
SOCIETY OF ACTUARIES
Quantitative Finance and Investment Advanced Exam

Exam QFIADV

MORNING SESSION

Date: Thursday, October 29, 2015

Time: 8:30 a.m. – 11:45 a.m.

INSTRUCTIONS TO CANDIDATES

General Instructions

1. This examination has a total of 100 points. It consists of a morning session (worth 60 points) and an afternoon session (worth 40 points).
 - a) The morning session consists of 10 questions numbered 1 through 10.
 - b) The afternoon session consists of 6 questions numbered 11 through 16.

The points for each question are indicated at the beginning of the question.
2. Failure to stop writing after time is called will result in the disqualification of your answers or further disciplinary action.
3. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions on the exam booklet.

Written-Answer Instructions

1. Write your candidate number at the top of each sheet. Your name must not appear.
2. Write on only one side of a sheet. Start each question on a fresh sheet. On each sheet, write the number of the question that you are answering. Do not answer more than one question on a single sheet.
3. The answer should be confined to the question as set.
4. When you are asked to calculate, show all your work including any applicable formulas. When you are asked to recommend, provide proper justification supporting your recommendation.
5. When you finish, insert all your written-answer sheets into the Essay Answer Envelope. Be sure to hand in all your answer sheets because they cannot be accepted later. Seal the envelope and write your candidate number in the space provided on the outside of the envelope. Check the appropriate box to indicate morning or afternoon session for Exam QFIADV.
6. Be sure your written-answer envelope is signed because if it is not, your examination will not be graded.

Tournez le cahier d'examen pour la version française.

****BEGINNING OF EXAMINATION****
Morning Session

1. (6 points) You are working with short rate models.

- (a) (1 point) Define an affine term structure model.
- (b) (1 point) Explain the advantage of using an affine term structure model to price a bond.

You are given the following equation for the short-rate risk-neutral dynamics:

$$dr(t) = r(t)(a - br(t))dt + c\sqrt{r(t)}dW_t$$

- (c) (2 points) Determine and justify whether this equation describes an interest rate model with an affine term structure.

Your colleague wants to use a one-factor Vasicek model. He presents you with the following equations:

$$\begin{aligned} \frac{\partial}{\partial t} B(t, T) - kB(t, T) + 1 &= 0, & B(T, T) &= 1, \\ \frac{\partial}{\partial t} [\ln A(t, T)] - kB(t, T) + \frac{1}{2} \sigma^2 B(t, T)^2 &= 0, & A(T, T) &= 0. \end{aligned}$$

- (d) (1 point) Identify and correct any mistakes in the equations above.
- (e) (1 point) Explain how you would use these equations to obtain the price of a zero-coupon bond.

2. (6 points) You are an investment actuary at AAA Life Company. You are responsible for managing the credit risk exposure in a credit portfolio that is made up of corporate bonds and credit default swaps (CDS).

The investment policy for this portfolio states that the Duration Times Spread (DTS) contribution from any single issue will not be greater than 4.

You are considering the following corporate bonds and their corresponding maturity-matching CDS for potential bond-CDS basis trades and as potential additions to the bond portion of the credit portfolio:

	Coupon	Price	Spread	Par Equivalent CDS Spread	Yield to Maturity	Duration	CDS Coupon	CDS Upfront Cost	CDS Risky Annuity
Bond A	2%	62	700 bps	1000 bps	13%	5	500 bps	300 bps	3
Bond B	15%	110	700 bps	800 bps	12%	4	500 bps	200 bps	4
Bond C	8%	81	800 bps	900 bps	13%	4	500 bps	0 bps	3
Bond D	8%	85	500 bps	700 bps	10%	9	500 bps	0 bps	8

- (a) (1 point) Identify four reasons why the full running spread of a CDS and the spread of the underlying bond are usually not the same value.

Assume accrued interest is zero.

- (b) For bond A and bond B:
- (i) (1 point) Determine an advantage and disadvantage of a basis trade for each bond.
 - (ii) (2 points) Recommend and describe which basis trade to pursue.
- (c) (1 point) Calculate the percentages of the portfolio's market value that can be invested in bond C and bond D and stay within the portfolio's credit limit.

The next day you notice that the upfront cost for a CDS on bond C is 100 bps and the upfront cost for a CDS on bond D is -100 bps.

You assume no changes in the Risky Annuity and ignore accrued interest.

- (d) (1 point) Evaluate the impact these changes are expected to have on spread volatility for each CDS.

- 3.** (5 points) You work for a real estate asset manager who wants to benchmark the performance of a portfolio of U.S. residential real estate properties. The portfolio is leveraged as the properties are funded with mortgages.

The following two indices are being considered to be used as a benchmark:

- I. A subgroup of FTSE EPRA/NAREIT Global Real Estate Index based on the market-cap-weighted performance of publicly traded REITs focusing on residential properties
 - II. National Council of Real Estate Investment Fiduciaries Property Index
- (a) (1 point) Explain two issues with using each of the above indices as a benchmark (two issues for each index is expected).

You are in talks with a foundation that is considering investing in your portfolio. The foundation currently only invests in publicly traded stocks and bonds.

- (b) (1 point) Explain the main role that this investment would play for the foundation and how it compares to using a REIT for the same purpose.

You are considering launching an infrastructure fund.

- (c) (2 points) Describe the market opportunity for infrastructure investments.

The new infrastructure fund has the opportunity to buy a 10% stake in an airport. The rest of the asset is held by other private equity investors. If it was publicly traded, the airport would be valued at \$500 million.

- (d) (1 point) Assess whether the fund should be willing to pay more, less, or exactly \$50 million for the 10% stake.

4. (6 points) SWC Corporation issued an IPO on January 3, 2014 with an initial share price of \$15. The company's first quarterly earnings report as a public company far exceeded analysts' estimates. The company also announced a dividend payment in June 2014. By July 1, 2014, the company's share price had increased to \$32, exceeding the analysts' consensus fundamental value of \$24.

- (a) (1.5 points) Identify and describe elements of investor behavior that could have contributed to SWC being overvalued on July 1, 2014.

On January 2, 2015, SWC's share price was \$36.

- (b) (1 point) Explain how investor behavior could have contributed to the share price increase since July 1, 2014.

On July 1, 2015, Ron (a 26-year old man) purchased shares of SWC Corporation for \$40 per share, and tracks its share price and related company and industry news online daily. By September 1, 2015, the share price has dropped to \$34.

- (c) (1 point) Explain why one might expect Ron's investment to underperform going forward compared to the average SWC shareholder.

On January 2, 2016, SWC's share price was \$25 vs. a fundamental value of \$30.

- (d) (1.5 points)

(i) (0.5 points) Describe how a rational investor might take advantage of the undervalued shares.

(ii) (1 point) Explain the various risks in the chosen strategy.

- (e) (1 point) Describe how SWC management might take advantage of its shares being undervalued, and the barriers that could prevent a successful implementation of this strategy.

5. (6 points) You are an investment actuary at ABC Life and are in the process of creating a mortgage backed security (MBS) using three independent loans with the following annual properties:

Loan	Exposure at Default (millions)	Probability of Default	Stressed Default Probability (95% confidence)	Stressed Default Probability (99% confidence)	Stressed Default Probability (99.9% confidence)
A	8	10%	15%	18%	20%
B	17	15%	19%	20%	22%
C	20	20%	26%	28%	30%

The security will be divided into three tranches, high, medium, and low, with the low tranche receiving payment if all loans are paid, the middle if at least two out of three are paid, and the high tranche if at least one loan is paid.

Tranche	Annual Return	Notional (millions)
High	4%	20
Medium	6%	15
Low	10%	10

- (a) (1 point) Describe the different types of off balance sheet entities that can be used to sell this security.
- (b) (2 points) Calculate the probability of payment within each tranche of the constructed mortgage backed security.

Assume maturity of all loans in the portfolio is 1 year.

- (c) (1 point) Calculate the credit value at risk of the portfolio over the next year assuming each loan's loss given default is 60% based on the BIS II guidance.

ABC Life is researching the impacts of default correlations for the portfolio of loans.

- (d) (1 point) Describe the differences between a general Bernoulli mixing model and a Poisson mixing.

Based on an implemented Bernoulli default correlation model, default correlations have been determined as follows:

ρ_{AB}	0.2
ρ_{AC}	0.0
ρ_{BC}	0.0

- (e) (1 point) Calculate the probability of High tranche getting paid.

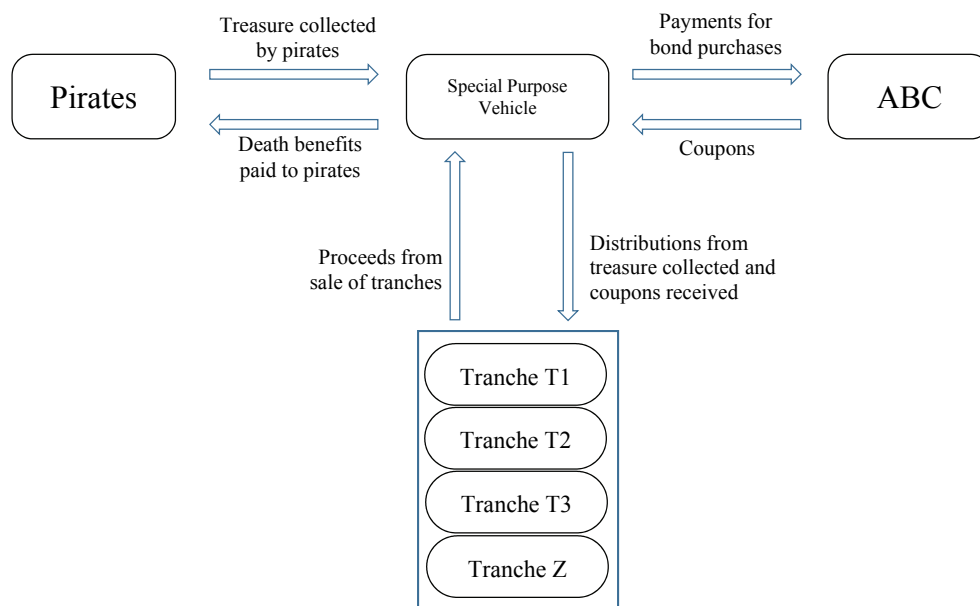
6. (8 points) A group of pirates, led by Captain Scott, acquires gold through their various pirating activities. As leader, Captain Scott collects a portion of each pirate's acquired gold and in exchange provides a death benefit that is paid to each pirate's beneficiary upon the pirate's death.

Piracy is a very lucrative but dangerous profession and the death benefit payment amounts paid by Captain Scott vary considerably month to month. The cash flow volatility in both the death benefits he pays and the gold he collects frustrates Captain Scott considerably.

To ensure all death benefits are met, Captain Scott invests the gold he collected in bonds issued by the ABC Trading company (ABC).

Captain Scott is hoping to keep his accumulated wealth and leave the pirating business within two years.

Captain Scott's financial advisor has recommended securitizing the gold he collects and then purchasing one of the tranches himself. This arrangement would help reduce his cash flow volatility and maintain his current wealth. The securitization scheme is illustrated in the diagram below:



6. Continued

Additional information about the structured deal:

- The primary currency used is gold pieces (gp);
- ABC pays 1 gp at the end of the month for every 100 gp invested at beginning of the month;
- Death benefits are 100 gp per death paid at the end of month, funded from the gold he collected and, if insufficient, from capital;
- Scheduled tranche coupons are paid first to T1, then T2, and then T3;
- Tranche Z receives all residual payments after all scheduled tranche coupons have been paid;
- Gold collected from pirates and ABC is used to fund scheduled tranche coupon payments;
- Scheduled tranche coupon and residual payments cannot be funded by capital;
- Any previously missed tranche coupon payments (Tranches T1, T2, T3) are deemed lost and will not be made up;
- At the end of the tranche's term all remaining capital will be paid back to the investor.

Tranche / Investment	Investment Amount (Capital)	Monthly tranche coupon payment	Term (months)
T1	1000 gp	10 gp	12
T2	1000 gp	30 gp	18
T3	700 gp	70 gp	36
Z	300 gp	Residual	

Consider the following scenario:

Month	Number of living pirates at beginning of month	Number of pirate deaths in month	Total amount of gold pieces collected by Captain Scott from the pirates during month	Total Capital Remaining at end of month
1	1000	9	1000	3000
2	991	12	900	2730
3	979	1	300	2730
4	978	3	X	2730

- (a) (1 point) Calculate the residual payment at the end of the first month.

Question 6 continued on next page

6. Continued

- (b) (3 points) Calculate the minimum value for X such that the total payments made to Tranche Z from months 1 to 4 equals their original investment (i.e. Tranche Z gets their original investment back at end of month 4).

Captain Scott has found three additional investors to participate in this new structured security with him:

Alan – a young wealthy man who is willing to throw his gold at anything if he thinks he has the slightest chance of winning big;

Becky – an older wealthy woman looking for a lucrative return with reasonable risk;

Clayton – a young man who is very tight with his money and only invests in very low risk assets.

- (c) (2 points) Recommend an investment pairing (one to one) between each of the four tranches (T1, T2, T3, Z) and each of the four investors (Captain Scott, Alan, Becky, Clayton) such that every investor is satisfied. Be sure to consider and discuss the risk of losing the investor's capital.
- (d) (1 point) Compare and contrast the concerns of the investors who purchased tranche T3 and Z if the scenario shown above occurs.

Captain Scott is seeking alternative ways to reduce the volatility of death benefit payments. He recently read an article on Credit Default Swaps (CDS) and is considering something similar.

- (e) (1 point) Develop a Diagram showing how a CDS could be redefined to a "Mortality Swap" and used by Captain Scott to eliminate the volatility of death benefit payments.

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7. (5 points) You have been hired by a major investment firm to assist with performance attribution. The firm's CFO has decided that the firm will adopt a bottom-up philosophy to investment management. The firm will select individual companies to look at and then make sector selection choices based on that.

You are given the following information for the portfolio:

Aggressive Equities

	Wp	Wb	Rp	Rb
Large Cap Equities	30.0%	30.0%	3.0%	2.5%
Mid-Cap Equities	25.0%	30.0%	5.0%	6.0%
Small Cap	25.0%	20.0%	7.0%	7.5%
Emerging Markets	20.0%	20.0%	15.0%	12.0%

Portfolio Return	Benchmark Return	Outperformance
6.90%	6.45%	0.45%

- The firm's portfolio manager is paid a bonus based on the outperformance achieved versus the benchmark.
 - The CFO calculates the outperformance as 35 bps.
 - The manager claims that the outperformance is 38 bps.
 - The manager makes the following statement to support his claim: "Because I started selecting securities first, I should own a greater share of outperformance."
- (a) (2 points) Critique the managers' statement.
- (b) (1 point) Briefly describe which principles of performance attribution are relevant to this situation.

You are given the following additional information:

- The Portfolio manager is not able to control the timing of cash inflows or outflows for his portfolio.
- The manager trades frequently in his portfolio.

Multi-Period Attribution		
	January-June	July-December
Period Return	15%	10%
Cash flows into portfolio at Beginning of Period	\$90M	\$20M

7. Continued

- (c) (1 point) Compute the portfolio's total return for the year, and justify your choice of method.
- (d) (1 point) Briefly describe the challenges associated with multi-period compounding.

8. (5 points) You work as a consulting actuary for ABC, a small insurance company, which has assets and liabilities as shown in the tables below:

Liabilities	Interest Rate Basis	Value	Withdrawal Provision
Bank A	Floating	50	7-day put
Bank B	Floating	20	7-day put
Bank C	Floating	10	None
Bank D	Floating	8	None
Bank E	Floating	6	None
Bank A	Fixed	4	7-day put
Bank B	Fixed	2	7-day put

Assets	Value
Real Estate	30
Real Estate	20
MBS (AAA-rated)	15
MBS (AAA-rated)	10
MBS (BB-rated)	10
US Treasury Bond	10
US Treasury Bond	5

ABC was recently downgraded from AAA to A rating. The CFO of ABC has requested that you analyze the company's liquidity position including addressing their rating and provide a memo to ABC's executives on your findings. You decide to construct your memo as follows:

- (a) (2 points) Explain concerns you have about ABC's liquidity position including key factors which may increase ABC's exposure to stress liquidity risk.
- (b) (1 point) Outline ways to quantify the liquidity of the assets.
- (c) (2 points) Recommend changes to the asset and/or liability profile to reduce ABC's stress liquidity risk.

9. (7 points) The portfolio manager at your company is proposing to invest in a 6-month bond and your supervisor has asked you to use a Q-matrix to evaluate the default risk of this investment.

(a) (1 point) Explain the reasons for using a Q-matrix to evaluate the default risk of this investment.

You have decided to base your analysis on data from Broody's rating agency, which breaks asset ratings down into 4 different classes (denoted as A, B, C and D). The bond is currently B-rated and the Broody's analyst has provided you with the following one-year transition matrix:

$$M = \begin{bmatrix} & A & B & C & D \\ A & 0.7 & 0.28 & 0.01 & 0.01 \\ B & 0.15 & 0.65 & 0.15 & 0.05 \\ C & 0.05 & 0.1 & 0.6 & 0.25 \\ D & 0 & 0 & 0 & 1 \end{bmatrix}$$

In addition, you used a spreadsheet to calculate the following series result:

$$\sum_{k=2}^{100} (-1)^{k+1} \frac{(M-I)^k}{k} = \begin{bmatrix} -0.1 & 0.15 & -0.05 & 0 \\ 0.07 & -0.15 & 0.1 & -0.02 \\ 0.01 & 0.05 & -0.13 & 0.07 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(b) (3 points) Calculate a generator Q of a Markov chain for ratings using

$$\tilde{Q} = \sum_{k=1}^{\infty} (-1)^{k+1} \frac{(M-I)^k}{k} \text{ and any necessary adjustments.}$$

You get a call from the analyst at Broody's saying that the transition matrix originally provided was based on faulty data and he sends you a revised transition matrix:

$$M = \begin{bmatrix} 0.54 & 0.33 & 0.1 & 0.03 \\ 0.13 & 0.6 & 0.18 & 0.09 \\ 0.03 & 0.08 & 0.59 & 0.3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

9. Continued

You then re-do your calculations from (b) above to calculate a revised Q-matrix as well as several results based upon this new Q-matrix:

$$Q = \begin{bmatrix} -0.7 & 0.6 & 0.09 & 0.01 \\ 0.24 & -0.6 & 0.3 & 0.06 \\ 0.05 & 0.1 & -0.55 & 0.4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$e^Q = \begin{bmatrix} 0.54 & 0.33 & 0.1 & 0.03 \\ 0.13 & 0.6 & 0.18 & 0.09 \\ 0.03 & 0.07 & 0.59 & 0.31 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\sum_{k=2}^{\infty} \frac{(0.5Q)^k}{k!} = \begin{bmatrix} 0.07 & -0.08 & 0 & 0.01 \\ -0.03 & 0.06 & -0.03 & 0.01 \\ 0 & -0.01 & 0.04 & -0.02 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

- (c) (1 point) Calculate the L^1 -norm as a measure of error for the revised Q-matrix.
- (d) (2 points) Calculate the probability that the bond defaults within 6 months.

10. (6 points) You are the Chief Risk Officer of a large life insurance company which has a substantial amount of residential mortgage loans in its investment portfolio. After a subprime mortgage crisis, your company had updated its mortgage credit risk valuation models. You have now been assigned to value the mortgage credit risk.

- (a) (1 point) Describe the three component models utilized in the valuation of mortgage credit risk.

One member of the company's Board of Directors has made the following comment:

“Home Price Appreciation (HPA) is one of the most important economic factors in determining subprime mortgage performance and in valuing mortgage credit risk. HPA not only directly affects the occurrence of a default, but other economic factors are also related to it especially surrounding leverage.”

- (b) (2 points) Critique the above statement.
- (c) (1 point) Identify an additional economic factor that the director is implying and explain its relationship to the HPA.
- (d) (1 point) Explain the complications that could arise in applying the factor identified above.

Another director of the Board has remarked:

“We should use the Asset-Backed Securities Index (ABX) to price mortgage credit.”

- (e) (1 point) Describe what information is contained in the ABX and how it could be used to price mortgage credit.

****END OF EXAMINATION****
Morning Session

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