g., Treasuries vs. corporates) or qualities to earn enough extra return to offset some of the indexing expense.

Call exposure positioning

A decline in yields will lead to underperformance for callable bonds relative to the effective duration model's prediction. This underperformance creates an opportunity for the portfolio manager to underweight these issues under these conditions.

12. Learning Objectives:

5. The candidate will understand and identify the variety of fixed instruments available for portfolio management. This section deals with fixed income securities. As the name implies the cash flow is often predictable, however there are various risks that affect cash flows of these instruments. In general the candidates should be able to identify the cash flow pattern and the factors affecting cash flow for commonly available fixed income securities. Candidates should also be comfortable using various interest rate risk quantification measures in the valuation and managing of investment portfolios.

Learning Outcomes:

- (5h) Construct and manage portfolios of fixed income securities using the following broad categories.
 - (i) Managing funds against a target return
 - (ii) Managing funds against liabilities.

Sources:

Managing Investment Portfolios: A Dynamic Process, Maginn & Tuttle, 3rd Edition

- Ch. 6, Fixed Income Portfolio Management

Quantitative Finance, Wilmott, Paul, 2nd Edition

- Ch. 14, fixed-income products and analysis: yield, duration and convexity

Commentary on Question:

The purpose of this question is to test the candidates' understanding on management of fixed income investment portfolio using immunization strategy and how to perform analysis on fixed income products using various quantification measures such as duration and convexity.

Solution:

- (a) Demonstrate that the surplus of the pension plan satisfying the above three conditions is immunized against small parallel interest rate changes.

Commentary on Question:

The question is to test candidates' understanding on the conditions for immunization theory. Candidates are expected to demonstrate the theory using the basic concept of duration, convexity and their relationship to the market value movement. Most of the candidates did not do well in this question. Candidates would receive partial credits if they attempted to explain the theory in words instead of mathematically.

- (a) Demonstrate that the surplus of the pension plan satisfying the above three conditions is immunized against small parallel interest rate changes.

12. Continued

Commentary on Question:

Candidates are expected to demonstrate the theory using the basic concept of duration, convexity and their relationship to the market value movement. Most of the candidates did not do well in this question. Candidates would receive partial credits if they attempted to explain the theory in words instead of mathematically.

Surplus = Asset – Liability

Since market value of asset \geq market value of liabilities, as long as we show that change in market value of assets will always be greater than or equal to change in market value of liabilities, then surplus is protected.

The Taylor series expansion gives

$$\frac{dV}{V} = \frac{1}{V} \frac{dV}{dy} \delta y + \frac{1}{2V} \frac{d^2V}{dy^2} (\delta y)^2 + \dots$$

where V is market value, y is interest rate, δy is change in interest rate.

$$\text{Duration (D)} = -\frac{1}{V} \frac{dV}{dy}$$

$$\text{Convexity (C)} = \frac{1}{V} \frac{d^2V}{dy^2}$$

Using that for a small parallel interest rate shift we have

$$\frac{dV}{V} = -D\delta y + \frac{1}{2} C(\delta y)^2$$

The difference in change in market value of assets and liabilities are

$$\begin{aligned} \frac{dV(A)}{V(A)} - \frac{dV(L)}{V(L)} &= -D(A)\delta y + \frac{1}{2} C(A)(\delta y)^2 - (-D(L)\delta y + \frac{1}{2} C(L)(\delta y)^2) \\ &= (-D(A) + D(L))\delta y + \frac{1}{2} (C(A) - C(L))(\delta y)^2 \\ &= \frac{1}{2} (C(A) - C(L))(\delta y)^2 \geq 0 \end{aligned}$$

since we have $D(A) = D(L)$ and $C(A) \geq C(L)$ and $(\delta y)^2 \geq 0$

$$\frac{dV(A)}{V(A)} - \frac{dV(L)}{V(L)} \geq 0$$

Therefore, excess of Asset - Liability can't go negative.

12. Continued

- (b) Show whether the immunization conditions hold for the ABC Pension portfolio.

Commentary on Question:

The question is to further test candidates on the definition of present value, duration, convexity and apply them in a simple scenario example. Most of the candidates did well on the present value and duration calculation, but only a handful of candidates received full credits for convexity.

$$PV(A) = \sum PV(CF_t) = \sum \frac{(CF_t)}{(1+i_t)^t} = \frac{907}{1.05^5} + \frac{1000}{1.05^7} = 1,421.34$$

$$PV(L) = \sum PV(CF_t) = \sum \frac{(CF_t)}{(1+i_t)^t} = \frac{1900}{1.05^6} = 1,417.81$$

$$PV(A) > PV(L)$$

$$\begin{aligned} \text{Duration} &= -\frac{1}{PV} \frac{dPV}{di} = -\frac{d \sum PV(CF_t)}{\sum PV(CF_t) di} = -\frac{d \sum \frac{(CF_t)}{(1+i_t)^t}}{\sum \frac{(CF_t)}{(1+i_t)^t} di} = -\frac{\sum (-t) * \frac{(CF_t)}{(1+i_t)^{t+1}}}{\sum \frac{(CF_t)}{(1+i_t)^t}} \\ &= \frac{\sum \frac{(CF_t)}{(1+i_t)^{t+1}} * t}{\sum \frac{(CF_t)}{(1+i_t)^t}} \end{aligned}$$

$$\text{Duration}(A) = \frac{\frac{907}{1.05^6} * 5 + \frac{1,000}{1.05^8} * 7}{\frac{907}{1.05^5} + \frac{1,000}{1.05^7}} = \frac{8121.96}{1421.34} = 5.71$$

$$\text{Duration}(L) = \frac{1900 * 6}{1.05^7} / \frac{1900}{1.05^6} = 5.71$$

For Duration, the following quantification is also being given some credits:

$$\text{Duration}(L) = 6 \quad (\text{a single payment at year 6})$$

On asset side, the portfolio consists of two zero-coupon bond with duration equals to 5 and 7. The portfolio's duration is equal to the weighted average of the durations of the bonds in the portfolio. The weight is proportional to how much of the portfolio consists of a certain bond.

12. Continued

$$\begin{aligned} \text{Duration (A)} &= W_1 D_1 + W_2 D_2 = \frac{PV(\text{bond1})}{PV(A)} * D_1 + \frac{PV(\text{bond2})}{PV(A)} * D_2 = \frac{\frac{907}{1.05^5}}{\frac{907}{1.05^5} + \frac{1,000}{1.05^7}} * 5 \\ &+ \frac{\frac{1,000}{1.05^7}}{\frac{907}{1.05^5} + \frac{1,000}{1.05^7}} * 7 = \frac{710.66}{1421.34} * 5 + \frac{710.68}{1421.34} * 7 = 6 \end{aligned}$$

$$\text{Duration (A)} = \text{Duration(B)}$$

$$\begin{aligned} \text{Convexity} &= \frac{1}{PV} \frac{d^2 PV}{di^2} = - \frac{1}{\sum \frac{(CF_t)}{(1+i_t)^t}} \frac{d \sum \frac{(CF_t)}{(1+i_t)^{t+1}} * t}{di} \\ &= - \frac{\sum -(t+1) * \frac{(CF_t)}{(1+i_t)^{t+2}} * t}{\sum \frac{(CF_t)}{(1+i_t)^t}} = \frac{\sum \frac{(CF_t)}{(1+i_t)^{t+2}} * (t^2 + t)}{\sum \frac{(CF_t)}{(1+i_t)^t}} \\ \text{Convexity(A)} &= \frac{\frac{907}{1.05^7} * (5^2 + 5) + \frac{1,000}{1.05^9} * (7^2 + 7)}{\frac{907}{1.05^5} + \frac{1,000}{1.05^7}} = \frac{55435.74}{1421.34} = 39.00 \end{aligned}$$

$$\text{Convexity(L)} = \frac{\frac{1,900}{1.05^8} * (6^2 + 6)}{\frac{1,900}{1.05^6}} = \frac{54011.78}{1417.81} = 38.10 \Rightarrow \text{Convexity(A)} >$$

Convexity(L)

The portfolio satisfies the following three conditions:

$$PV(A) > PV(L); \text{Duration(A)} = \text{Duration(L)}; \text{Convexity (A)} > \text{Convexity (L)}$$

Therefore, the immunization condition holds for the ABC portfolio.

- (c) Critique the above statements.

Commentary on Question:

The question is testing candidates' understanding of how to manage fixed income investment portfolio using immunization strategies and what kind of risks are associated with immunization. Candidates need to provide correct explanation for each statement in order to receive full credits. Most of candidates did well on this question.

12. Continued

1. This statement is incorrect. The duration of portfolio will change as the market yield changes or simply because of passage of time. One needs to rebalance the portfolio duration whenever interest rates change and as time elapses. Immunization also does not protect from non-parallel movement in interest rate changes.
2. This statement is only true if the yield curve is downward sloping. If the yield curve is upward sloping, then this statement is not true as the immunization target rate of return would be less than the yield to maturity because of the lower reinvestment return.
3. The statement is incorrect. The portfolio described would be exposed to the risk of a change in interest rates that results in a change in the shape of the yield curve.
4. The statement is incorrect. Immunized portfolios need to be rebalanced; the liquidity of securities used to construct an immunized portfolio is a relevant consideration. Illiquid securities involve high transaction costs and make portfolio rebalancing costly.
5. The statement is correct. The entire portfolio does not have to be turned over to rebalance it because shifting a small set of securities from one maturity range to another is generally enough. Also, to avoid excessive transaction costs, rebalancing is usually not done on a daily basis, which could involve excessive transaction costs.
6. The statement is incorrect. If the portfolio has interim cash flows, it is exposed to reinvestment risk. E.g. Bond transactions at year 5 need to be reinvested. The reinvestment rates could be lower at that time.

13. Learning Objectives:

6. The candidate will understand the variety of equity investments and strategies available for portfolio management.

Learning Outcomes:

- (6a) Explain the nature and role of equity investments within portfolios that may include other asset classes.
- (6b) Demonstrate an understanding of the basic concepts surrounding passive, active, and semi active (enhanced index) equity investing, including managing exposures.
- (6d) Demonstrate an understanding of equity indices and their construction, including distinguishing among the weighting schemes and their biases.
- (6e) Identify methods for establishing passive exposure to an equity market.

Sources:

Managing Investment Portfolios: A Dynamic Process, Maginn & Tuttle, 3rd Edition
Ch. 7 Equity Portfolio Management

Commentary on Question:

The question tested candidate's knowledge of the definition of equity indexing, and how it differs from other types of passive and active investing strategies. Candidates were required to apply such knowledge in assessing the justification of investment results in a hypothetical scenario.

Overall, candidates did well on the question. Maximum points were awarded for a full description of the advantages of equity indexing (13a), alternative passive investment vehicles (13a), and the description of alternative approaches to constructing an indexed portfolio (13c) including limitations and considerations.

Candidates struggled in justifying their criticism of the manager's justification (13b) based on the facts presented in the question. Full credits were awarded if the properly identified the appropriateness of the manager's justification and the cause of the observed investment performance.

Solution:

- (a) Describe the advantages of equity indexing, and list alternative passive investment vehicles available.

13. Continued

Commentary on Question:

Candidates generally identified that low transaction costs and management fees were advantages in equity indexing. Candidates were awarded full credit if they also identified either tax efficiency or that it is a good way to access unfamiliar markets. Candidates were awarded partial credit if they only identified indexed ETFs or indexed mutual funds as alternative passive vehicles. Full credit was awarded for noting long position in cash plus long position in futures or swaps.

Advantages of equity indexing:

- Low portfolio turnover / low transaction costs
- Low management fees
- Low tracking error
- High tax efficiency
- Good way to access unfamiliar markets

Alternative passive vehicles:

- Investment in an indexed portfolio (indexed ETF or mutual fund)
- Long position in cash plus a long position in futures contracts on the underlying index
- Long position in cash plus a long position in a swap of the index

- (b) Critique your equity manager's justification

Commentary on Question:

Candidates generally noticed that full replication will create lagged returns given the transaction costs that are not reflected in the benchmark index returns. However, it was necessary to identify it was an anomaly since the index is value-weighted. Some candidates identified that the cause was the large number of stocks in the index, which required a larger-than-usual number of transactions to fully replicate the benchmark. Full credit was awarded if candidates identified that the justification was inaccurate and indicated the cause of the lagged returns.

Criticism required the statement of the following facts:

- Since the index is value-weighted, the high-frequency trading is anomalous, not the business-as-usual aspect of implementing a fully replicated indexed portfolio
- The cause of such large trading activity is driven by the large number of stocks in the index (3,000), which increases the number of portfolio adjustments, reinvestment/disinvestment activity, and thus drag on performance
- Full replication is not advisable for indices with large number of stocks (>1000)

13. Continued

- (c) Describe one alternative approach to full replication for constructing an indexed portfolio.

Commentary on Question:

Candidates generally identified Stratified Sampling as an alternative approach to constructing an indexed portfolio. Partial credits were awarded for noting the essential features of the stratified sampling or optimization. Full credits were awarded if they also note additional considerations and limitations.

Alternatives include Stratified Sampling or Optimization

Stratified sampling

- Divide the index along a number of dimensions (e.g., market capitalization, industry)
- Place each index stock into the dimension cell that best describes it
- Determine weight on each cell
- Select a random sample of stocks in each cell
- Ensure that the sum of the weights of the stocks purchased from each cell corresponds to the cell's weight in the index
- Allows the manager to build a portfolio that retains the basic characteristics of the index without having to buy all the stocks in the index
- The greater number of dimensions and the finer the divisions, the more closely the portfolio will resemble the index
- May be required to follow regulatory requirements for fund diversification
- Implicitly assumes risk factors are mutually uncorrelated
- Depends on portfolio size and the availability of active trading in an index basket

Optimization

- Use a multifactor risk model to measure risk exposures
 - Model takes into account covariances among factors
- Calculate securities needed to minimize tracking risk subject to appropriate constraints
- Seeks to exploit any risk differences among securities, even if they just reflect sampling error
- Cannot exactly capture risks associated with a given stock
- Depends on portfolio size and the availability of active trading in an index basket

14. Learning Objectives:

6. The candidate will understand the variety of equity investments and strategies available for portfolio management.
7. The candidate will understand how to develop an investment policy including governance for institutional investors and financial intermediaries.

Learning Outcomes:

- (6c) explain the basic active equity selection strategies including value, growth and combination approaches.
- (6f) compare techniques for characterizing investment style of an investor.
- (6j) describe the process of identifying, selecting, and contracting with equity managers.
- (7a) Explain how investment policies and strategies can manage risk and create value.
- (7c) Determine how a client's objectives, needs and constraints affect investment strategy and portfolio construction. Include capital, funding objectives, risk appetite and risk-return trade-off, tax, accounting considerations and constraints such as regulators, rating agencies, and liquidity.

Sources:

Managing Investment Portfolios: A Dynamic Process, Maginn & Tuttle, 3rd Edition Ch. 7

Managing Investment Portfolios: A Dynamic Process, Meginn & Tuttle, 3rd Edition Ch. 1

Liquidity as an Investment Style, Ibbotson, et. Al FAJ Vol 69, 2013 QFIC-109-15

Commentary on Question:

The question was relatively easy. The majority of the candidates got the maximum score in part a). On the other hand, many candidates lost points in part is c). They described the goals, objectives and constraints (time horizon, regulatory, taxation...) instead of listing the IPS elements.

Solution:

- (a) Identify and describe the portfolio manager's investment styles today and one year ago.

14. Continued

One year ago the portfolio reflected a growth investment style. A growth-oriented portfolio exhibits a bias towards high P/Es, high P/Bs and low dividend yields. Growth portfolios typically have lower dividend payout ratios because growth companies want to retain most of their earnings to finance future growth and expansion. A growth-oriented portfolio also tends to hold companies experiencing above-average and/or increasing earnings growth rates. In addition, the portfolio had a small cap bias.

Today the portfolio reflects a value investment style. A value-oriented portfolio exhibits a bias towards low P/Es, low P/Bs and high dividend yields. A value-oriented investor believes a company's earnings will revert back to the mean and will therefore benefit from buying when it is at a low P/E and P/B. EPS is also below average indicating the portfolio is not growth oriented. In addition, the portfolio had a large cap bias.

- (b) Identify the issues relating to this manager's investment practices and recommend whether or not to use this portfolio manager to manage your clients' funds.

It is not recommended to use this portfolio manager. The portfolio manager exhibits style drift over the one year period. Investors generally dislike style drift because they want to achieve a particular exposure and style drift may expose them to an undesired style. Portfolio managers are also hired for their area of expertise and style drift may indicate the manager is no longer operating in their area of expertise.

The number of stocks also decreased dramatically from 50 to 28 implying a high turnover. High turnover is associated with high transaction costs.

- (c)
- (i) Describe the purpose of the investment policy statement (IPS).
 - (ii) List the elements of the IPS.
 - (iii) Identify and describe those elements that could be specified to avoid the issues of this portfolio manager.
- (i) The IPS serves as the governing document for all decision making. Also, the IPS forms the basis for strategic asset allocation.
- (ii) The elements of the IPS are:
- Brief client description
 - Purpose of establishing policies and guidelines
 - Duties and investment responsibilities of parties involved
 - Statement of investment goals, objectives and constraints

14. Continued

- Schedule for review of investment performance
- Performance measures and benchmarks
- Considerations for strategic asset allocation
- Investment strategies and investment styles
- Guidelines for rebalancing portfolio based on feedback
- Manager fees

- (iii) The IPS should specifically state:
- Desired investment style. The IPS needs to specify which investment style is expected.
 - Duties and investment responsibilities of investment managers. The IPS should describe the manager mandate.
 - Frequency of investment communication. Investment performance should be reviewed most frequently than once a year to ensure style drift is not occurring.
 - Guidelines for rebalancing. Within the guidelines for rebalancing the IPS should state any limits on transaction costs to ensure that turnover isn't higher than expected or desired.

- (d) Identify four criteria that characterize a benchmark style and explain why the liquidity should be considered as a benchmark style.

The four criteria are:

- Identifiable before the fact
- Not easily beaten
- Viable alternative
- Low in cost

Liquidity should be considered as a benchmark style because:

Identifiable using stock turnover, bid-ask spreads, or price impact of unit trade, or average price impact relative to daily trading volume

Ibbotson shown the historical returns were hard to beat

Viable alternative to size, value and momentum styles

Less liquid portfolios could be formed at low cost

15. Learning Objectives:

6. The candidate will understand the variety of equity investments and strategies available for portfolio management.

Learning Outcomes:

- (6a) Explain the nature and role of equity investments within portfolios that may include other asset classes.

Sources:

Maginn and Tuttle, Chapter 3 Managing Institutional Investor Portfolios

Commentary on Question:

Many candidates got partial credit for explaining the concepts involved in this question in general terms. Full marks were given for applying those concepts specifically to the constraints and context of the lines of business in the problem.

Solution:

- (a) State the advantage of the segmentation of portfolios.

Commentary on Question:

There were many valid answers to this question. The below list is by no means comprehensive, but would have counted for full marks.

Segmentation Benefits:

- Promote competitive crediting rates for each segment
 - Satisfy the multiple returns objectives of the different lines of business
 - Different investment horizons
 - Different liquidity requirement
- (b) Draft the risk objective section of the IPS for the UL line.
- The risk objective section of the IPS for the UL line should address:
- The ability of the company to fund future policyholder benefits and claims
 - The ability of the company to absorb modest loss of principal by maintaining adequate reserves
 - Reinvestment risk – the company may have to reinvest coupons or principal at a rate below the original coupon or purchase rate
 - The potential mismatch of duration of assets and liabilities
 - Controlling credit risk through broad diversification
 - Loss or delay of income through cash flow volatilities (surrender, late payments, etc.)
- (c) Explain why effective duration is more appropriate for the UL line instead of modified duration.

15. Continued

In order to use modified duration, cash flows should be deterministic and not varied.

Universal life can have varied cash flows due to their interest rate sensitivity from changes in the crediting rate. Different crediting rates will affect the surrender rates which will in turn affect the surrender cash flows.

Therefore, using the effective duration is a better approach to measuring the interest rate sensitivity of this product.

- (d) Comment on the liquidity requirement in the IPS for the SPDA line.

The SPDA line is exposed to disintermediation risk. A mismatch in duration between assets and liabilities can lead to a net loss under rising interest environment. Not having market value adjustment amplifies this risk. The product is also exposed to asset marketability risk. If this line is backed by non-liquid assets, the company may be forced to sell assets at a loss to fund the liabilities in the event of surrender.

- (e) Critique the CIO's statement.

Commentary on Question:

Partial credit was given for explaining why increasing the equity allocation could also be justified.

If we were to increase the equity allocation of the pension plan, this would also significantly increase the correlation between the pension plan and VA line of business (as the VA line is highly correlated with equity market). This higher correlation between pension assets and VA line of business leads to a lower risk tolerance, as there is now less diversification in the within all of ABC Life's investment portfolio. In this sense, a lower equity allocation in pensions can be viewed as a hedge for the VA line. Therefore, lower equity allocation is justified.

16. Learning Objectives:

8. The candidate will understand the theory and techniques of portfolio asset allocation.

Learning Outcomes:

- (8a) Explain the impact of asset allocation, relative to various investor goals and constraints.
- (8b) Propose and critique asset allocation strategies.
- (8c) Evaluate the significance of liabilities in the allocation of assets.
- (8d) Incorporate risk management principles in investment policy and strategy, including asset allocation.

Sources:

Managing Investment Portfolios: A Dynamic Process, Maginn & Tuttle, 3rd Edition
Ch. 5 Asset Allocation

Commentary on Question:

The question tested candidates' knowledge of asset allocation in the specific context of a pension plan. Candidates generally performed well, but some failed to address their responses in the context of pension plans. Candidates generally were unable to clearly describe the utility and limitations of a tangency portfolio. Maximum points were received for complete descriptions when required (beyond just mentioning essential components).

Solution:

- (a) Describe the asset-only approach currently employed.

Commentary on Question:

Candidates did well on this question. Full credits were awarded if they noted that the asset-only approach considers asset returns relative to asset risks (as measured by volatility) and that it excludes liabilities.

Asset-only approach:

- Aims to maximize return while minimizing return volatility (i.e., Sharpe ratio)
- Does not explicitly involve modeling liabilities.

- (b) Identify and describe one alternative approach to strategic asset allocation.

16. Continued

Commentary on Question:

Candidates did well on this question. Most candidates noted the asset-liability management as the alternative approach. Full credits were awarded if candidates listed the consideration of liabilities and surplus as the target metric. Some candidates noted that an alternative to strategic asset allocation was tactical asset allocation. Credits were also awarded in this case.

Asset-Liability Management

- Explicitly models liabilities and adopts the optimal asset allocation in relationship to liabilities
- Seeks to minimize risk with respect to net worth or surplus (assets minus liabilities)

- (c) Assess whether the current approach is appropriate for VNT.

Commentary on Question:

Candidates did generally well on this part, as it was mainly the summation of the previous parts.

The current approach is not appropriate for funding pension liabilities, especially for plans with a deficit.. Controlling the risk related to funding future liabilities is a key investment objective for pension plan investors. Pension plan may want to minimize the future risk-adjusted value of pension deficit.

Situations where the ALM approach is more appropriate

- Low risk tolerance
- High penalty for not meeting return requirement
- Market value of liability is interest sensitive
- Legal and regulatory requirements favor fixed income.

- (d) Determine and justify the strategic asset allocation most appropriate for VNT based on mean-variance analysis as well as VNT's risk objective and constraints.

Commentary on Question:

Candidates were generally able to identify the portfolio that met the IPS requirements. Credits were awarded for calculating the required components. Full credits were awarded for properly identifying the portfolio and how it meets the IPS conditions. No credits were awarded if candidates discarded the portfolios with incomplete data.

16. Continued

First need to calculate Roy's safety first criterion based on Portfolio I or II and then derive the required return (6%). Then calculate the missing values in the table

	I	II	III	IV	V
Exp Return	12.00%	11.00%	10.00%	9.00%	8.00%
Std. Dev	21.00%	16.0%	12.00%	8.00%	6.00%
Roy's SF	0.286	0.313	0.333	0.375	0.333
RA return	3.18%	5.88%	7.12%	7.72%	7.28%

Portfolios I and II need to be discarded given that they don't meet the maximum standard deviation requirement.

The remaining portfolios meet the risk objective (standard deviation less than 14%), return objectives (at least 6% risk-adjusted return), and no short-selling allowed (positive weights). The portfolio with the highest Roy's safety ratio needs to be selected: Portfolio IV

- (e) Identify the corner portfolio most likely to be the tangency portfolio.

Commentary on Question:

Candidates who answered part d) correctly did generally well on this question. Full credits were awarded if they identified that the tangency portfolio is the corner portfolio with the highest Sharpe ratio. No credits were awarded for partial or incomplete solutions.

Derive risk-free rate from the given Sharpe ratios (3%).

The tangency portfolio is the perceived highest-Sharp-ratio efficient portfolio.

	I	II	III	IV	V
Sharpe ratio	0.429	0.500	0.583	0.750	0.833

Based on the Sharpe ratios of each portfolio, the tangency portfolio is Corner Portfolio (V)

- (f) Describe the utility and the limitations of the tangency portfolio in the process of selecting an optimal strategic asset allocation.

16. Continued

Commentary on Question:

Most candidates were able to identify the utility of the tangency portfolio, but failed to note the limitations. Partial credits were awarded for identifying the utility or the limitations.

Utility

If the investor can borrow or lend at the risk-free rate, he can use margin to

- a) Leverage the position in the tangency portfolio to achieve a higher expected return, or
- b) Split money between the tangency portfolio and the risk-free asset to achieve a lower risk position

Limitations

- a) Many investors face restrictions against buying risky assets on margin, particularly pension plans
 - b) Even without the formal constraint against using margin, a negative position in cash equivalents may be inconsistent with an investor's liquidity needs
- (g) Critique the Board member's statement and recommend potential change(s) in the investment policy, if any. Justify your answer.

Commentary on Question:

Candidates performed irregularly on this question. Candidates generally identified that the Board member statement is inaccurate or wrong, but failed to justify their answer in the given context of a pension plan. Some candidates incorrectly assumed that because the funding ratio is greater than 100%, the investment manager could increase its risk tolerance.

Full credits were awarded for noting that the Board member's recommendation was against the key objectives of the plan; that changes to IPS would actually require to reduce the risk tolerance of the plan; and recommend changes such as incorporating funding ratio (or surplus) as the target metric in investment management and update specific aspects in the IPS to achieve this purpose.

- Given current investment management policy is asset-only, it is difficult for the board member to understand how the funding status can change over time
- For a pension plan, the primary investment objective is not to maximize returns but to have the ability to fund current and future pension obligations. Thus, the Board members' statement is misguided in the context of a pension plan.
- Achieving a funding status greater than 100% might indicate that there is no need to assume higher risks and, in fact, reducing the risk tolerance might be better aligned with a pension plan's objectives

16. Continued

- Might consider other changes in the investment policy, such as: restrictions on margin, risk-adjustments to returns (i.e., RA in Roy's safety criterion), the return objective, changes in asset class allocation to achieve better diversification benefits.

17. Learning Objectives:

7. The candidate will understand how to develop an investment policy including governance for institutional investors and financial intermediaries.

Learning Outcomes:

- (7a) Explain how investment policies and strategies can manage risk and create value.
- (7b) Identify a fiduciary's obligations and explain how they apply in managing portfolios.
- (7c) Determine how a client's objectives, needs and constraints affect investment strategy and portfolio construction. Include capital, funding objectives, risk appetite and risk-return trade-off, tax, accounting considerations and constraints such as regulators, rating agencies, and liquidity.
- (7d) Incorporate financial and non-financial risks into an investment policy, including currency, credit, spread, liquidity, interest rate, equity, insurance product, operational, legal and political risks.

Sources:

QFIC-106-13: Chapter 10 of Modern Investment Management: An Equilibrium Approach

Commentary on Question:

This question was testing the importance of liabilities when selecting an asset portfolio.

Solution:

- (a) Assess which asset portfolio should be selected, according to the Sharpe ratio, in an asset-only approach to strategic asset allocation. Justify your answer.

Commentary on Question:

Most candidates answered the Sharpe Ratio question correctly.

The traditional Sharpe Ratio measure considers only the risk and return of assets, ignoring the presence of any liability stream.

Risk Free rate	1%
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	Exp. Return	Std. Dev	Corr.	Sharpe Ratio
Portfolio A	6%	12%	0.65	0.42
Portfolio B	7%	15%	0.35	0.40

Sharpe ratio of Portfolio A is greater than the Sharpe ratio of Portfolio B. Therefore, Portfolio A should be selected in an asset-only approach.

17. Continued

- (b) Assess which asset portfolio should be selected, according to RACS, in an Asset/Liability Management approach to strategic asset allocation.

Commentary on Question:

Most candidates were able to locate the RACS formula, but many did not compute the correct answer. Credit was given for a partially correct formula and a recommendation.

$$RACS_t = \frac{E_t \left[A_t (1 + R_{A,t+1}) - L_t (1 + R_{L,t+1}) - (A_t - L_t) (1 + R_f) \right]}{\sigma_t \left[A_t (1 + R_{A,t+1}) - L_t (1 + R_{L,t+1}) \right]}$$

Portfolio A:			
Surplus(t)	=Asset(t) - Liability(t)	=120-100	20
Surplus(t+1)	=Asset(t+1) - Liability(t+1)	=120*1.06-100*1.05	22.2
Excess Surplus	=Surplus(t+1) - Surplus rf return	=22.2-20*1.01	2.0
Asset volatility	=Asset(t) * Asset Vol	=120*0.12	14.4
Liab volatility	=Liability(t) * Liab Vol	=100*0.1	10.0
Surplus volatility	=asset vol ² +liab vol ² - 2*corr*asset_vol*liab_vol	=14.4 ² +10 ² - 2*14.4*10*0.65	120.16
RACS(Portf A)	=Excess surplus/Surplus Vol	=2/sqrt(120.16)	0.18

Portfolio B:			
Surplus(t)	=Asset(t) - Liability(t)	=120-100	20
Surplus(t+1)	=Asset(t+1) - Liability(t+1)	=120*1.07-100*1.05	23.4
Excess Surplus	=Surplus(t+1) - Surplus rf return	=23.4-20*1.01	3.2
Asset volatility	=Asset(t) * Asset Vol	=120*0.15	18.0
Liab volatility	=Liability(t) * Liab Vol	=100*0.1	10.0
Surplus volatility	=asset vol ² +liab vol ² - 2*corr*asset_vol*liab_vol	=18 ² +10 ² - 2*18*10*0.35	298
RACS(Portf A)	=Excess surplus/Surplus Vol	=3.2/sqrt(298)	0.19

Because RACS for Portfolio B is greater than the RACS for Portfolio A, therefore Portfolio B should be selected in an ALM approach.

- (c) Compare the appropriateness of the two approaches when managing the assets of the pension plan.

Commentary on Question:

Most candidates answered RARCS was better than Sharpe ratio and gave reasons why RARCS was better. Some of them did not give reasons why Sharpe ratio was not appropriate, so partial points were given.

17. Continued

Sharpe ratio ignores liabilities hence based on this approach a portfolio with a higher surplus volatility may be selected

The RACS approach recognizes liabilities by considering the expected change (mean) in surplus and the uncertainty (standard deviation) in the change in surplus

The RACS measures the dollar return on surplus that is in excess of the risk-free rate of return against the risk taken relative to the risk-free strategy.
Hence RACS is more suitable when managing the assets of this pension plan.