QFI ADV Model Solutions Fall 2015

1. Learning Objectives:

- 1. The candidate will understand the standard yield curve models, including:
 - One and two-factor short rate models
 - LIBOR market models

The candidate will understand approaches to volatility modeling.

Learning Outcomes:

- (1a) Identify and differentiate the features of the classic short rate models including the Vasicek and the Cox-Ingersoll-Ross (CIR) models.
- (1b) Understand and explain the terms Time Homogeneous Models, Affine Term Structure Models and Affine Coefficient models and explain their significance in the context of short rate interest models.

Sources

Brigo, D and Mecurio F, Interest Rate Models – Theory and Practice, 2nd Edition, Section 3.2.4, p.57-70.

Commentary on Question:

The question is tested the candidates understanding of identifying and differentiating the features of short rate models. In addition, the question tested the candidate's ability to understand and explain an affine term structure and its significance in the context of short rate models.

Solution:

(a) Define an affine term structure model.

Commentary on Ouestion:

The candidates performed as expected on this section. Successful candidates provided the definition while unsuccessful candidates, for the most part, did not provide an answer.

An interest rate model has an affine term structure if its continuously compounded interest rate can be written as an affine function of the short rate r(t):

$$R(t,T) = \alpha(t,T) + r(t)\beta(t,T),$$

where $\alpha(t,T)$ and $\beta(t,T)$ are deterministic functions of time.

(b) Explain the advantage of using an affine term structure model to price a bond.

Commentary on Question:

The candidates performed below average on this section. Successful candidates wrote about the price of a zero coupon bond. However, the majority of candidates missed the notation of solving two differential equations.

When the interest rate model has an affine term structure, the price of the zero-coupon bond can be written as $P(t,T)=A(t,T)exp\{-B(t,T)r_t\}$, where A(t,T) and B(t,T) are deterministic functions of time.

Equations A(t,T) and B(t,T) can be obtained by solving two differential equations.

(c) Determine and justify whether this equation describes an interest rate model with an affine term structure.

Commentary on Question:

The candidates performed as expected on this section. Successful candidates included a note about a quadratic function. Many candidates missed the justification portion of the question.

Given the risk neutral dynamics of the short rate

$$dr(t) = b(t, r(t))dt + \sigma(t, r(t))dW_{\star}$$

if the coefficients b(t,r(t)) and $\sigma^2(t,r(t))$ are themselves affine functions, then the model has an affine term structure.

In our case, b(t,r(t)) is quadratic in r(t), hence it is not an affine function.

Thus the model does not have an affine term structure.

(d) Identify and correct any mistakes in the equations above.

Commentary on Ouestion:

The candidates performed above average on this section. Successful candidates discovered both mistakes in the equations. Candidates were less successful at identifying $\eta(t) = k\Theta$.

We have $P(T,T)=A(T,T)exp\{-B(T,T)r_t\}$, we must have A(T,T)=1 and B(T,T)=0. The 2nd formula missing Θ and the correct one should be

$$\frac{\partial}{\partial t}[\ln A(t,T)] - k\theta B(t,T) + \frac{1}{2}\sigma^2 B(t,T)^2 = 0, \text{ as } \eta(t) = k\Theta$$

(e) Explain how you would use these equations to obtain the price of a zero-coupon bond.

Commentary on Question:

The candidates performed above average on this section. Successful candidates provided a formula for the price. Candidates commonly missed solving the differential equation.

Since this system of equation represents the Vasicek one-factor model, it can be solved analytically.

To obtain B(t,T), solve the first differential equation.

B(t,T) can be used to solve the second equation and obtain A(t,T).

Then P(t,T) is obtained using $P(t,T)=A(t,T)exp\{-B(t,T)r_t\}$

2. The candidate will understand and be able to apply a variety of credit risk theories and models.

Learning Outcomes:

- (2f) Demonstrate an understanding of modeling approaches for correlated defaults.
- (2g) Demonstrate and understanding of and be able to apply the concept of Duration Times Spread (DTS).

Sources:

Ben-Dor et al Quantitative Credit Portfolio Management Chapters 1 and 3

QFIA-103-13 Bond CDS-Handbook

Commentary on Question:

This question tested the candidate's understanding of duration times spread (DTS) and applying DTS to credit default swaps. The question also tested a candidates understanding of basis trades involving credit default swaps and bonds. Candidates performed as expected on the question.

Solution:

(a) Identify four reasons why the full running spread of a CDS and the spread of the underlying bond are usually not the same value.

Commentary on Question:

Candidates performed as expected on this question. One common mistake was to provide one reason and restate it multiple ways. In this case it only counted as one reason.

- CDS and bond markets have different investor bases
- Cheapest to deliver option should increase CDS spreads relative to bonds to compensate sellers for giving buyers this option.
- A variety of bond covenants give lenders and bond holders rights which CDS protection buyers and sellers do not enjoy.
- Companies offering bonds in times of low liquidity and deteriorating credit conditions may need to offer wider spreads to incentivize investors.

These were the four most common answers, however many other answers were accepted as well.

- (b) For bond A and bond B:
 - (i) (1 point) Determine an advantage and disadvantage of a basis trade for each bond.
 - (ii) (2 points) Recommend and describe which basis trade to pursue.

Commentary on Question:

Candidates performed as expected on this section. Candidates performed better on part (ii). A common mistake on part (i) was providing advantages and disadvantages on the bond rather than on the basis trade. A common mistake on part (ii) was using the wrong spreads.

(i) Bond A is low price and low coupon: (Advantage) low price means that a negative basis trade will have a favorable jump to default exposure; (Disadvantage) low coupon means that a negative basis trade will have a negative carry profile over the life of the trade.

Bond B is high price and high coupon: (Advantage) high coupon means that the negative basis trade will have a positive carry profile; (Disadvantage) high price means the negative basis trade will have an unfavorable jump to default exposure early on.

(ii) CDS Spread = (upfront cost – Accrued Interest)/(RA) + Coupon CDS Spread Bond A = (300-0)/3 + 500 = 600 bps CDS Spread Bond B = (200-0)/4 + 500 = 550 bps Bond A CDS Basis = CDS Spread Bond A – Bond A Par Equivalent CDS Spread = 600 bps – 1000 bps = -400 bps Bond B CDS Basis = CDS Spread Bond B – Bond B Par Equivalent CDS Spread = 550 bps – 800 bps = -250 bps

Recommend the negative basis trade for Bond A since it has the more negative Bond CDS basis then Bond B.

The negative basis trade is executed by buying the bond and buying the CDS protection.

(c) Calculate the percentages of the portfolio's market value that can be invested in bond C and bond D and stay within the portfolio's credit limit.

Commentary on Question:

Candidates performed as expected on this section. Many candidates changed the units from bps to percent and came up with an answer that was off by a factor of 100. These candidates received most of the points. The other common problem was using the wrong spread.

Bond C: DTS Limit/(Duration X Spread) = 4/(4*800) = 0.125%Bond D: 4/(5*900) = 0.089%

(d) Evaluate the impact these changes are expected to have on spread volatility for each CDS.

Commentary on Question:

Candidates performed as expected on this section. Candidates that identified the linear relationship between spread volatility and spread level generally got full credit. Those candidates that did not identify that relationship generally got no credit.

There is a linear relationship between the expected spread volatility and spread level.

If the upfront cost increase than the spread level increases according to the formula: CDS Running Spread = (Upfront cost – AI)/RA + Coupon. Since we can ignore accrued interest and the risky annuity did not change, the CDS running spread increases if the upfront cost increases and decreases if the CDS upfront cost decreases.

Since spread level for CDS on bond C has increased spread volatility should increase.

Since spread level for CDS on bond D has decreased spread volatility should decrease.

- The candidate will understand and be able to describe the variety and assess the role of alternative assets in investment portfolios. The candidate will demonstrate an understanding of the distinguishing investment characteristics and potential contributions to investment portfolios of the following major alternative asset groups:
 - Real Estate
 - Private Equity
 - Commodities
 - Hedge Funds
 - Managed Futures
 - Distressed Securities
 - Farmland and Timber

Learning Outcomes:

- (6a) Demonstrate an understanding of the types of investments available in each market, and their most important differences for an investor.
- (6b) Demonstrate an understanding of the benchmarks available to evaluate the performance of alternative investment managers and the limitations of the benchmarks.
- (6c) Demonstrate an understanding of the investment strategies and portfolio roles that are characteristic of each alternative investment.
- (6d) Demonstrate an understanding of the due diligence process for alternative investments.

Sources:

Maginn & Tuttle, Managing Investment Portfolios, 3rd Ed. 2007, Ch. 8

Commentary on Question:

This question tested the candidates' understanding of the concept of investment and market value for real estate properties and how it applies to a transaction.

Solution:

(a) Explain two issues with using each of the above indices as a benchmark (two issues for each index is expected).

Commentary on Question:

The candidates performed as expected on this section. Many candidates recognized that the benchmark was an appraisal-based valuation, and that the global benchmark was not appropriate for US based properties. However, only a few candidates noted that the REIT benchmark was exposed to equity market volatility. Another key difference often overlooked was that the portfolio was leveraged but the benchmark was not.

NAREIT benchmark:

- exposed to equity market volatility while portfolio is not
- is global while portfolio is US focused

NCREIT benchmark:

- exposed to equity market volatility while portfolio is not
- underestimating the volatility of the underlying values because of appraisalbased valuations
- (b) Explain the main role that this investment would play for the foundation and how it compares to using a REIT for the same purpose.

Commentary on Question:

The candidates performed above average on this section.

Most candidates recognized that the main role of the investment was to diversify from the bond and stock portfolio. Some candidates fail to mention REIT is less effective at diversifying because it's still exposed to market movements

Main role of the investment was to diversify from the bond and stock portfolio. Investment is more effective at diversifying b/c REIT is still exposed to stock market movements

(c) Describe the market opportunity for infrastructure investments.

Commentary on Question:

The candidates performed poorly on this section.

A few candidates actually described what infrastructure investments were and who was typically involved in the process. Very few recognized that the type of financing started in UK, and is rapidly growing in other markets around the world.

Infrastructure investments are real estate investments by private companies in public sector projects (e.g. airports, roads, etc). They take on the risks of building & financing the infrastructure projects by raising debt or equity capital, and the government leases it over a typically long period. This type of financing was pioneered in UK, and is rapidly growing in other markets around the world

(d) Assess whether the fund should be willing to pay more, less, or exactly \$50 million for the 10% stake.

Commentary on Question:

The candidates performed below average on this section.

About half of the candidates correctly stated that they would pay less than \$50 mil because of the liquidity discount. A common mistake was to state that it depended on the investment value (IV).

10% stake is \$50mil. Should pay less because should apply a discount to its value for illiquidity. Also, we would have no control since only hold minority stake.

5. The candidate will understand the behavior characteristics of individuals and firms and be able to identify and apply concepts of behavioral finance.

Learning Outcomes:

- (5a) Explain how behavioral characteristics of individuals or firms affect the investment or capital management process.
- (5b) Describe how behavioral finance explains the existence of some market anomalies.

Sources:

QFIA-109-13: A Survey of Behavioral Finance, by Barberis & Thaler

Commentary on Question:

This question tested the candidates' understanding of the behavioral characteristics of individuals or firms that affect the investment decisions and drive some market anomalies.

Solution:

(a) Identify and describe elements of investor behavior that could have contributed to SWC being overvalued on July 1, 2014.

Commentary on Question:

The candidates performed as expected in this section. Most candidates identified "Representativeness" and "Dividend" effect, but missed "Index inclusion" and "Limitations to arbitrage".

There are four elements that could have contributed to overvaluation of SWC

- 1. Representativeness: law of small numbers and/or narrow framing
 - Investors place too much weight on a small sample of recent positive earnings and extrapolate as the new norm
 - Stocks with surprise positive earnings announcements outperform over the next 60 days
- 2. Dividend initiation: Studies have shown that firms who initiate dividends significantly outperform over the subsequent year
- 3. Index inclusion: The company may have been added to an index (such as S&P500) after its IPO and has very few identifiable

- 4. Limitations to arbitrage:
 - Prices may have stayed above fundamental value because there are few identifiable good substitutes of the new company for arbitragers to take advantage of the over valuation.
 - Company information is now more available, and leads to investor overconfidence.
- (b) Explain how investor behavior could have contributed to the share price increase since July 1, 2014.

Commentary on Question:

The candidates performed as expected in this section. Most candidates identified two of the three behaviors in the illustrative solution.

There are three behaviors to consider

- 1. Momentum has been proven to exist for stocks that outperform over the past 6 months to continue outperforming in the short term
- 2. Driven by law of small numbers placing excess weight on a couple of strong quarters in a row, creating optimism and overconfidence
- 3. Positive feedback trading
- (c) Explain why one might expect Ron's investment to underperform going forward compared to the average SWC shareholder.

Commentary on Question:

The candidates performed poorly in this section. Few candidates correctly explained any one of the three main reasons below. Many candidates justified Ron's underperformance from the high price Ron paid for the shares and the expected price decline due to mean reversion phenomenon. Partial credit was given if the explanation is reasonable.

Ron is expected to underperform the average investor due to

- 1. Overconfidence: as a male Ron is more likely to be overconfident and thus trade excessively, incurring higher transaction costs than female investors. Since SWC shareholders will be a mix of male and female investors, he will underperform the average.
- 2. Prospect theory, and specifically Narrow temporal framing tracking the share price daily is much more frequent than the average investor (annually), and leads to excessive trading.
- 3. Overconfidence online investors have access to more information and are more likely to act irrationally as a result than phone investors or the average investor.

(d)

- (i) Describe how a rational investor might take advantage of the undervalued shares.
- (ii) Explain the various risks in the chosen strategy.

Commentary on Question:

The candidates performed as expected in this section. Many candidates correctly proposed to purchase the undervalued shares, but did not mention shorting a similar "substitute" to hedge the risk. Also, many candidates identified at least one of the risks shown below.

(i) How to take advantage of the undervalued shares

An investor could purchase SWC shares and short shares of a similar 'substitute' stock to hedge the risk of the entire industry declining. This puts pressure on SWC' shares to rise back to fundamental value, after which the investor sells the shares for a gain.

- (ii) Risks include
 - 1. Fundamental risks:
 - New information about SWC could cause its price to fall further
 - There are no perfect substitute shares to short, so risk remains
 - Short selling is either forbidden or costly
 - 2. Noise Trader risk:
 - Irrational traders' actions can keep the price below the fundamental value longer than expected. This may force fund managers to liquidate positions for redemption request from impatient investors.
- (e) Describe how SWC management might take advantage of its shares being undervalued, and the barriers that could prevent a successful implementation of this strategy.

Commentary on Question:

The candidates performed as expected in this section. Many candidates correctly proposed share repurchase, but identified barriers that did not fully correspond to the syllabus material. Partial credit is given if the identified barriers were reasonable.

SWC management could start share re-purchase, as this leads to undersupply and in theory will influences the share price upward. Studies have shown that firms that re- purchase shares outperform over the following 4 years

Barriers include:

- 1. Implementation costs
- 2. It is very difficult for anyone to know for certain that the shares are truly undervalued.

2. The candidate will understand and be able to apply a variety of credit risk theories and models.

Learning Outcomes:

- (2b) Demonstrate an understanding of the basic concepts of credit risk modeling such as probability of default, loss given default, exposure at default, and expected loss.
- (2f) Demonstrate an understanding of modeling approaches for correlated defaults.

Sources:

QFIA -102-13 Credit Risk Measurement in and out of the Financial Crisis Saunders/Allen Ch 1 pgs 8-15

Introduction To Credit Risk Modeling, Bluhm, Christian, 2nd Edition, Chapter 1, pg 12-15

Managing Credit Risk: The Great Challenge for Global Financial Markets, Caouette, John B., et. al., 2nd Edition, 2008, Ch. 20, pg 402-404

Introduction To Credit Risk Modeling, Bluhm, Christian, 2nd Edition, Chapter 2 pg 55-63

Commentary on Question:

This question tested concepts in credit risk. Conceptually it was a construction of a mortgage backed security using three distinct mortgages, with and without co-variance between them. Loss given Default (LGD) and a simple credit VAR calculation are included.

Solution:

(a) Describe the different types of off balance sheet entities that can be used to sell this security.

Commentary on Question:

The candidates performed as expected on this question. Most candidates identified SPVs and SIVs. However, many candidates did not mention the others below.

OBS structured solutions include SPVs, SIVs, syndicated loans, and proprietary entities such as hedge funds and trusts.

(b) Calculate the probability of payment within each tranche of the constructed mortgage backed security.

Commentary on Question:

The candidates performed brilliantly on this question.

```
Low Tranche = 90% x 85% x 80% = 61%
Middle Tranche = 61% + 95% x 85% x 20% + 95% x 15% x 80% + 10% x 85% x 80% = 94%
High Tranche = 94% + 90% x 15% x 20% + 10% x 85% x 20% + 10% x 15% x 80% = 99.7%
```

(c) Calculate the credit value at risk of the portfolio over the next year assuming each loan's loss given default is 60% based on the BIS II guidance.

Commentary on Question:

The candidates performed as expected on this question. Most candidates did the calculation correctly. However few candidates correctly used the 99.9th percentile to do the calculation.

```
Credit VAR @ 99.9<sup>th</sup> percentile = 60% x [8 x (20%-10%)+17 x (22%-15%) + 20 x (30%-20%)] = 2.394
```

(d) Describe the differences between a general Bernoulli mixing model and a Poisson mixing.

Commentary on Question:

The candidates performed above average on this question. Most candidates identified the differences in the tail and correlation but many did not provide a deeper explanation of the comparison.

Bernoulli distribution is systematically different and has higher default correlations and fatter tails than Poisson, but if the law of small numbers holds the two distributions can be comparable. Poisson also allows for more than one default.

(e) Calculate the probability of High tranche getting paid.

Commentary on Question:

Candidates performed poorly on this question. Most candidates did not answer the question or did not calculate the new correlation.

```
Probability of default(A,B) = 10% x 15% + 20% x (10% x 90% x 15% x 85%)^(0.5)=3.6% Total Probability of Default = 3.6% x 20% = 0.72% Probability of high tranche payment = 1-0.72%=99.28%
```

2. The candidate will understand and be able to apply a variety of credit risk theories and models.

Learning Outcomes:

- (2h) Demonstrate an understanding of credit default swaps (CDS) and the bond-CDS basis, including the use of CDS in portfolio and trading contexts.
- (2m) Understand the rationale, markets and risks of structured finance.

Sources:

QFIA-101-13: Managing Credit Risk, The Great Challenge for Global Financial Markets, Caouette, John B. et al., 2nd edition 2008, chapter 24

Commentary on Question:

This question tested the candidates understanding of how structured securities can be applied in a non-standard financial situation. The candidates needed to demonstrate how structured securities operate when presented with a situation where the structure is different than the standard investment situation.

Solution:

(a) Calculate the residual payment at the end of the first month.

Commentary on Question:

The candidates performed above average on this section. Most candidates received full credit for calculating the correct distribution. A few candidates forgot the interest payment from the EIT Investment coupon payment when calculating the residual payment.

Consider the following table:

			Total													
		Number	amount of													
	Number of	of pirates	treasure													
	living	killed	(in gp)		Net gold	Capital										Capital
	pirates at	during	collected	Total DB	collected	Balance	Coupons	Total gold							Total	Balance
	beginning	the	during	paid to	from	(Beg. Of	from EIT	available for		Capital					distribn's	(End Of
Month	of month	month	month	pirates	pirates	Mo.)	Investment	distribution	Distributions	Payout	Tranch Payments			nts	paid to Z	Mo.)
	Col A	Col B	Col C	Col D	Col E	Col F	Col G	Col H	Coll	Col J	T1	T2	Т3	Z		[Col P]
				= 100	= [Col C] -			=[Col E] +	=MAX([Col H],	=MIN([Col H],						= [Col F] +
				x[Col B]	[Col [D]		= 1% * [Col F]	[Col G]	0)	0)						[Col I]
1	1000	9	1000	900	100	3000	30	130	130	0	10	30	70	20	20	3000
2	991	12	900	1200	-300	3000	30	-270	0	-270	0	0	0	0	20	2730
3	979	1	300	100	200	2730	27.3	227.3	227	0	10	30	70	117	137.3	2730
4	978	3	545.4	300	245.4	2730	27.3	272.7	273	0	10	30	70	163	300	2730

- 1) First calculate the net gold collected from pirates [Col E];
- 2) Next, calculate the opening capital balance for the month [Col F]: [Capital @ time t+1] less [Capital @ time t];
- 3) Next, calculate the amount of gold received from the EIT bond [Col F];
- 4) Now, calculate the total amount of gold that can be distributed to the tranches [Col G]: [Net Gold collected from pirates] + [Gold paid by EIT bond];
- 5) If net gold available for distribution is positive, distribute to the tranches using a waterfall structure; otherwise reduce capital to pay out pirate death benefits. [Col H] and [Col I];

As shown in the table, the residual payment to Z at the end of month 1 is 20 gold pieces

(b) Calculate the minimum value for *X* such that the total payments made to Tranche Z from months 1 to 4 equals their original investment (i.e. Tranche Z gets their original investment back at end of month 4).

Commentary on Question:

The candidates performed as expected on this section. The candidates that did well systematically calculated all the key elements as outlined in the steps shown in the solution above for Part (a). The candidates that did poorly did not account for either the EIT Investment coupon payment(s) or the Death Benefit payments.

Repeating the steps found in the solution for part (a), the table can be filled out for the first 4 months. The description below describes the 4th month's calculation.

The total capital initially invested by Tranche Z is 300 gp. We need to calculate the amount of treasure collected by the pirates in month 4 to ensure that Tranche Z earns back all 300 gp after 4 months.

Based on the table shown above, the total amount paid to Tranche Z in the first three years is

$$20 + 0 + 117.3 = 137.3$$
 gp

Thus, in year 4 there needs to be a residual payment made to Tranche Z of 300 - 137.3 = 162.7 gp

Other required payments in month 4 include:

Death benefits: $3 \times 100 = 300 \text{ gp}$ T1, T2, T3 Coupons: 10 + 30 + 70 = 110 gp

The total amount of payouts that need to be made for DB, coupons and the tranche Z residual payment is

$$300 + 110 + 162.7 = 572.7$$
 gp

However, the EIT Investment provides an interest coupon of $2730 \times 1\% = 27.3 \text{ gp}$

Thus the total amount of gold that needs to be collected by the pirates in order that Tranche Z earn back it's original investment is

527.7 - 27.3 = 545.4 gp (rounded answers of 545 and 546 were also accepted as gold pieces typically are not fractional)

(c) Recommend an investment pairing (one to one) between each of the four tranches (T1, T2, T3, Z) and each of the four investors (Captain Scott, Alan, Becky, Clayton) such that every investor is satisfied. Be sure to consider and discuss the risk of losing the investor's capital.

Commentary on Question:

The candidates performed brilliantly on this section. The candidates that did well listed all <u>four</u> investors, paired them with a specific investment, and provided one or two supporting points. The candidates that did poorly either just listed their pairings without any supporting information, or did not include all <u>four</u> investors in the analysis.

Candidates received full marks for any pairing as long as reasonable supporting information was provided. Below is one possible solution.

Scott – Tranche T2

• We have no knowledge of Scott's income appetite, but do know he wants something more stable and has a two year horizon. For these reasons Tranche T2 is the best investment

Alan – Tranche Z

- There could be a large payout if the pirates are lucrative and deaths are minimal
- He is not worried too much if he was to lose his initial investment

Becky – Tranche T3

- Becky has a higher tolerance for risk and Tranche T3 provides a higher level of return for this risk;
- Tranche T2 is also a good choice for Becky since the return is still reasonably high but there is a lot more protection, however, T3 is not a good choice for Captain Scott since he wants his money back within 2 years;

Clayton - Tranche T1

- This is a good choice for Clayton as there is very little risk in losing his capital; other investments provide a higher risk of losing capital.
- (d) Compare and contrast the concerns of the investors who purchased tranche T3 and Z if the scenario shown above occurs.

Commentary on Question:

The candidates performed poorly on this section. The candidates that did well explicitly listed the two tranches being reviewed (T3 and Z) and two or three points for each investor describing the concerns of that investor given the specific scenario presented in the question. The candidates that did poorly either did not consider both investments or did not provide comments specific to the scenario presented in the question.

The information below is based on observations from the table shown in the Part (a) solution.

Primary concerns under the given scenario for T3 investor:

- Most of Tranche Z's capital has been lost (270 of the 300) so if 30 more capital is lost, then T3's capital will start to be lost;
- T3's return could be less than the initial IRR of 10% per month if capital is lost:
- Since prior lost coupons are not made up, there is no way for T3 to make back the lost capital;

Primary concerns under the given scenario for Z investor:

- Most of Tranche Z's capital has been lost (270 of the 300);
- After month 4, Z's IRR = 0
- Tranche Z could have a very lucrative position going forward even if future capital is lost since all future residual cash flows are 'excess' payments
- (e) Develop a Diagram showing how a CDS could be redefined to a "Mortality Swap" and used by Captain Scott to eliminate the volatility of death benefit payments.

Commentary on Question:

The candidates performed poorly on this section. The candidates that did well sketched a diagram showing all three parties involved in the transaction and identified all cash flows to show how the 'mortality swap' eliminated the volatility of death benefit payments. The candidates that did poorly left out the cash flow transactions or one of the key counterparties.

There is more than one way a 'mortality swap' could be integrated within this situation. The example below is just one possible solution. Candidates providing other correct solutions received full marks.

Captain Scott could pay a monthly premium to the swap counterparty for the risk of pirate deaths and in return the swap counterparty will cover the death payments. This is equivalent to switching the 'credit' risk within a CDS with 'mortality' risk. Alternatively, it can be viewed as mortality reinsurance through the swap market.

Diagram of the situation is shown below:



4. The candidate will understand important quantitative techniques for analyzing financial time series Performance Measurement and Performance Attribution.

Learning Outcomes:

- (4f) Calculate and interpret performance attribution techniques.
- (4g) Explain the limitations of attribution techniques

Sources:

Handbook of Fixed Income Securities, Fabozzi, F.J., 8th Edition, 2012, Ch. 69

Commentary on Question:

This question tested the basic principles of performance attribution. Overall, candidates performed below the average on this questuon.

Solution:

(a) Critique the managers' statement.

Commentary on Question:

Candidates performed below average on this section. Most candidates just restated how the manger got their answer or said the manager's statement is correct or not.

Manager used the top down approach (69-5)to get outperformance 38bpts:

Security Selection
$$\sum_{s}^{s} w_{s}^{P} \cdot (R_{s}^{P} - R_{s}^{B})$$
 (69–5)

CFO used the bottom-up approach to get outperformance 35bpts:

Security selection
$$\sum_s w_s^B \cdot (R_s^P - R_s^B)$$
.

In this case, the firm has decided to use the bottom up approach. The managers' return is actually only 35 bps, because he has had the opportunity to select the securities first.

(b) Briefly describe which principles of performance attribution are relevant to this situation.

Commentary on Ouestion:

Candidates performed as expected on this section. More candidates identified the fairness principle than the completeness principle.

1. Completeness – The total return should always be attributed into components that can explain the entire result.

- 2. Fairness The allocation of outperformance to different interacting agents is performed in a way that is perceived to be fair by all agents.
- (c) Compute the portfolio's total return for the year, and justify your choice of method.

Commentary on Question:

Candidates performed as expected on this section. Some candidates used dollar weighted return instead of time weighted return, or provided the correct answer but did not state the reasons.

The preferred method is Time Weighted Return because the portfolio manager has no influence on the timing of cash flows.

$$R = (1+15\%)*(1+10\%) - 1 = 26.5\%$$

(d) Briefly describe the challenges associated with multi-period compounding.

Commentary on Question:

Candidates performed below average on this section. Most candidates stated that it is difficult depending on the choice of methods to calculate the return. Also some candidates stated how cash flow affects the return of the whole period.

- The problem of splitting multi period returns.
- Additivity is not preserved
- If the frequency of attribution is too low, part of portfolio performance may remain unexplained.

3. Candidate will understand the nature, measurement and management of liquidity risk in financial institutions.

Learning Outcomes:

- (3a) Understand the concept of liquidity risk and the threat it represents to financial intermediaries and markets.
- (3c) Understand the levels of liquidity available with various asset types, and the impact on a company's overall liquidity risk.
- (3g) Create liquidity risk management plans and procedures, including addressing appropriate product design, investment guidelines, and reporting given a desired liquidity risk level.

Sources:

QFIA-105-13: Report of the Life Liquidity Work Group of the American Academy of Actuaries to the Life Liquidity Risk Working Group of the NAIC (5 points)

Commentary on Question:

This question tested the candidates' knowledge of the impact of the liquidity risk embedded in various asset types and various liability termination procedures.

Important considerations for receiving maximum points include explaining concerns of liquidity positions instead of just listing high level points.

Solution:

(a) Explain concerns you have about ABC's liquidity position including key factors which may increase ABC's exposure to stress liquidity risk.

Commentary on Question:

The candidates performed above average on this section.

Most candidates pointed out key factors which may increase ABC's exposure to stress liquidity risk – such as 7 days notice constrain on the liability side, the illiquid assets such as low rated MBS and real estate. Implication of Experience (timing) and Predictability were also commonly answered.

Some candidates overlooked the fact that ABC got a recent downgrade and its implication to its liquidity concern. Some candidates noted the concern of size of contract holders. Most candidates did not mention the size of company may limit the funding choices available to ABC.

Liabilities – majority offer the ability to demand repayment on 7 days notice Recent downgrade creates incentive for some to call back funds Assets – real estate and low rated MBS highly illiquid Likely difficult to liquidate enough to meet the potential demand for cash

Size of contract holders – ABC has two creditors that control 70% of the liabilities – this is a very significant risk to ABC

Size of company – ABC is very small company (100mm) – this may limit the funding choices available to ABC

Predictability – Having 7-day puts in the contracts exposes ABC to the possibility of a sudden large cash demand

Experience (Timing) – The exercising of the 7-day puts limits the options available to ABC due to the very short time before the cash outflow

(b) Outline ways to quantify the liquidity of the assets.

Commentary on Question:

The candidates performed below average on this section.

Most candidates identified the projection of future expected asset principal. Most candidates also mentioned the calculation of haircuts to determine realized asset value.

Very few candidates pointed out the projection of future unexpected asset principal and interest payments, specifically the prepayment risk from MBS.

Project future expected asset principal and interest payments Project future unexpected asset principal and interest payments,

- Some products, such as the MBS, may have prepayment risks. Determine bid/ask spreads on the assets.
- The larger the bid/ask, the more the haircut should be.
- The bid/ask spread depends on type and quality rating of asset
- The volatility of fair value returns over time needs to be considered Calculate haircuts to determine realizable asset value
- (c) Recommend changes to the asset and/or liability profile to reduce ABC's stress liquidity risk.

Commentary on Question:

The candidates performed above average on this section

Most candidates recommended the changes outlined below, especially the elimination of liability 7-day put, decrease of real estate/MBS proportions, increase of cash/other liquid assets.

Few candidates did not recommend increasing the number of funding sources nor the asset liability management.

Liabilities:

Increase the number of sources of funding / reduce the amount of funding coming from any one source – to reduce the exposure the impact of any one funding provider

Try to not have 7-day puts be include in the funding sources – due to unpredictable nature of being redeemed

Assets:

Reduce the amount of illiquid asset classes (real estate & low-rated MBS) – due to difficulty in selling to meet unpredictable cash demands Increase the diversification of the assets / reduce the amount in any one asset – to reduce impact of any one asset

ALM

Increase the amount of fixed-rate funding sources given fixed-rate assets - to reduce exposure to increases in the rates to be paid on liabilities while the rates earned on assets remain fixed or Increase the amount of floating-rate assets to better align with the amount of floating-rate funding sources - to better offset increases in liability rates

2. The candidate will understand and be able to apply a variety of credit risk theories and models.

Learning Outcomes:

(2e) Demonstrate an understanding of the term structure of default probability.

Sources:

Introduction to Credit Risk Modeling, Bluhm, Christian, 2nd Edition, Chapter 6, pages 235-243

Commentary on Question:

This question tested the understanding and application of Q-matrices with respect to evaluating the term structure of default probabilities. The candidate needed to explain the reasons for using a Q-matrix and demonstrate their knowledge by evaluating the short-term default probability of a sample investment using a simplistic transition matrix.

Solution:

(a) Explain the reasons for using a Q-matrix to evaluate the default risk of this investment.

Commentary on Question:

The candidates performed below average on this section. Many candidates provided a reasonable definition of a Q-matrix and explained its application to derive a term structure of default probabilities. Very few candidates pointed out the deficiencies in transition matrices that lead to the need for Q-matrices.

- Q-matrices are used to derive a full term of structure of default probabilities by generating transition matrices from a continuous Markov process
- The provided transition matrix is based upon 1-year default data but we need to evaluate 6-month default risk for this bond; a Q-matrix provides default probabilities for shorter time periods.
- Fractional powers of the transition matrix cannot be used because its roots are not stochastic.
- Fractional powers of the transition matrix cannot be used because it's unclear how to handle multiple roots.
- (b) Calculate a generator Q of a Markov chain for ratings using

$$\tilde{Q} = \sum_{k=1}^{\infty} (-1)^{k+1} \frac{(M-I)^k}{k}$$
 and any necessary adjustments.

Commentary on Question:

The candidates performed as expected on this section. Many candidates derived a Q-matrix using the formulas. Many candidates also recognized the need to adjust the Q-matrix for negative entries, but few made appropriate adjustments. Also, some did not attempt the question or made incorrect attempts at answering the question and thus scored zero points.

$$Q = \sum_{k=1}^{\infty} (-1)^{k+1} \frac{(M-I)^k}{k}$$

$$= (M-I) + \sum_{k=2}^{\infty} (-1)^{k+1} \frac{(M-I)^k}{k}$$

$$= \begin{bmatrix} -0.3 & 0.28 & 0.01 & 0.01 \\ 0.15 & -0.35 & 0.15 & 0.05 \\ 0.05 & 0.1 & -0.4 & 0.25 \\ 0 & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} -0.1 & 0.15 & -0.05 & 0 \\ 0.07 & -0.15 & 0.1 & -0.02 \\ 0.01 & 0.05 & -0.13 & 0.07 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -0.4 & 0.43 & -0.04 & 0.01 \\ 0.22 & -0.5 & 0.25 & 0.03 \\ 0.06 & 0.15 & -0.53 & 0.32 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

We then need to adjust for a negative off-diagonal entry, i.e. $q_{13} = -0.04$. One approach is to allocate the negative result proportionately based on absolute weights:

$$q_{11} = -0.4 - 0.04 \times \frac{0.4}{0.84} = -0.42$$

$$q_{12} = 0.43 - 0.04 \times \frac{0.43}{0.84} = 0.41$$

$$q_{13} = 0.01 - 0.04 \times \frac{0.01}{0.84} = 0.01$$

This then leads to the following adjusted Q-matrix:

$$Adjusted Q = \begin{bmatrix} -0.42 & 0.41 & 0 & 0.01 \\ 0.22 & -0.5 & 0.25 & 0.03 \\ 0.06 & 0.15 & -0.53 & 0.32 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Other methods to adjust for a negative entry were permissible.

(c) Calculate the L^1 -norm as a measure of error for the revised Q-matrix.

Commentary on Question:

The candidates performed poorly on this section. A small number of candidates got full credit for this question but most candidates did not know how to calculate an L^1 -norm and therefore scored zero points. Note that only 2 differences produced non-zero results and thus the math was quite simple.

$$L^{1}$$
-norm = $||M - e^{Q}||_{1} = \sum_{i,j} |m_{i,j} - q_{i,j}|$
= $|0.08 - 0.07| + |0.3 - 0.31| = 0.02$

(d) Calculate the probability that the bond defaults within 6 months.

Commentary on Question:

The candidates performed below average on this section. Some candidates got the correct answer and received full credit. However, some candidates only received partial credit by performing a few of the preliminary steps or simply identifying that they need to obtain the value in the second row and fourth column. Many did not attempt the question or made incorrect attempts at answering the question and thus scored zero points.

$$M(t) = e^{tQ} = \sum_{k=0}^{\infty} \frac{(tQ)^k}{k!}$$

$$= I + tQ + \sum_{k=2}^{\infty} \frac{(tQ)^k}{k!}$$

$$M(0.5) = I + 0.5Q + \sum_{k=2}^{\infty} \frac{(0.5Q)^k}{k!}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} -0.35 & 0.3 & 0.045 & 0.005 \\ 0.12 & -0.3 & 0.15 & 0.03 \\ 0.025 & 0.05 & -0.275 & 0.2 \\ 0 & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0.07 & -0.08 & 0 & 0.01 \\ -0.03 & 0.06 & -0.03 & 0.01 \\ 0 & -0.01 & 0.04 & -0.02 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0.72 & 0.22 & 0.045 & 0.015 \\ 0.09 & 0.76 & 0.12 & 0.04 \\ 0.025 & 0.04 & 0.765 & 0.18 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\rightarrow m_{24} = 0.04$$

2. The candidate will understand and be able to apply a variety of credit risk theories and models.

Learning Outcomes:

(2j) Demonstrate an understanding of mortgage default models in the valuation of MBS.

Sources:

QFIA-100-13: "Modeling of Mortgage Defaults", Jan 22, 2008, pp. 5-38 excl. pp. 13-25 (background only)

Commentary on Question:

This question tested the candidate's understanding of the mortgage insurance.

Solution:

(a) Describe the three components a models utilized in the valuation of mortgage credit risk.

Commentary on Question:

The candidates performed poorly on this section. Most candidates did not list or describe the components.

- Default when a loan becomes delinquent and is paid off with a loss to the servicer
- Prepayments when a loan is prepaid without a loss, e.g., due to refinance
- Loss severity the amount of loss when a mortgage defaults
- (b) Critique the above statement.

Commentary on Question:

The candidates performed below average on this section. Most candidates agreed HPA was an important factor but did not provide adequate reasoning. Most of the other areas to critique were omitted

Generally agree that HPA is one of the most important factors

HPA could still have a strong effect on mortgage default rates, beyond the impact of HPA on LTV

There are several reasons for this:

 housing market may be a proxy for general economic conditions, so a weakening housing sector can mean a deteriorating situation for struggling borrowers

- servicers may decide to cut their losses when they think home prices are going to fall and provide incentives to a delinquent borrower to do a short sale (that is, sell the home for less than the mortgage balance)
- declining home prices may have a psychological impact on struggling homeowners, who may decide that it is not worthwhile to try to hold on to the property
- (c) Identify an additional economic factor that the director is implying and explain its relationship to the HPA.

Commentary on Question:

The candidates performed below average on this section. Many candidates made comments on the 2008 subprime mortgage crisis which was not the intent of the question. Those that did comment on LTV did not fully explain the relationship to HPA.

Loan-to-value (LTV) ratio measures the amount of equity the borrower has in the home – thus determines how feasible it will be for the borrower to refinance or sell if he can no longer make the monthly payments

The current loan-to-value (LTV) = the initial LTV + the cumulative HPA, along with amortization

(d) Explain the complications that could arise in applying the factor identified above.

Commentary on Question:

The candidates performed poorly on this section. About half left the question blank. Virtually no one commented on inflated appraisals or silent 2nds.

Inflated appraisals: were widespread in the housing boom of recent past, especially for cash-out re-financings, and so the reported initial LTV may need to be adjusted

Silent 2nds: If the borrower has taken out a second mortgage on the property, then the combined LTV should be used. But we may not know there is a second lien on the property. An educated guess has to be made of the likely proportion of properties that have loans with second liens

(e) Describe what information is contained in the ABX and how it could be used to price mortgage credit.

Commentary on Question:

The candidates performed poorly on this section. There were some general comments about the ABX but virtually no one commented specifically about the market expectation or the average over possible HPA scenarios.

The ABX index incorporates information about market expectations for home price appreciation and subprime credit performance time just as the yield curve incorporates market expectations for the economy and inflation Just as forward rates are in some sense a probability weighted average of possible rates, ABX Index valuations can be thought of as an average over possible HPA scenarios

Thus, ABX indexes can be used to price mortgage credit.

- 6. The candidate will understand and be able to describe the variety and assess the role of alternative assets in investment portfolios. The candidate will demonstrate an understanding of the distinguishing investment characteristics and potential contributions to investment portfolios of the following major alternative asset groups:
 - Real Estate
 - Private Equity
 - Commodities
 - Hedge Funds
 - Managed Futures
 - Distressed Securities
 - Farmland and Timber

Learning Outcomes:

(6b) Demonstrate an understanding of the benchmarks available to evaluate the performance of alternative investment managers and the limitations of the benchmarks.

Sources:

QFIA-111-13-Maginn & Tuttle, Managing Investment Portfolios, 3rd Ed. 2007, Ch. 8

Commentary on Question:

Overall, the candidates performed below average on this question. This question tested the candidate's understanding of how different statistics relate to hedge funds. The candidate was expected to be able to read through and beyond the different stats and indices published. The candidate also needed to demonstrate a basic knowledge of what makes two hedge funds comparable or different.

Solution:

(a) Describe four of the main differences in the constructions of the major manager-based hedge fund indices.

Commentary on Question:

The candidates performed below average on this section. The question asked for a brief description of each different element. Simply listing each different element only got partial credit.

Selection criteria:

- How and which basis a hedge fund is include in the index.
- Decision rules to determine which hedge funds are included in the index.

Style classification:

 Indices have various approaches to how hedge funds is assigned to specific index.

Weighting schemes:

- How much weight a particular fund's return is given in the index.
- Equal weight, dollar weight, etc.

Rebalancing scheme

• Frequency at which indices are rebalance.

Investability/replicability

- Does the index is directly or indirectly investable? Is it possible to replicate the index?
- (b) Describe four different biases that an index may present including commentary on the direction of the bias.

Commentary on Question:

The candidates performed as expected on this section. Most got two of the bias correct with a satisfactory description. A correct description of the impact was necessary to get full credit.

Reporting bias:

- Most index database are self-reported: different hedge fund manager chooses which database to report and provide the return data.
- Then the index overestimates the performance of the market.

Surivorship bias:

 Manager with poor track records exit the business and are dropped from the index while those with a good track record stay.

Stale price bias:

- Infrequent trading or absence of trading makes it difficult for an index to represent the value at a given time. Values used to build the index may not be up to date.
- This will underestimate the volatility of the index.

Backfill bias:

- Missing past return data for a component of an index are filled at the discretion of the component when it joins the index.
- Results may look too good, only component with good past results are likely to be included.
- (c) Explain why different fund managers may obtain different results than you in this context.

Commentary on Question:

The candidates performed below average on this section. It was explicit in the question that the data was not an issue and that there were no exclusions, omissions or illicit manipulations of data.

A key element here was to recognize that the sharpe ratio can be "gamed" and to explain how this is done without any manipulating the data.

Another key element is that the sharpe ratio is time dependent and this needed to be identified and explained.

The Sharpe ratio is time dependent.

- Sharpe ratio increase proportionally with the square root of time.
- An annual Sharpe ratio will be $\sqrt{12}$ bigger than a monthly Sharpe ratio.

The Sharpe ratio can be gamed, the reported ratio can be increased without the investment.

- By lengthening the measurement interval. This will underestimate the volatility.
- Compounding the monthly return but calculating the standard deviation from the not compounded monthly returns.
- Smoothing of returns. Infrequent marking to market of illiquid assets and pricing models that understate monthly gains or losses can reduce reported volatility.
- (d) Describe the formula and calculate the Sortino ratio for this fund.

Commentary on Question:

The candidates performed below average on this section. Many candidates used the average rate of return rather than the risk free rate. Calculating the annualized rate of return was expected to be a trivial calculation, but most had this trivial element wrong. The question explicitly stated that the rates given were already annualized, thus the annualized rate of return was obtained by a geometric average rather than an arithmetic average. The sortino ratio formula was given in the formula sheet provided with the exam.

Sortino ratio = (annualized rate of return – risk free return)/Downside deviation = (ARR –Rf)/DD

DD =
$$(\min(0.1\%-3\%)^2 + \min(0.4.35\%-3\%)^2 + \min(0.2\%-3\%)^2 + \min(0.5.05\%-3\%)^2)/(n-1)$$

$$DD = (-2\%^2 + 0\%^2 + -1\%^2 + 0\%^2)/3 = 1.290994\%$$

$$ARR = -1 + ((1+1\%)x(1+4.35\%)x(1+2\%)x(1+5.05\%))^{1/4} = 3.0867\%$$

Sortino ratio =
$$(3.0867\% - 3.0\%)/1.290994\% = 0.0671$$

- 1. The candidate will understand the standard yield curve models, including:
 - One and two-factor short rate models
 - LIBOR market models

The candidate will understand approaches to volatility modeling.

Learning Outcomes:

- (1p) Describe and contrast several approaches for modeling smiles, including: Stochastic Volatility, local-volatility, jump-diffusions, variance-gamma and mixture models.
- (1q) Describe and explain various issues and approaches for fitting a volatility surface.

Sources:

Volatility Correlation-The Perfect Hedger and the Fox, R., 2nd Edition, Section 7.3.1, p.206

Volatility Correlation - The Perfect Hedger and the Fox, Rebonato, R., 2nd Edition, Section 8.1, p. 237-239

Volatility Correlation - The Perfect Hedger and the Fox, Rebonato, R., 2nd Edition, Sections 8.3 and 8.5, p. 241-265

Commentary on Question:

This question tested candidate's ability to describe and contrast several approaches for modeling smiles.

Solution:

(a) Identify four basic facts of equity smiles.

Commentary on Question:

The candidates performed brilliantly on this section. A few candidates made the following mistakes about Fact 5 below: an increase of asymmetry is different than an increase of smiles, and a market turbulence is different from a bad economy.

We are showing 5 Facts for completeness sake.

Fact 1: Smiles have greatly increased in magnitude after the 1987 equity market crash.

Fact 2: The magnitude of the smile as a function of a fixed money strike tends to decrease for increasing option expiries: short maturities display pronounced smiles, and distant maturities give rise to shallow smiles. What constitutes a 'short' or a 'long' maturity, can, of course, change over time.

Fact 3: The magnitude of the smile as a function of the degree of out-of-themoney-ness is much more constant across different option expiries.

Fact 4: The smile is much more pronounced going from the ATM level towards out-of-the-money puts than in the opposite direction. Going towards out-of-the-money calls the smile either becomes less steep, or is monotonically decreasing, or, sometimes, it is even absent.

Fact 5: The asymmetry in the smile tends to increase during periods of market turbulence.

(b) Discuss whether you would expect changes in implied volatility to be positively or negatively correlated with the level of the underlying and whether you would expect this correlation to be stronger with longer or shorter options.

Commentary on Question:

The candidates performed below average on this section. Most candidates identified that the correlation is weak and becomes weaker as the option expiry increases. However, some candidates incorrectly provided an answer of positive correlation. Furthermore, most candidates did not note that the negative correlation was observed in 2 different studies.

A strong negative correlation has been noted between the level of the 3-month ATM implied volatility and the level of the index for the S&P500 during the period September 1997 to November 1998.

The correlation is weak and becomes weaker as the option expiry increases. The correlation is negative but rather weak, and becomes progressively weaker as the option expiry increases: 35%, 34%, 30%, 28%, 10% for 1-month, 3-month, 6-month, 1-year and 5-year options, respectively.

This was observed in two different studies. Rebonato extended the observation by estimating the strength of the correlation between changes in the implied volatility and in the level of the index for the the S&P500 for several maturities during the period 4-Feb-2000/31-May-2001.

(c) Describe and compare a fully-stochastic-volatility model and a local volatility model.

Commentary on Question:

The candidates performed below average on this section. Most candidates did not provide the full mathematical formula for a fully-stochastic-volatility model.

Description of a fully-stochastic-volatility model: A fully-stochastic-volatility model is a model of the following form:

$$\begin{split} dS &= \mu_S \ (S,V,t) dt + \sigma_S \ (S,V,T) \ dZ_t, \\ dV &= \mu_V \ (S,V,t) dt + \sigma_V \ (S,V,T) d, \\ W_t \ V &= \sigma^2 2, \\ E[dWdZ] &= \rho dt, \end{split}$$

where Z_t and W_t are two different Brownian motions.

Difference between a fully-stochastic-volatility model and a local volatility model:

In a local volatility model, the stochasticity only arises due to the functional dependence of the volatility on the underlying; in a fully-stochastic-volatility model, a second source of uncertainty (usually a second Browian motion) affects the evolution of the stock price. "

(d) Explain the cause of the smile effect with a fully-stochastic-volatility model.

Commentary on Question:

The candidates performed poorly on this section. Most candidates were unable to explain any behavior of the Black-Scholes formula at at-the-money or out-of-money levels, in connection to its convexity.

At the money the Black-Scholes formula is virtually exactly linear in the root-mean-squared volatility, but cease to be so away from the at-the-money level.

Therefore, at the money the effect of a high volatility path will be almost exactly cancelled by the associate, 'miror-image', equi-probability low-volatility path.

Away from the at-the-money level, the average of Black-Scholes options prices obtained with a higher- and lower-than-average root-mean-squared volatility will be higher than then option price evaluated with the average volatility.

Therefore, the smile effect with stochastic volatility models is a direct consequence of the convexity of the Black-Scholes formula.

(e) Identify a problem surrounding using a diffusion-based fully-stochastic-volatility model to model smiles for short maturities and explain what could lead to such a problem.

Commentary on Question:

The candidates performed as expected on this section. Most candidates did not explain why the convexity of the smile stems from the convexity of the Black-Scholes formula.

The potential problem:

The fully stochastic volatility model tends to produce rather shallow smiles for short maturities. This is at odds of the empirical findings.

The causes of the problem: since the convexity of the smile stems from the convexity of the Black-Scholes formula, in general the effect will be more pronounced when there is a high probability of finding widely dispersed values of the root-mean-squared volatility. With Brownian-diffusion-based stochastic-volatility models, however, today's volatility can only continuously diffuse away from its initial value. Therefore, for 'reasonable choices' of the coefficients, fully-stochastic-volatility models tend to produce rather shallow smiles for short maturities.

(f) Explain two possible solutions to solve the problem in (e).

Commentary on Question:

The candidates performed below average on this section. Most candidates did not justify why the 2 possible solutions will be helpful. For example, most candidates did not mention that a combination of the two approaches (see illustrative solution below) would seem to give the best of both worlds: sharp, short-maturity smiles that persist over intermediate maturities.

Two possible ways to overcome the problems:

- Use a jump-diffusion model;
- Use a combined jump-diffusion plus fully-stochastic-volatility model

Jump-diffusion models:

As in fully-stochastic-volatility models, the smile-producing mechanism of jump diffusion models is due to the non-linearity of the Black-Scholes function in the realized 'volatility'. However, owing to the discontinuous nature of the process, and to the fact that the size of the jump does not scale with the size of the time-step, jump-diffusion processes give a much higher probability of 'large' moves of the underlying for short times after 'today'. Therefore, the non-linearity of the Black-Scholes is allowed to 'kick' in much earlier, and the smile can be steepter at shorter maturities.

Mixing processes:

The basic idea of using a mixing process is that jump-diffusion description is 'almost right', and that the local-volatility addition should only be used as a 'correction term' to get an extra fit to market data. Joining a stochastic diffusive component with jumps can fix two often-observed problems: stochastic-volatility models produce appreciable smiles for intermediate maturities, but have difficulties in producing the sharp smiles for short maturities often observed in the market. Jump-diffusion processes, on the other hand, do produce sharp smiles at the short end of the expiry spectrum, but these tend to decay too rapidly. A combination of the two approaches would there seem to give the best of both worlds: sharp, short-maturity smiles that persist over intermediate maturities.

(g) Recommend one of the solutions in (f) and justify your recommendation.

Commentary on Question:

The candidates performed below average on this section. Most candidates did not explain some weakness of jump-diffusion (decay too fast) and a mixed process (be difficult for traders to judge whether a good fit to market data indicate a well-specified modeling approach).

Jump-diffusion processes decay too fast, although it is more parsimonious than mixing processes.

Mixing processes allows sharp smiles for short maturities and do not decay too fast; but it has more parameters than jump-diffusion models.

Therefore, mixing process models are not parsimonious and it could be difficult for traders to judge whether a good fit to market data indicate a well-specified modeling approach, or is simply the result of the flexibility of the chosen set of parameter functions.

Recommendation can be either one of the two as long as the justification is sound.

4. The candidate will understand important quantitative techniques for analyzing financial time series Performance Measurement and Performance Attribution.

Learning Outcomes:

(4h) Understand and apply various techniques of adjusting auto correlated returns for certain asset classes.

Sources:

QFIA-110-13: CAIA Level II, Advance Core Topics in Alternative Investment, 2nd Edition, 2012, Ch.16

Commentary on Question:

The question tested the following:

- Understand the difference between smoothed and unsmoothed returns
- Understand why returns of certain asset classes are "smoothed"
- Application of a first order autocorrelation adjustment to unsmoothed returns
- Understand the implications and inherent risks of only looking at smoothed returns and calculating risk adjusted measures

Solution:

(a) List the four primary explanations for delayed price changes and smoothed prices.

Commentary on Question:

Candidates performed below average on this part. Only a few candidates correctly listed all four primary explanations. Many candidates only mentioned that appraiser observes price changes on a delayed basis.

Price index is being based on observed prices of the most recent transactions of each component of the index and that old or stale prices are being used for index components that have not recently traded.

Appraiser observes price changes on a delayed basis and only on those properties transacted (or may exhibit anchoring).

Even current transaction prices in an efficient market may be selected such that they signal lagged price responses. (Reluctance to buy property types with higher price increases, and reluctance to sell property types that declined the most.)

Delay between the setting of a price on a real estate transaction and the reporting of the transaction.

(b) Rank the five return series in order of increasing delay in price response and justify your ranking.

Commentary on Question:

Candidates performed as expected on this part. Candidates in general could identify C as the least delay in price response with the correct reason. Some candidates did not rank A before B. Many candidates did not explain why B is ranked before E and some candidates did not rank E before D. At times the order was correct but the explanations were not complete.

The order from the least decay in price response: C A B E D

C is likely to have the least delay in price response.

Because it is tradable, and it has no transaction cost or trading barriers

If any consistent delay in price response exits, arbitrageurs will try to exploit profit opportunities by trading in the same direction before the delayed prices respond (i.e. if asset price rises, arbitrageurs will take a long position, if asset price declines, arbitrageurs will take a short position in the asset with delayed response). Arbitrage will eliminate all profits.

Competition between arbitrageurs will force prices to respond fully and immediately in the absence of transaction costs.

D is likely to have more delay in responses.

D has lower standard deviation when compared to the unsmoothed prices.

The standard deviation of smoothed series is substantially lower, because smoothing causes the largest outliers of the unsmoothed series to be muted.

A is ranked earlier than B because A has a lower ρ , meaning less correlation between the price (return) reported in the previous period and the return/price reported in current period.

B is ranked earlier than E because B has lower ρ.

E is ranked before D, because of the standard deviation as a percentage of unsmooth prices is lower for D. This makes D the series with the most delay in price (return) response.

(c) Estimate the unsmoothed return for the current period using first-order autocorrelation.

Commentary on Question:

Candidates performed brilliantly on this part, however some did not write down the correct formulas and some made mistakes in the calculations.

$$\begin{split} \hat{\rho} &= \frac{\sigma_{i,j}}{\sigma_i \cdot \sigma_j} \\ \hat{\rho} &= 0.024 / (\sqrt{0.045} \sqrt{0.04} \,) = 0.024 / \, (0.21213)(0.2) = 0.566 \\ R_{t,true} &= (R_{t,reported} - \rho \, R_{t-1,reported}) / (1 - \rho) \\ &= (3\% - 0.566*10\%) / (1 - 0.566) = -6.12\% \end{split}$$

(d) Describe potential issues with using smoothed returns when calculating risk or performing asset allocation.

Commentary on Question:

Candidates performed above average on this part. They generally pointed out that smoothed returns understate volatility and correlation. Some candidates did not mention that smoothed returns inflate estimates of risk adjusted returns such as Sharpe ratio.

Smoothed returns mute the largest outliers of the unsmoothed series, hence understating the volatility

Smoothed returns inflate estimates of risk adjusted returns that use volatility as a denominator, such as Sharpe ratio.

Smoothed returns understate correlation with other assets. An asset with smoothed returns may appear to be a good diversifying asset due to its low correlation, when the correlation is actually understated due to smoothing.

3. Candidate will understand the nature, measurement and management of liquidity risk in financial institutions.

Learning Outcomes:

- (3a) Understand the concept of liquidity risk and the threat it represents to financial intermediaries and markets.
- (3b) Measure and monitor liquidity risk, using various liquidity measurement tools and ratios.
- (3c) Understand the levels of liquidity available with various asset types, and the impact on a company's overall liquidity risk.
- (3g) Create liquidity risk management plans and procedures, including addressing appropriate product design, investment guidelines, and reporting given a desired liquidity risk level.

Sources:

Liquidity Risk: Measurement and Management – A Practitioner's Guide to Global Best Practices, Matz, Leonard & Neu, Peter Ch. 2 (4 points)

Reflections on Northern Rock: The Bank Run that Heralded the Global Financial Crisis (2 points)

Commentary on Question:

This question tested the candidates' understanding of liquidity risk, including the levels of liquidity available within various asset classes and liquidity measurement tools and ratios.

Solution:

(a) Calculate PBJ's Liquidity Ratio under both the going concern scenario and the panic scenario.

Commentary on Question:

The candidates performed brilliantly on this section. Many candidates were awarded full credit for the correct answer. A small number of candidates lost credit by incorrectly calculating the panic demand liability amount.

```
Cash and short term assets = 360 + 500 = 860
Going Concern demand liabilities = 1,000 * 5\% + 3,500 * 10\% = 400
Going Concern liquidity ratio = 860 / 400 = 2.15
Panic demand liabilities = 5,000 * 10\% + 1,000 * 90\% + 3,500 * 80\% = 4,200
Panic liquidity ratio = 860 / 4200 = 0.2047
```

(b) Describe the problems with the ERM VP's proposed liquidity management approach.

Commentary on Question:

The candidates performed below average on this section. Many candidates received partial credit for identifying one or two of the problems. Some candidates provided answers that were not related to the proposed approach as shown in the question.

Problems with proposed liquidity management approach:

- Only considers one stress scenario not sufficient to capture the range of disintermediation risks on both the asset and liability side
- Quarterly review not frequent enough
- Time horizon of each asset/liability is ignored
- Off balance sheet exposure not considered
- Process is impacted by accounting rules which do not actually affect the company's liquidity position
- Marketability of the assets not considered
- (c) Critique the CIO's recommendation including consideration of the change in liquidity ratio.

Commentary on Question:

The candidates performed as expected on this section. Some candidates received partial credit by correctly calculating the liquidity ratio under the proposal and discussing the potential implications of the lower ratio. Other candidates did not discuss any of the impacts on the liquidity ratio, despite it being specifically asked for in the question.

Not advisable to move the cash position into MBS In a panic scenario, the MBS market could become illiquid. The panic scenario liquidity ratio under this recommendation – 500/4,200 = 0.119

A liquidity ratio below 15% would put PBJ at risk of being downgraded

(d) Explain both the rationale for, and criticism against, a raw leverage constraint.

Commentary on Question:

The candidates performed poorly on this section. Some candidates received some credit for the criticism provided. Most candidates provided answers that demonstrated a lack of familiarity with the concept of a raw leverage constraint and therefore received no credit.

Rationale – Bank can survive a run if it has sufficient liquid assets or if it has sufficiently stable liabilities. Too much illiquidity on the liabilities side of the balance sheet would undermine the incentive effects. The idea would be to strike a balance between mitigating the externalities and preserving incentives.

Argument for raw leverage constraint – It can act as a binding constraint on the way up when the banks increase leverage on the back of permissive lending conditions. The buildup of excessive leverage makes the system vulnerable to a shock that raises the implicit haircuts.

Criticism against raw leverage constraint – It does not take into account the riskiness of the assets.

- 1. The candidate will understand the standard yield curve models, including:
 - One and two-factor short rate models
 - LIBOR market models

The candidate will understand approaches to volatility modeling.

Learning Outcomes:

(1h) Explain the set up and motivation of the Lognormal Forward LIBOR Model (LFM).

Sources:

Interest Rate Models – Theory and Practice, 2nd Edition, Brigo, D and Mecurio F, Sections 4.1-4.2, page number: 144 – 169

Commentary on Question:

This question tested the candidates' understanding of an interest rate model from a practical point of view including calibration, parameter generation, correlation, expected value and variance of short rates, and probability of negative rates. Overall the candidates performed as expected on this question.

Solution:

(a) List the considerations to determine the appropriate instruments for two-factor short rate model calibration.

Commentary on Ouestion:

The candidates performed below average on this section. Most identified that correlation is the key consideration for calibration, however many did not clearly explain how swaptions and caps would impact calibration differently. Also most did not explain what would happen if swaptions and caps were used together for calibration.

- 1. Cap prices do not depend on the correlation of forward rates, so that even a one-factor model that implies perfectly-correlated rates can fit caps data well in many situations. It often happens that the ρ value is quite close to minus one, which implies that the G2++ model tends to degenerate into a one-factor (non-Markov) short rate process
- 2. Swaption prices contain information on the correlation between forward rates, and indeed a possible way to incorporate such information into the model consists in assigning a non-trivial value to the ρ coefficient.
- 3. When in need to price a particular product that is influenced only by a certain set of swap rates, it may be reasonable to calibrate the model only to the relevant swaptions.

- 4. A joint calibration to caps and swaptions usually is not complete satisfactory. This may be due both to misalignments between the two markets and to the low number of parameters in the model.
- (b) Calculate the G2++ parameters equivalent to the parameterization of the Hull-White two-factor model above.

Commentary on Question:

The candidates performed brilliantly on this section. Most of them understood what formulas to use to calculate G2++ parameters. A few of them did the calculation wrong.

To transform the HW two-factor model into the G2++ model, one must set

$$a = \overline{a} = 0.7$$

$$b = \overline{b} = 0.1$$

$$\eta = \frac{\sigma_2}{\overline{a} - \overline{b}} = 0.01667$$

$$\sigma = \sqrt{\sigma_1^2 + \frac{\sigma_2^2}{\left(\overline{a} - \overline{b}\right)^2} + 2\overline{\rho} \frac{\sigma_1 \sigma_2}{\overline{b} - \overline{a}}} = 0.02455$$

$$\rho = \frac{\sigma_1 \overline{\rho} - \eta}{\sigma} = -0.801$$

(c) Calculate the instantaneous correlation between 1-month forward rates and 10-year forward rates at time 0.

Commentary on Question:

The candidates performed as expected on this section. Most of them identified what formula to use to calculate the correlation between two rates. The calculation was time consuming and a few did not perform the calculation however those that showed some of the steps got partial credit.

The instantaneous correlation between the two forward rates f(0,1M) and f(0,10Y) is calculated by formula in page 152 (page 5 in the formula sheets).

$$\begin{split} &Corr\Big(df(t,T_1),df(t,T_2)\Big)\\ &=\frac{\sigma^2e^{-a(T_1+T_2-2t)}+\eta^2e^{-b(T_1+T_2-2t)}+\rho\sigma\eta(e^{-aT_1-bT_2+(a+b)t}+e^{-bT_1-aT_2+(a+b)t})}{\sigma_f(t,T_1)\sigma_f(t,T_2)} \end{split}$$

$$\sigma_r^2(t) = Var[r_t] = \frac{\sigma^2}{2a}(1 - e^{-2at}) + \frac{\eta^2}{2b}(1 - e^{-2bt}) + 2\frac{\rho\sigma\eta}{a+b}(1 - e^{-(a+b)t})$$
(4.6)

$$\sigma_f(t,T) = \sqrt{\sigma^2 e^{-2a(T-t)} + \eta^2 e^{-2b(T-t)} + 2\rho\sigma\eta e^{-(a+b)(T-t)}}.$$
 (4.16)

By replacing t = 0, T_1 =1/12, T_2 =10, a=0.7, b=0.1, η =0.01667, σ = 0.02455, and ρ =-0.801 from part b), we get $\sigma_f(0,1/12)$ = 1.40% $\sigma_f(0,10)$ = 0.61%

$$Corr\left(df\left(0, \frac{1}{12}\right), df(0, 10)\right) = -14.23\%$$

- (d) At time 0, the 1-month forward rate $f^{M}(0, 1/12)$ is 0.5%.
 - (i) Determine the formula for calculating the expected short rate at time t = 1/12.
 - (ii) Determine the formula for calculating the variance of the short rate at time t = 1/12.

Commentary on Question:

The candidates performed as expected on this section. Most determined what formulae to use to calculate the expected value and variance of the short rate. To get the full points, candidates needed to simplify the formulae using s=0 which demonstrated an understanding of the concept.

For any time t, r_t the instantaneous short rate, in this case, 1M rate follows a normal distribution with (4.7, 4.6 from page 144 and 4.12 from page 146) (page 5 in the formula sheets).

$$E\{r(t)|\mathcal{F}_s\} = x(s)e^{-a(t-s)} + y(s)e^{-b(t-s)} + \varphi(t),$$

$$r(t) = x(t) + y(t) + \varphi(t), \quad r(0) = r_0,$$
 (4.4)

where the processes $\{x(t): t \geq 0\}$ and $\{y(t): t \geq 0\}$ satisfy

$$dx(t) = -ax(t)dt + \sigma dW_1(t), \quad x(0) = 0,$$

$$dy(t) = -by(t)dt + \eta dW_2(t), \quad y(0) = 0,$$
(4.5)

$$\begin{split} \varphi(T) &= f^{M}(0,T) + \frac{\sigma^{2}}{2a^{2}} \left(1 - e^{-aT}\right)^{2} \\ &+ \frac{\eta^{2}}{2b^{2}} \left(1 - e^{-bT}\right)^{2} + \rho \frac{\sigma \eta}{ab} \left(1 - e^{-aT}\right) \left(1 - e^{-bT}\right), \quad (4.12) \end{split}$$

Since s = 0

$$E[r_t] = \varphi(t) = f^M(0,t) + \frac{\sigma^2}{2a^2} (1 - e^{-at})^2 + \frac{\eta^2}{2b^2} (1 - e^{-bt})^2 + \rho \frac{\sigma \eta}{ab} (1 - e^{-at}) (1 - e^{-bt})$$

 $E[r_t]$ will be obtained by replacing t = 1/12, a=0.7, b=0.1, η =0.01667,= 0.02455, and ρ =-0.801.

$$\operatorname{Var}\{r(t)|\mathcal{F}_{s}\} = \frac{\sigma^{2}}{2a} \left[1 - e^{-2a(t-s)} \right] + \frac{\eta^{2}}{2b} \left[1 - e^{-2b(t-s)} \right] + 2\rho \frac{\sigma\eta}{a+b} \left[1 - e^{-(a+b)(t-s)} \right]. \tag{4.6}$$

Since s = 0,

$$\sigma_r^2(t) = Var[r_t] = \frac{\sigma^2}{2a} (1 - e^{-2at}) + \frac{\eta^2}{2b} (1 - e^{-2bt}) + 2\frac{\rho \sigma \eta}{a + b} (1 - e^{-(a+b)t})$$
(4.6)

 $Var[r_t]$ will be obtained by replacing t = 1/12, etc..

(e) Calculate the probability that the short rate at time t = 1/12 is negative.

Commentary on Question:

The candidates performed above average on this section. Most understood what formulae to use to calculate probability of negative short rates. A few did the calculation wrong.

Remark 4.2.2. (Short-rate distribution and probability of negative rates). By fitting the currently-observed term structure of discount factors, i.e. by applying (4.12), we obtain that the expected instantaneous short rate at time t, $\mu_r(t)$, is

$$\mu_r(t) := E\{r(t)\} = f^M(0,t) + \frac{\sigma^2}{2a^2} \left(1 - e^{-at}\right)^2 + \frac{\eta^2}{2b^2} \left(1 - e^{-bt}\right)^2 + \rho \frac{\sigma\eta}{ab} \left(1 - e^{-at}\right) \left(1 - e^{-bt}\right),$$

while the variance $\sigma_r^2(t)$ of the instantaneous short rate at time t, see (4.6), is

$$\sigma_r^2(t) = \text{Var}\{r(t)\} = \frac{\sigma^2}{2a} \left(1 - e^{-2at}\right) + \frac{\eta^2}{2b} \left(1 - e^{-2bt}\right) + 2\frac{\rho \sigma \eta}{a+b} \left(1 - e^{-(a+b)t}\right).$$

This implies that the risk-neutral probability of negative rates at time t is

$$Q\{r(t) < 0\} = \varPhi\left(-\frac{\mu_r(t)}{\sigma_r(t)}\right),\,$$

 $E[r_{1/12}] = 0.44\%$ and $Var[r_{1/12}] = 0.1367\%$

$$Q{r(t) < 0} = \Phi(-1.19) = 11.61\%$$

4. The candidate will understand important quantitative techniques for analyzing financial time series Performance Measurement and Performance Attribution.

Learning Outcomes:

- (4f) Calculate and interpret performance attribution techniques.
- (4g) Explain the limitations of attribution techniques

Sources:

Fabozzi, F.J., Handbook of Fixed Income Securities, 8th

Edition, Chapter 70, pages 1672 – 1679, 1682 – 1688 and 1697 -1710.

Commentary on Question:

The question asked the candidates to recall the performance attribution methods and applied the formulae to analyze the outperformance. The question also asked the candidates to identify the risks embedded with new additions of assets class and assess the appropriateness of each PA method. The candidates were generally able to identify and apply the correct formulae, but fell short on identifying the risks and further critique each PA method.

Solution:

(a) Describe how SG's excess return can be broken down by return factors.

Commentary on Question:

The candidates performed below average on this section. Most candidates only identified the methods (security selection, asset allocation, top level exposures), but were not able to further identify the return factors used to analyze the excess return.

- Yield curve
- Implied volatility
- Spread
- Inflation
- Residual
- (b) Calculate the following using the total return method:
 - (i) X, Y and Z in the above table
 - (ii) The top-level exposure

Commentary on Question:

The candidates performed above average on this section. Most candidates were able to identify the formulae and apply it correctly. A number of errors were due to arithmetic mistakes.

(i) To find X, we need the total benchmark excess return:

$$TR^{B} = 326 * 33\% + 268 * 21\% + 321 * 23\% + 162 * 15\% + 0 * 8\% = 262$$

$$TR^P = 442 * 38\% + 354 * 28\% + X * 12\% + 237 * 21\% + 0 * 1\% = 262 + 87 = 349$$

Solve for X, we get X = 268.

$$Y = (W_s^P - W_s^B) * (TR_s^B - TR^B) = (21\% - 15\%) * (162 - 262) = -6$$

$$Z = (TR_s^P - TR_s^B) * W_s^P = (268 - 321) * 12\% = -6.36$$

(ii) Top-Level Exposure
$$(w^P - w^B) \cdot TR^B$$
 (70–3)

Absent leverage, the top level weights of the portfolio and the benchmark are both equal to one, so the top-level exposure term is always 0.

(c) Identify risk factors that result from these additions to the portfolio.

Commentary on Question:

The candidates performed as expected on this section. Most candidates were able to identify some risk factors, but only a few had identified a sufficient number of them. The question asked the candidate to identify the risks associated with the addition of the two new assets. No credit was given for risk factors that already existed in the portfolio.

ILS: risks associated with the reference inflation indices: inflation spread, inflation rate carry, rate accretion, rate surprise.

MBS: risk associated with the mortgages: prepayment, mortgage spread change, home price appreciation.

- (d) Assess whether each of the following performance attribution methods is appropriate for this portfolio:
 - (i) The Total Return Method
 - (ii) The Excess Return Method
 - (iii) The Fully Analytical Method

Commentary on Question:

The candidates performed as expected on this section. Most candidates were able to identify the shortfalls and advantages of the three methods. However, many candidates did not associate the reasoning to the new risks introduced to the portfolio. Some candidates ranked the three methods while a few just identified the most suitable method. However, the question asked for an assessment of each of the three methods.

- (i) Total Return Method is not desirable because it treats the total return as a single allocation factor. The performance can only be broken down in asset allocation and security selection framework, using market value weightings.
 - It does not allow common factors to be analyzed separately which is desirable in this case due to the risk factors introduced by addition of ILS and MBS.
- (ii) Excess return method allows the performance to be broken down into common factors, such as inflations. While it is suitable for the existing portfolio and ILS, it is not suitable for MBS, since MBS has other performance components that are not common factors, such as home price appreciation, prepayment.
- (iii) Fully Analytical method allows the outperformance to be broken down into various components and allows manager to analyze these components individually. The components can be common or specific to each asset.