MLC Fall 2015 Written Answer Questions

Model Solutions

MLC Fall 2015

Question 1 Model Solution

Learning Outcome: 1(a)

Chapter References: AMLCR 2.2-2.5

Overall, this question was done reasonably well, although few candidates received maximum credit.

(a) (i) For any age x, the survival function $S_x(t)$ must satisfy the following

1. $S_x(0) = 1$
2. $\lim_{t\to\infty} S_x(t) = 0$
3. $S_x(t)$ must be a non-increasing function of t

(ii) **b = -0.2**:

1. $S_{40}(0) = 1.0$
2. $\lim_{t\to\infty} S_{40}(t) = \lim_{t\to\infty} 0.75e^{-0.2(t-25)} = 0$

For the third criterion, we show that $S_{40}(t)$ is non-increasing before age 65, after age 65 and at age 65:

$$\frac{d}{dt}S_{40}(t) = -0.0008t < 0 \quad \text{for} \quad 0 \le t < 25$$

$$\frac{d}{dt}S_{40}(t) = -0.2 \times 0.75 \times e^{-0.2(t-25)} < 0 \quad \text{for} \quad t \ge 25$$

and $\lim_{t\uparrow 25^-} S_{40}(t) = 0.75 = \lim_{t\downarrow 25^+} S_{40}(25)$

Hence, $b = -0.2$ is a valid parameter.

***b*=0.0**:

$$\lim_{t\to\infty} S_{40}(t) = \lim_{t\to\infty} 0.75 \neq 0$$

Hence, $b = 0.0$ is not a valid parameter.

***b*=0.2**:

$$\lim_{t\to\infty} S_{40}(t) = \lim_{t\to\infty} 0.75e^{0.2(t-25)} = \infty$$

Hence, $b = 0.2$ is not a valid parameter.

For full credit, candidates were expected to verify all three criteria from (a)(i), for $b = -0.2$. Few candidates verified, for example, that the function is non-increasing at $t = 25$. For the invalid values of b, candidates could justify the conclusion by indicating any criterion that is not satisfied.

(b) (i)

$$\mu_{40+t} = -\left(\frac{1}{S_{40}(t)}\right)\frac{d}{dt}S_{40}(t)$$
$$\Rightarrow \mu_{60} = \frac{1}{0.84}\left(2 \times 0.02^2 \times 20\right) = 0.01905$$

Alternative:

$$\begin{aligned}\mu_{40+t} &= -\frac{d}{dt}\log S_{40}(t)\\ &= -\frac{d}{dt}\log\left(1-(0.02t)^2\right) \qquad \text{for } t \leq 25\\ \Rightarrow \mu_{60} &= \frac{2(0.02)^2\,20}{1-((0.02)(20))^2} = 0.01905\end{aligned}$$

(ii)

$$\begin{aligned}\mu_{40+t} &= -\frac{d}{dt}\log S_{40}(t)\\ &= -\frac{d}{dt}\left(\log 0.75e^{2.5} + \log e^{-0.1t}\right) \ \text{ for } t > 25\\ \Rightarrow \mu_{70} &= -\frac{d}{dt}(-0.1t) = 0.1\end{aligned}$$

Alternative:

$$\begin{aligned}\mu_{40+t} &= -\left(\frac{1}{S_{40}(t)}\right)\frac{d}{dt}S_{40}(t)\\ &= \frac{(0.75)\,0.1\,e^{-0.1(t-25)}}{0.75\,e^{-0.1(t-25)}} = 0.1 \qquad \text{for } t > 25\\ \Rightarrow \mu_{70} &= 0.1\end{aligned}$$

(iii)

$$\begin{aligned}
\mathring{e}_{40:\overline{35}|} &= \int_0^{25} S_{40}(t)\,dt + \int_{25}^{35} S_{40}(t)\,dt \\
&= \int_0^{25} (1 - 0.0004t^2)dt + \int_{25}^{35} 0.75\,e^{-0.1(t-25)}dt
\end{aligned}$$

$$\int_0^{25} (1 - 0.0004t^2)dt = \big[t - 0.0004t^3/3\big]_0^{25} = 22.917$$

$$\int_{25}^{35} 0.75\,e^{-0.1(t-25)}dt = \int_0^{10} 0.75\,e^{-0.1u}du = 4.741$$

$$\text{so} \quad \mathring{e}_{40:\overline{35}|} = 22.917 + 4.741 = 27.658$$

Most candidates who attempted this part correctly calculated the first integral on the right hand side above. Only the stronger candidates successfully set up and evaluated the second integral.

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Question 2 Model Solution

Learning Outcomes: 1(b), 1(d), 2(a), 4(a)

Chapter References: AMLCR 8.4-8.7, 9.6

This proved to be one of the most challenging questions on the paper.

(a) The future lifetimes of (x) and (y) are **dependent**, because the force of mortality for each is different depending on whether the other is alive or not, as $\mu^{01}_{x+t:y+t} \neq \mu^{23}_{x+t}$, and $\mu^{02}_{x+t:y+t} \neq \mu^{13}_{y+t}$. That means that the distribution of the time to death of (x) is different if (y) dies early than if (y) dies later (and vice versa), which means the future lifetime random variables T_x and T_y are dependent.

The key point, which around one-third of the candidates identified, is that the force of mortality for (x) from state 2 is different than the force from state 0, and the force of mortality for (y) is different from state 1 than from state 0, which indicates dependence. The fact that there is no common shock transition does not imply independence. When justifying dependence, candidates were expected to compare appropriate pairs (i.e. $\mu^{01}_{x+t:y+t}$ and $\mu^{23}_{x+t:y+t}$, which both concern the death of (x), and $\mu^{02}_{x+t:y+t}$ and $\mu^{13}_{x+t:y+t}$, which both concern the death of (y)).

(b) (i)

$$\frac{d}{dt}\,{}_tp^{00}_{xy} = -{}_tp^{00}_{xy}\left(\mu^{01}_{x+t:\,y+t} + \mu^{02}_{x+t:\,y+t}\right)$$

Boundary condition: ${}_0p^{00}_{xy} = 1.0$

(ii)

$$\frac{d}{dr}\,{}_rp^{00}_{xy} = -{}_rp^{00}_{xy}\left(\mu^{01}_{x+r:\,y+r} + \mu^{02}_{x+r:\,y+r}\right)$$

$$\Rightarrow \frac{1}{{}_rp^{00}_{xy}}\frac{d}{dr}\,{}_rp^{00}_{xy} = -\left(\mu^{01}_{x+r:\,y+r} + \mu^{02}_{x+r:\,y+r}\right)$$

$$\Rightarrow \frac{d}{dr}\log\,{}_rp^{00}_{xy} = -\left(\mu^{01}_{x+r:\,y+r} + \mu^{02}_{x+r:\,y+r}\right)$$

Integrate both sides from 0 to t

$$\int_0^t \frac{d}{dr} \log {}_r p_{xy}^{00}\, dr = -\int_0^t \mu_{x+r:\,y+r}^{01} + \mu_{x+r:\,y+r}^{02}\, dr$$

$$\Rightarrow \log {}_t p_{xy}^{00} - \log {}_0 p_{xy}^{00} = -\int_0^t \mu_{x+r:\,y+r}^{01} + \mu_{x+r:\,y+r}^{02}\, dr$$

from the boundary condition $log\, {}_0 p_{xy}^{00} = 0,$ so

$$\log {}_t p_{xy}^{00} = -\int_0^t \mu_{x+r:\,y+r}^{01} + \mu_{x+r:\,y+r}^{02}\, dr$$

$$\Rightarrow {}_t p_{xy}^{00} = \exp\left(-\int_0^t \mu_{x+r:\,y+r}^{01} + \mu_{x+r:\,y+r}^{02}\, dr\right)$$

Most candidates scored partial credit for this part. Some candidates wrote down a few steps, but missed the key parts of the proof. An acceptable alternative approach was to plug the given solution into the Kolmogorov equation and demonstrate that the integral equation for ${}_t p_{xy}^{00}$ satisfies the Kolmogorov differential equation and the boundary condition. This approach was awarded full credit if done correctly.

(c) (i) At time 10, (x) is age 60 and (y) is age 65. The value at time 10 of the joint and reversionary annuities is

$$50,000\, \bar{a}_{60:\,65}^{00} + 30,000\, \bar{a}_{60:\,65}^{01} + 30,000\, \bar{a}_{60:\,65}^{02}$$
$$= 50,000(8.8219) + 30,000(1.3768) + 30,000(3.0175)$$
$$= 572,924$$

(ii) The EPV of the deferred annuity for the case where both lives survive 10 years uses the result from (i):

$${}_{10}p_{50:\,55}^{00} \times v^{10} \times 572,924$$
$$= 0.86041 \times 0.61391 \times 572,924 = 302,627$$

The EPV of the deferred annuity for the case where (y) survives 10 years but (x) does not, is

$${}_{10}p_{50:\,55}^{01} \times v^{10} \times (30,000\, \bar{a}_{65}^{11})$$
$$= 0.04835 \times 0.61391 \times 30,000 \times 10.1948 = 9078$$

The EPV of the deferred annuity for the case where (x) survives 10 years but (y) does not, is

$$
\begin{aligned}
&{}_{10}p^{02}_{50:55} \times v^{10} \times (30,000\,\bar{a}^{22}_{60}) \\
&\quad = 0.08628 \times 0.61391 \times 30,000 \times 11.8302 \\
&\quad = 18,799
\end{aligned}
$$

Let P denote the premium. Then the EPV of the benefit paid on second death during the deferred period is

$$
3P\bar{A}^{03}_{50:55:\overline{10}|} = 3P\,(0.003421) = (0.010263)\,P
$$

Putting these together gives the premium

$$
\begin{aligned}
&P = 302,627 + 9,078 + 18,799 + 0.010263\,P \\
&\Rightarrow P = \frac{330504}{0.989737} = 333,931
\end{aligned}
$$

Only the top few candidates achieved full credit for this part. Many candidates did not allow for the possibility that only one life would survive the deferred period. The lower scoring candidates used combinations of probabilities and annuities that indicated less than adequate understanding of multiple state models and notation. For example, the expression $({}_{10}p^{01}_{50:55}\,\bar{a}^{01}_{60:65})$ *is meaningless, as it requires the lives to be in state 0 and also in state 1 at time 10. The second superscript of* ${}_tp^{ij}$ *in this type of combination must be the same as the first superscript of* $\bar{a}^{jk}$*.*

(d) (i)

$$
\begin{aligned}
{}_{10}V^{(0)} &= 572,924 \quad \text{from (c)(i)} \\
{}_{10}V^{(1)} &= 30,000\,\bar{a}^{11}_{65} = 30,000 \times 10.1948 = 305,844 \\
{}_{10}V^{(2)} &= 30,000\,\bar{a}^{22}_{60} = 30,000 \times 11.8302 = 354,906
\end{aligned}
$$

(ii) For $t \geq 10$, and $\delta = \log(1.05)$,

$$
\begin{aligned}
\frac{d}{dt}\,{}_tV^{(0)} = \delta\,{}_tV^{(0)} - 50,000 &- \mu^{01}_{50+t:\,55+t}\left({}_tV^{(1)} - {}_tV^{(0)}\right) \\
&- \mu^{02}_{50+t:\,55+t}\left({}_tV^{(2)} - {}_tV^{(0)}\right)
\end{aligned}
$$

(iii) We need $\mu^{01}_{60:65} = A + Bc^{60} = 0.009076$ and $\mu^{02}_{60:65} = A + Bc^{65} = 0.015919$. Then

$$
\begin{aligned}
{}_{t+h}V^{(0)} &\approx {}_tV^{(0)} + h\left(\delta\, {}_tV^{(0)} - 50{,}000 - \mu^{01}_{50+t:\,55+t}\left({}_tV^{(1)} - {}_tV^{(0)}\right)\right. \\
&\qquad\qquad\qquad \left. -\mu^{02}_{50+t:\,55+t}\left({}_tV^{(2)} - {}_tV^{(0)}\right)\right) \\
&\approx 572{,}924 + 0.5\Big\{0.048790(572{,}924) - 50{,}000 \\
&\qquad\qquad\qquad - 0.009076(305{,}844 - 572{,}924) \\
&\qquad\qquad\qquad - 0.015919(354{,}906 - 572{,}924)\Big\} \\
&\approx 564{,}848
\end{aligned}
$$

As in (c), a few of the the very best candidates achieved full credit for this part. Many students correctly calculated the three reserves in (i). Some did not understand that, for example, ${}_tV^{(1)}$ is the reserve assuming the policy is in state 1 at t, that is that (50) has already died, so the correct annuity factor must be a^{11}_{65}, not $a^{01}_{60:65}$. Common errors in (ii) and (iii) included omitting the release of reserve terms involving ${}_tV^{(0)}$ on the right hand side, omitting the annuity paid in state 0, including the annuities paid in the other states (these are implicitly valued in ${}_tV^{(1)}$ and ${}_tV^{(2)}$), and getting one or more signs wrong.

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Question 3 Model Solution

Learning Outcomes: 3(c), 4(c), 4(g)

Chapter References: AMLCR 13.1-13.3

Overall, this question was done well, with many candidates achieving at least 9/10 points.

(a)

$$P\ddot{a}_{50} = 100,000A_{50} + 1000 + 50\ddot{a}_{50}$$
$$\ddot{a}_{50} = 13.2668 \qquad A_{50} = 0.24905$$
$$\Rightarrow P = \frac{26,568}{13.2668} = \frac{24905 + 1000}{13.2668} + 50 = 2002.6$$

This part was very well done by most candidates.

(b) The profit test table for $t = 0, 1, 2, 3$ using notation from the textbook, is as follows.

t	${}_{t-1}V$	P_t	E_t	I_t	EDB_t	ECV_t	E_tV	Pr_t
0*			1000					-1000.0
1	0	2002.6	50	97.6	592	0.0	708.3	749.9
2	750	2002.6	50	189.2	642	29.8	1415.9	804.1
3	1500	2002.6	50	241.7	697	69.5	2122.6	805.2

Explanation and useful numbers

$I_t = i_t({}_{t-1}V + P_t - E_t), \quad i_t = 0.05$ for $t = 1$, $i_t = 0.07$ for $t = 2, 3$

$EDB_t = q_{50+t-1}\,100,000$

$ECV_t = p_{50+t-1} \times 0.05 \times CV_t$

$E_tV = p_{50+t-1} \times 0.95 \times {}_tV$

$q_{50} = 0.00592 \quad q_{51} = 0.00642 \quad q_{52} = 0.00697$

$p_{50} = 0.99408 \quad p_{51} = 0.99358 \quad p_{52} = 0.99303$

Most candidates received partial credit for this part. Other than calculation errors, the most common mistakes were (i) Missing or incorrect use of surrender probabilities when calculating the expected surrender values and/or reserves, and (ii) Using the previous year profit instead of the reserve when calculating the interest income and annual profit.

(c) (i)

$$NPV(0) = -1000$$
$$NPV(1) = -1000 + 749.9v_{10\%} = -318.3$$
$$NPV(2) = -318.3 + 804.1 \times {}_1p_{50}^{(\tau)} \times v_{10\%}^2$$
$${}_1p_{50}^{(\tau)} = 0.99408 \times 0.95 = 0.94438$$
$$\Rightarrow NPV(2) = 309.3$$

So the third year is the first year with $\Pr_t > 0$ and $NPV(t-1) > 0$.

(ii) The insurer pays out 85% of the profit in year 3, which is

$0.85 \times 805.2 = 684.4$ per policy in force at $t = 2$

Only policies in force at $t = 3$ participate. The probability that a policy in force at $t = 2$ is still in force at $t = 3$ is

${}_1p_{52}^{(\tau)} = 0.95p_{52} = 0.94338$

Hence the projected dividend payment per policy in force at $t = 3$ is

$684.4/0.94338 = 725.5$

Most candidates received partial credit for this part. The most common mistake was ignoring or using incorrect probabilities when discounting the profit vector. Also, some candidates multiplied the distributed profit by ${}_1p_{52}^{(\tau)}$*, instead of dividing by it.*

(d) **Possible Advantages:**

- Maintain more assets under management if policyholders choose to convert, which gives potential for future profits.
- Reduce liquidity requirements.
- Increase attractiveness of the contract.
- More tax-efficient distribution than cash dividends
- Less expensive to operate than cash dividends

Possible Disadvantages:

- Adverse selection – sicker lives will choose the option, which will then cost more than anticipated.
- More complex for policyholders to understand
- Extra cost of administration, acquisition and maintenance expenses.

Candidates were only required to mention one advantage and one disadvantage for full credit. Partial credit was awarded for some answers where the candidate showed some understanding, but did not complete the explanation. Some candidates discussed advantages and disadvantages of offering a participating policy, instead of discussing the dividend option. These answers received no credit.

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Question 4 Model Solution

Learning Outcomes: 3(a)

Chapter References: AMLCR 6.4, 6.8

This question was the highest scoring on the paper, with a large number of candidates achieving full credit.

(a) Death in Year 1:

$L_0|\text{Event} = (1000 + G(1+i))v - G = 1000v = 943.4$
Probability $= 0.06$

Withdrawal in Year 1:

$L_0|\text{Event} = -G$
Probability $= 0.04$

Death in Year 2:

$L_0|\text{Event} = (1000 + G((1+i) + (1+i)^2))v^2 - G(1+v) = 1000v^2 = 890.0$
Probability $= 0.9 \times 0.12 = 0.108$

Survival in force to end of Year 2:

$L_0|\text{Event} = -G(1+v) = -1.9434G$
Probability $= 0.9 \times 0.88 = 0.792$

In table form:

Event	Value of L_0 given Event	Probability of Event
Death in Year 1	943.4	0.06
Withdrawal Year 1	$-G$	0.04
Death in Year 2	890.0	0.108
Neither death nor withdrawal	$-1.9434G$	0.792

Most candidates did well in this part. Some candidates confused dependent and independent rates of mortality and withdrawal. A few calculated an equivalence principle premium, even though it was not required or relevant. Amongst candidates who did not achieve full marks, the most common problem was determining the amount of the return of premiums benefit.

(b) (i)

$$
\begin{aligned}
E[L_0] &= 943.4(0.06) - G(0.04) + 890.0(0.108) - 1.9434G(0.792) \\
&= 152.7 - 1.579G \\
&\Rightarrow a = 152.7 \qquad b = 1.579
\end{aligned}
$$

(ii)

$$
\begin{aligned}
E[L_0^2] &= 943.4^2(0.06) + G^2(0.04) + 890.0^2(0.108) + (1.9434G)^2(0.792) \\
&= 138947 + 3.0312G^2 \\
E[L_0]^2 &= (152.7 - 1.579G)^2 = 23317 - 482.2G + 2.4932G^2 \\
V[L_0] &= E[L_0^2] - E[L_0]^2 = 115,630 + 482.2G + 0.538G^2 \\
&\Rightarrow c = 0.538 \quad d = 482.2 \quad e = 115,630
\end{aligned}
$$

The table in part (a) was used by stronger candidates to answer part (b), as the examiners' intended. Standard variance formulas for level benefit term insurance do not work in this case, and candidates who tried to use memorized formulas received little or no credit.

(c) For each policy

$$
\begin{aligned}
E[L_0] &= a - bG = -52.57 \\
V[L_0] &= cG^2 + dG + e = 187,408
\end{aligned}
$$

So for the aggregate loss

$$
\begin{aligned}
E[L_{agg}] &= 200 \times (-52.57) = -10514 \\
V[L_{agg}] &= 200 \times 187,408 = 37481600 = 6122.2^2 \\
\Rightarrow \Pr[L_{agg} > 0] &= 1 - \Phi\left(\frac{0 - (-10,514)}{6122.2}\right) \\
&= 1 - \Phi(1.72) = 1 - 0.9573 = 0.0427
\end{aligned}
$$

Some candidates used the wrong tail of the Normal distribution for this part. Otherwise, this was done well.

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Question 5 Model Solution

Learning Outcomes: 3(c), 4(e)

Chapter References: AMLCR 13.4

Many candidates scored full marks for parts (a), (b), and (c)(i). Only the best candidates managed to score full marks for (c)(ii) and (d).

(a) $P_2 = 1000$:

$$\begin{aligned} AV_2 &= (AV_1 + 0.9P_2 - 10 - 200v)\,(1.06) \\ &= (165 + 0.9P_2 - 10)\,(1.06) - 200 \\ &= 918.3 \end{aligned}$$

Generally well done. Errors included incorrect discounting of the CoI, and incorrect treatment of the surrender charge.

(b)

$$\begin{aligned} AV_2 &= (165 + 0.9P_2 - 10)\,(1.06) - 200 \\ &= 0.954P_2 - 35.7 \\ AV_3 &= (AV_2 + 0.9P_3 - 10)\,(1.06) - 300 \\ &= (0.954P_2 - 35.7 + 0.9P_3 - 10)\,(1.06) - 300 \\ &= 1.0112P_2 + 0.954P_3 - 348.4 \\ &\Rightarrow a = 1.0112 \quad b = 0.954 \quad c = -348.4 \end{aligned}$$

Generally well done.

(c) (i) Calculate the different possible values for AV_3, using the result from (b),

$$AV_3 = 1.0112P_2 + 0.954P_3 - 348.4$$

as follows:

P_2	P_3	Prob.	AV_3
1000	1000	0.36	1616.80
1000	200	0.24	853.6
200	1000	0.08	807.8
200	200	0.32	44.6

which gives

$$\begin{aligned}E[DB_3] =& 0.36 \times 101,616.80 + 0.24 \times 100,853.6 + 0.08 \times 100,807.8 \\ &+ 0.32 \times 100,044.6 \\ =& 100,865.8\end{aligned}$$

(ii)

$$E[CV_3] = 1516.8 \times 0.36 + 753.6 \times 0.24 + 707.8 \times 0.08 + 0 \times 0.32 = 783.5$$

The main issue in this part was recognizing that CV_3 *cannot be less than zero, so it is not appropriate simply to subtract 100 from* $E[AV_3]$.

(d) Let $AV_{10}^{(1)} = 5114$ denote the AV assuming all premiums paid; $AV_{10}^{(2)}$ denotes the AV at time 10 assuming all premiums except the last are paid, and $AV_{10}^{(3)}$ denotes the AV at time 10 assuming all premiums except the third are paid.

Then

$$\begin{aligned}AV_{10}^{(2)} &= AV_{10}^{(1)} - 1000(0.95)(1.06) = 4107.0 \\ AV_{10}^{(3)} &= AV_{10}^{(1)} - 1000(0.9)(1.06)^8 = 3679.5 \\ &\Rightarrow E[AV_{10}] = 0.5\,AV_{10}^{(2)} + 0.5\,AV_{10}^{(3)} = 3893.3\end{aligned}$$

This part was done well by those who made a substantive attempt. Many candidates omitted this part.

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Question 6 Model Solution

Learning Outcomes: 2(a), 2(b)

Chapter References: AMLCR 4.4, 11.4

This was the lowest scoring question on the paper. There was some evidence that candidates were pressed for time, but also evidence that candidates struggled with some of the concepts covered.

Generally, passing candidates understood what to do with a force of interest that varies with time (from Exam FM), and, further, understood that conditional variance was needed, and knew how to find it.

(a) Let Y_j denote the PV of benefits for the jth life, $Y = \sum_{j=1}^{100} Y_j$.

Let $v(5)$ denote the discount factor for time 5. That is

$$v(5) = \begin{cases} \exp\left(-\int_0^5 0.03t^{\frac{1}{2}}dt\right) & \text{with prob. } 0.6 \\ \exp\left(-\int_0^5 0.02dt\right) & \text{with prob. } 0.4 \end{cases}$$

$$\int_0^5 0.03\,t^{0.5}dt = 0.02\,t^{\frac{3}{2}}\Big]_0^5 = 0.22361$$

$$\Rightarrow e^{-\int_0^5 0.03\,t^{0.5}dt} = 0.79963$$

$$\Rightarrow v(5) = \begin{cases} 0.79963 & \text{with prob. } 0.6 \\ 0.90484 & \text{with prob. } 0.4 \end{cases}$$

$$\text{Also } {}_5p_{85} = \frac{1058491}{2358246} = 0.44885$$

$$\Rightarrow E[Y_j|v(5)] = 0.44885\,v(5)$$

$$\Rightarrow E[Y_j|v(5)] = \begin{cases} 0.35891 & \text{with prob. } 0.6 \\ 0.40613 & \text{with prob. } 0.4 \end{cases}$$

$$\Rightarrow E[Y|v(5)] = \begin{cases} 35.891 & \text{with prob. } 0.6 \\ 40.613 & \text{with prob. } 0.4 \end{cases}$$

$$\Rightarrow E[Y] = E[E[Y|v(5)]] = 37.780$$

The most common error was incorrect calculation of $v(5)$.

(b)

$$\begin{aligned}
V[Y] &= E\left[V[Y|v(5)]\right] + V\left[E[Y|v(5)]\right] \\
E[Y|v(5)] &= 100 v(5) {}_5p_{65} \\
V[Y|v(5)] &= 100\,(v(5))^2\ {}_5p_{85}(1-\ {}_5p_{85})
\end{aligned}$$

so we have

$$E[Y|v(5)] = \begin{cases} 35.891 & \text{with prob. } 0.6 \\ 40.613 & \text{with prob. } 0.4 \end{cases}$$

$$V[Y|v(5)] = \begin{cases} 15.818 & \text{with prob. } 0.6 \\ 20.254 & \text{with prob. } 0.4 \end{cases}$$

So

$$\begin{aligned}
E\Big[V[Y|v(5)]\Big] &= 0.6 \times 15.818 + 0.4 \times 20.254 = 17.592 \\
V\Big[E[Y|v(5)]\Big] &= (35.891)^2 \times 0.6 + (40.613)^2 \times 0.4 - (37.78)^2 = 5.352 \\
&\Rightarrow V[Y] = 17.592 + 5.352 = 22.944 = 4.790^2
\end{aligned}$$

So, the required probability is

$$\begin{aligned}
\Pr[Y \leq 30] &= \Phi\left(\frac{30 - 37.78}{4.790}\right) \\
&= \Phi(-1.62) = 1 - \Phi(1.62) = 1 - 0.9474 = 0.0526
\end{aligned}$$

The most common error for this part was omitting the second part of the conditional variance calculation.

(c) (i) A risk is diversifiable if adding more units of risk reduces the standard deviation of the mean loss per contract, with a limit of zero as the number tends to infinity.

Mathematically, a portfolio of risks X_j is diversifiable if and only if

$$\lim_{N\to\infty} \frac{\sqrt{V\left[\sum_{j=1}^{N} X_j\right]}}{N} = 0$$

(ii) The risk is not diversifiable, because the uncertainty about the discount factor cannot be eliminated by increasing the number of policies.

Any verbal or mathematical explanation of diversification that incorporates the idea that for a diversifiable portfolio, adding policies reduces the variance or standard deviation of the mean loss, with a limit of 0, was given full credit. It was not sufficient however, to say that the variance of the portfolio goes to 0 as $N \to \infty$; the variance of the portfolio as a whole tends to ∞, it is the variance of the mean that tends to 0.

For (ii) it was acceptable to say that the mortality risk is diversifiable, but the interest rate risk is not.