Course 6
Spring 2005

ANSWER KEY

Question #	*Answer*		*Question #*	*Answer*
1	B		21	D
2	B		22	D
3	E		23	A
4	A		24	B
5	B		25	E
6	A		26	B
7	C		27	C
8	B		28	B
9	C		29	D
10	A		30	D
11	E		31	E
12	A		32	C
13	B		33	D
14	B		34	E
15	C		35	D
16	C		36	D
17	B			
18	E			
19	B			
20	D			

Spring 2005 Course 6 Written-Answer Solutions

1. (a) Full Replication Approach

Purchase every bond in an index with the same weighting as the index

Sampling Approach

The securities used to match the index are selected randomly from the bonds that make up the chosen index

Stratified Sampling Approach

1. Divide the index "universe" into homogenous classes
2. Select one security from each class
3. Determine the appropriate amount to hold in each security

(b) Full Replication Approach

Advantage: Ensures you will match the index very closely

Disadvantage: High transaction costs to purchase bonds and rebalance portfolio

Disadvantage: Not practical because so many bonds in each index

Sampling Approach

Advantage: Can get returns close to the index with a relatively small number of securities (usually fewer than 40)

Disadvantage: Does not work well if the securities in the index are diverse. Index may not represent optimal portfolio

Stratified Sampling Approach

Advantage: Can match duration and convexity to the index

Disadvantage: Requires judgment in determining the number of classes/issues to use

(c) -Active strategies provide the greatest opportunity for the highest possible return
-Good for total return-maximizing investors who are willing to accept greater risk
-Used by managers willing to make assumptions about the future
-Active strategy is based on Interest rate anticipation and sector/security selection

(d) Credit Analysis

Assess default risk

Spread Analysis

Invest in sector(s) that will display the strongest relative price movements, concentrating on spreads.

Valuation Analysis

Determine whether a bond is "rich" or "cheap"

2. (a)

Investment:	Advantages:	Disadvantages:
Short Term T-bills	-low credit risk -liquid, so can easily sell if annuity surrenders	-very short maturity compared to the annuity
real-return public bonds	-coupons are indexed by CPI, so they will increase as the annuity payments do -low credit risk -liquid	-safe investments, so return will be fairly low
corporate bonds	-longer maturities than T-bills (closer to maturity of annuity) -diverse range of investment choices -higher return than Treasuries	-higher credit risk (not good ∵ annuity has guaranteed payments) -may not be as liquid as treasury securities
real estate	-high returns (but they may not be worth the extra risk)	-illiquid -higher credit risk

(b) I would recommend investing primarily in real return public bonds. Their income will vary with the annuity payments, and they are liquid, which is needed in case of withdrawals.

I would avoid real estate investments because they are illiquid.

Could include some T-bills to help with liquidity.

A small portion of the portfolio could be invested in corporate bonds, to diversify the portfolio. These are riskier than the government securities, so you should look at the investor's risk tolerance.

(c) -accumulate asset and liability CF to calculate an accumulated surplus
-repeat the projections under various interest rate shocks – different levels and shapes

-advantage: reinvestments and renewals are explicitly modeled.
-disadvantage: difficult to incorporate an active management strategy into the analysis

-key components:
- -assume assets and liabilities mature at the next date when they reset to market values
- -model reinvestment rates
- -include investment expenses
- -model expected asset defaults
- -adjust the initial portfolio if it is not the "normal" portfolio expected to be used

(d) -objective – qualitative or quantitative

-description of the liabilities
  -payments linked to CPI, expected withdrawals
  -interest rate guarantees implied in annuity

-risk tolerance
  -how will this product impact the firm's overall profitability

-liquidity / marketability requirements
  -need enough liquidity to deal with potential large numbers of surrenders

-regulatory, accounting, tax constraints
  -eg. MCCSR, RBC

-portfolio management constraints
  -limits on investments in specific sectors, issuers, maximum lot sizes

-performance goals and objectives
  -for fund (compare to an index – here, maybe the CPI is appropriate) and managers (compare to their peers, monitor compliance with policy).

-procedures and authorities
  -who is responsible for what, asset purchase approval limits

-asset mix target

3. (a) Bond A is callable bond, only callable bonds have negative convexity

B is option-free bond priced below par, below par implies lower coupon which implies higher duration

C is option-free bond priced above par, higher coupon, lower duration

D is putable bond, high convexity implies put option.

(b) duration only captures very small interest rate shifts accurately, can be misleading for large shifts

doesn't address volatility of interest rates and the factors that drive changes.

can change quickly

convexity is an adjustment to duration estimates that takes into account the shape of yield curve. Precisely, it is the second derivative of the price/yield function.

(c) Modified convexity can only be used for optionless bonds. Can be calculated exactly.

$$C = \frac{1}{A}\sum \frac{t^2 A_t}{(1+y)^{t+1}}$$

Effective convexity is a convexity approximation that can be used on all bonds, including option-embedded. Only meaningful when convexity-adjustment defined. Fabozzi's measure is:

$$C = \frac{V_+ - 2V_0 + V_-}{2V_0(\Delta y)^2}$$

Relies upon good model to value V_+ & V_-

(d) $D = \dfrac{V_- - V_+}{2(V_0)(\Delta y)}$

adjustment: $-D \times \Delta y \times V_0 + C \times (\Delta y)^2 \times V_0$

For A $100(-3.1)(-0.005) + (0.005)^2(-41.7)(100)$

$= 1.44575\%$

For B $(100)(-4.5)(-0.005) + (0.005)^2(23.4)(100)$

$= 2.3085\%$

4. (a) Spot Rate = Yield to Maturity of zero-coupon treasury with the same maturity

$S_{0.5}$ = Yield to Maturity of A = 3.0% (no coupon so yield to maturity = spot rate)

$S_{1.0}$ = Yield to Maturity of A = 3.2% (no coupon so yield to maturity = spot rate)

$S_{1.5}$ = Need to calculate price…

$$\text{Price} = \frac{(0.06/2)}{(1+0.035/2)+(1+0.035/2)^2+(1+0.035/2)^3} =$$

$$= 1.0362248$$

$$\text{Spot Rate } S_{1.5} = \frac{(0.06/2)}{(1+0.03/2)} + \frac{(0.06/2)}{(1+0.032/2)^2} + \frac{1.03}{(1+S_{1.5/2})^3} = 1.0362248$$

$$S_{1.5} = 3.51098\%$$

$S_{2.0}$ = Need to calculate price…

$$\text{Price} = \frac{(0.05/2)}{(1+0.036/2)+(1+0.036/2)^2+(1+0.036/2)^3+(1+0.036/2)^4} =$$

$$= 1.026783971$$

$$\text{Spot Rate } S_{2.0} =$$

$$\frac{(0.05/2)}{(1+0.03/2)} + \frac{(0.05/2)}{(1+0.032/2)^2} + \frac{(0.05/2)}{(1+0.032/2)^3} + \frac{1.025}{(1+S_{2.0/2})^4} = 1.026783971$$

$$S_{2.0} = 3.610807\%$$

(b) Arbitrage Profits can be made by creating zero-coupon securities from coupons of the 2-year bond.

Profits will be made if the present value of the coupons sold separately is greater than the price of the two-year bond.

Profits will be made if the coupons are sold for more than the rates used in the spot rate calculation.

Profits could arise if the zero-coupon treasuries are sold for more than yield to maturity.

Coupon stripping does not work well in practical applications.

(c) One year forward rate one year from today = Yield on 1-year Treasury at time $T+1$

Forward Rate = Investor is indifferent between one 2-year bond and two 1-year bonds

$$(1+F/2)^2 = \frac{(1+S_{2.0}/2)^4}{(1+S_{1.0}/2)^2} =$$

$$F = \frac{(1+0.036108/2)^2}{(1+0.032/2)} - 1 =$$

$$F = 0.04022$$

4. Continued

(d) i. Pure expectations theory states that expected future rates are strictly a function of forward rates

Forward rates represent expected future rates

Rising term structure indicates expectation of rising interest rates

Decreasing term structure indicates expectation of declining interest rates

Flat structure indicates an expectation of constant interest rates

ii. Broad interpretation – investors expect return to be the same regardless of maturity strategy

Local Expectations Theory – Returns will be the same over short term

Return to Maturity expectations – Returns are rolling over short-term bonds = same as zero coupon bond.

iii. Shortcomings:
-Does not account for risks inherent in investing in bonds
-There is uncertainty about bond returns – investment horizon
-There is uncertainty about reinvestment risk

(e) Liquidity premium theory –
-Liquidity Premium implied in forward rates
-Increasing premium for higher maturities

Preferred habitat theory –
-Every investor has a preferred maturity
-Does not necessarily rise uniformly with maturity
-Some lenders may be induced to shift maturities for a premium

Market segmentation theory –
-Yield curve is determined by supply and demand only
-Asset/Liability strategies and constraints create an unwillingness to shift
-Investors not willing to shift to other maturities due to matching

5. (a) 1. No dividends on underlying security
2. Can borrow and lend at risk-free rate
3. Short selling and borrowing are allowed and can trade in fractional shares.
4. No transaction costs or taxes
5. No arbitrage opportunities

(b) $C = SN(d_1) - Ke^{-r\tau}N(d_2)$

$$d_1 = \frac{\ln\left(S/K\right) + \left(r + \sigma^2/2\right)\tau}{\sigma\sqrt{\tau}} = 0.204 \rightarrow d_2 = d_1 - \sigma\sqrt{\tau} = 0.004$$

$$\Rightarrow N(d_1) = 0.6 \times 0.5793 + 0.4 \times 0.5832 = 0.58086$$

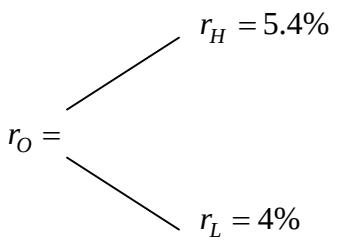
$$N(d_2) = 0.6 \times 0.5 + 0.4 \times 0.504 = 0.5016$$

$$C = 50 \times 0.58086 - 52 \times e^{-0.06} \times 0.5016 = 4.4788$$

(c) from put-call parity $\Rightarrow C + Ke^{-r\tau} = P + S$

$$\Rightarrow P = C + Ke^{-r\tau} - S = 3.45056$$

6.



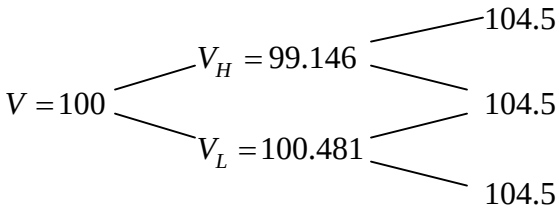
$\sigma = 15\%$
$t = 1$

$$r_H = r_L \times e^{2\sigma}$$
$$= 4\% \times e^{2(0.15)}$$
$$= 5.4\%$$

(a) Using Bond A to calculate the one-year spot rate:

$$V_i = \left(\frac{V_{i+1} + C}{1+y} + \frac{V_i + C}{1+y}\right)\left(\frac{1}{2}\right)$$

$MV = 100$
$Par = 100$
$C = 4.5\%$ annually



$$V_H = \left(\frac{104.5 + 104.5}{1.054}\right)\left(\frac{1}{2}\right) = 99.146$$

$$V_L = \frac{(104.5 + 104.5)\left(\frac{1}{2}\right)}{1.04} = 100.481$$

$$V = 100 = \left(\frac{99.146 + 4.5}{1+S_1} + \frac{100.481 + 4.5}{1+S_1}\right)\frac{1}{2}$$

$$100 = \frac{104.3135}{1+S_1} \qquad 1+S_1 = 1.043135$$

$$S_1 = r_O = 4.3135 \quad \frac{1}{2}$$

(b) $$100 = \frac{4.5}{1.043135} + \frac{104.5}{(1+S_2)^2}$$

$$95.6860809 = \frac{104.5}{(1+S_2)^2}$$

$$(1+S_2)^2 = 1.092112865$$

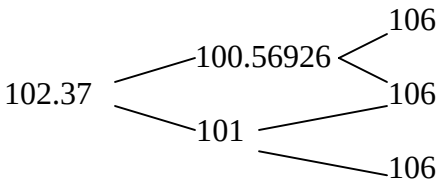
$$S_2 = 4.504\%$$

6. Continued

(c) $1+ f_1 = \dfrac{(1+S_2)^2}{1+S_1} = \dfrac{1.092112865}{1.043135} = 1.04696$

$f_1 = 4.696\%$

(d) Value of Call Option in Bond B.
Bond will be called if bond value is greater than \$101

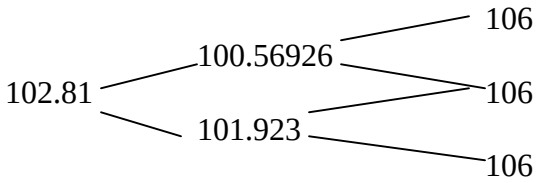


$V_H = \dfrac{106}{1.054} = 100.56926$

$V_L = \dfrac{106}{1.04} = 101.923$ (Bond will be called)

$V = \left(\dfrac{100.56926+6}{1.043135} + \dfrac{101+6}{1.043135} \right) \dfrac{1}{2} = 102.37$

Value of Bond B if no call option:



$V = \left(\dfrac{100.56926 + 101.9231^{\frac{1}{2}} + 6}{1.043135} \right)$

$= 102.81$

Call Option Value = Option-less Bond – Callable Bond

$= 102.81 - 102.37$

$= 0.44$

7. Value of Callable Bond = Value of option-less bond – Value of Call Option
Value of Option-less bond changes inversely with rate changes
Value of Call Option increases as interest rates decrease.
Value of Call Option increases as interest rate volatility increases

Risks

1. Market or Interest Rate Risk - net value of bond will decrease (increase) if interest rates rise (fall)

2. Yield Curve Risk - yield curve might shift in an unparallel fashion

3. Credit / default risk - corporations may not make all of the payments

4. *Liquidity Risks (especially)
   -might have higher transaction costs when sell
   -many think that private placements are less liquid than normally issued bonds

5. Event Risk - Something might happen at corporation that makes them unable to pay on debt - key employee quit, natural disaster

6. Volatility Risk – interest rate volatility increase option value and ∴ decreases bond value

7. Exchange Rate / Currency Risk – for bonds that have foreign currency components… not this one…

8. Reinvestment Risk - might not be able to reinvest coupon cash flows at a high enough interest rate

9. *Timing / Call Risk (especially) - bond could be called way before your planning horizon
   -this could limit your capital gain
   -subjects you to more reinvestment risk because now must reinvest the call amount (whole bond price)

10. Inflation Risk - may achieve desired nominal return, but not desired real return because inflation increased

11. Political / Legal Risk - tax treatment of coupon income could change

12. Sector Risk - something could happen to the whole industry that could affect all of the companies' ability to pay on debt.
    -perceived creditworthiness of sector
    -collapse of major industry participant cause skepticism in market

8.

1. Loyalty
   - -act in the best interests of the plan participants
   - -acting on behalf of the benefit of the plan sponsor, without benefiting the plan participants, is a violation

2. Care and diversify
   - -should take 'prudent' care of assets
   - -should make appropriate diversification, unless it is clearly prudent not to do so
   - -investing solely on one specific security of one issuer is a violation.

3. Delegate
   - -can delegate authority, but not responsibility
   - -blaming an external manager of the trust is a violation since the responsibility still belongs to the trustee.

4. Impartiality
   - -should not favor any specific age group
   - -older participants may want to preserve capital while younger ones want to maximize returns.
   - -investing solely on high risk assets such as equity is a violation because it does not take into consideration the concerns of the older group

5. Make assets productive
   - -should strive to earn a return on the assets
   - -keeping all the assets in a money market account is a violation because the trustee should be capable of earning a higher return without increasing a lot of risk

6. Follow statutory constraints
   - -avoid prohibited transactions such as transactions between plan sponsor and the trust, transferring assets from the trust to the sponsor
   - -invest the assets solely on the stock of the plan sponsor is a violation

7. Act in accordance with trust agreement
   - -but not if it violates other duties
   - -blindly following a trust agreement such as investing solely on derivatives fulfill 5. but not 2. is a violation

8. Regarding co-trustees
   - -should report to authorities and plan sponsor if other co-trustees violate any of their duties
   - -collaborate with other co-trustees who violate any of the duties is a violation

9. (a) i. interest rate forwards – dealer OTC, Over the Counter market
interest rate futures – exchange

interest rate swaps, interest rate caps, interest rate floors } cap and floor usually embedded in securities like CMO — dealer OTC markets

ii. interest rate forwards – lower liquidity, since customized
interest rate futures – higher liquidity, since standardized in exchange

interest rate swaps, interest rate cap, interest rate floor } less liquid when compare to futures since they are often customized products in dealer market, not easy to find another buyer who likes the terms of particular swaps/cap/floor

iii. Contract form

interest rate forwards – customized contract, depending on the need of participants in transaction

interest rate futures – standardized term, embedded features, size, maturity date, sticker price

interest rate swaps, interest rate caps, interest rate floors } similar to forwards which contrast vs customized

iv. interest rate forwards – higher transaction cost than futures, they are usually dealer bid – ask spread

interest rate futures – lower transaction cost semi higher liquidity when traded in exchange.

interest rate swaps, interest rate caps, interest rate floors } similar to forwards so termination cap and floor would be expensive due to bid ask volatility

(b) i. Enter into the swaps which pay fixed and receive floating

In this way, floating income from asset can be swapped info a fixed income and can use the fixed income received to pay off the fixed rate liability in the annuities. A spread may be locked in through this use of swap. The cash flow for asset and liability in this case is tightly matched, this reduces the interest rate risk regarding reinvestment and disinvestment.

9. Continued

ii. Long interest rate floor, fund the long position by setting interest rate cap.

Long position interest rate floor will pay when the designated floating rate decreases below the predetermined floor. This can ensure the earned rate from the floating rate asset not to fall below the floor. The risk of floating rate drop below the fixed liability is minimized as we are compensated for any floating interest rate below a certain level.

To find this long floor, have to sacrifice the upside return when the floating rate earned on asset would increase when the interest rate is high.

Both cover, thus reduce the exposure to interest rate risk by limit the earned rate to be between the cap and floor level.

10. (a) $\rho_{AB} = \dfrac{Cov(r_A, r_B)}{\sigma_A \, \sigma_B}$

For A

$E(r_A) = 50\%(25) + 30\%(10) + 20\%(-30) = 9.5\%$

$E(r_A^2) = 50\%(25^2) + 0.3(10^2) + 0.2(-30)^2 = 522.5\%$

$\sigma_A^2 = 522.5 - 9.5^2 = 432.25\%$

$\sigma_A = 20.80\%$

For B

$E(r_B) = 50\%(20) + 30\%(-20) + 20\%(25) = 9\%$

$E(r_B^2) = 0.5(20^2) + 0.3(-20^2) + 0.2(25^2) = 445\%$

$\sigma_B^2 = 445 - 81 = 364$

$\sigma_B = 19.08\%$

$Cov(r_A, r_B) = E(r_A \, r_B) - E(r_A) \cdot E(r_B)$

$E(r_A, r_B) = 0.5(25)(20) + 0.3(10)(-20) + 0.2(-30)(25)$

$= 40$

$Cov(r_A, r_B) = 40 - 9.5(9) = -45.5\%$

$\rho_{AB} = \dfrac{-45.5}{20.8(19.08)} = -0.115$

(b) $A \to D \qquad B \to E$

$$w_D = \frac{\left(E(r_D) - r_f\right)\sigma_E^2 - \left(E(r_E) - r_f\right)Cov(r_D, r_E)}{\left(E(r_D) - r_f\right)\sigma_E^2 + \left(E(r_E) - r_f\right)\sigma_D^2 - \left(E(r_D) - r_f + E(r_E) - r_f\right)Cov(r_D \;\; r_E)}$$

$$w_A = \frac{\left(E(r_A) - r_f\right)\sigma_B^2 - \left(E(r_B) - r_f\right)Cov(r_A, r_B)}{\left(E(r_A) - r_f\right)\sigma_B^2 + \left(E(r_B) - r_f\right)\sigma_A^2 - \left[\left(E(r_A) - r_f\right) + \left(E(r_B) - r_f\right)\right]Cov(r_A, r_B)}$$

$$= \frac{(9.5-3)(364) - (9-3)(-45.5)}{(9.5-3)(364) + (9-3)(432.25) - [9.5-3+9-3][-45.5]} = \frac{2639}{5528}$$

$= 47.7\% \approx 48\%$

$w_B = 52\%$

$E(r_p) = w_A E(r_A) + w_B E(r_B)$

$= 48\%(9.5) + 52\%(9) = 9.24\%$

$\sigma_p^2 = w_A^2 \, \sigma_A^2 + w_B^2 \, \sigma_B^2 + 2 w_A w_B \, Cov(r_A, r_B)$

10. Continued

$$= (0.48)^2 (432.25) + (0.52)^2 (364) + 2(0.48)(0.52)(-45.5)$$
$$= 175.3$$
$$\sigma_p = 13.24\%$$

(c) Slope of CAL $= \dfrac{E(r_p) - r_f}{\sigma_p}$

$$= \frac{9.24 - 3}{13.24} = 0.47$$

(d) $U = E(r) - 0.025\ \sigma^2$ $\qquad u = E(r) - 0.005A\ \sigma^2$

$0.005A = 0.025$ $\qquad A = 5$

$$y^* = \frac{E(r_p) - r_f}{0.01A\ \sigma_p^2}$$
$$= \frac{9.24 - 3}{0.01(5)(175.3)} = 71.2\%$$

So

For A $\quad Wtg\ A = 71.2\%(0.48) = 34.18\%$

For B $\quad Wtg\ B = 71.2\%(0.52) = 37.02\%$

For T-bill $\quad Wtg\ T = 1 - 71.2\% = 28.8\%$

(e) Q

$E(r_Q) = 10\%$

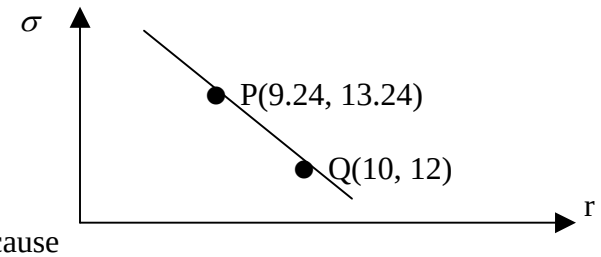
$\sigma_Q = 12\%$

P

$E(r_p) = 9.24\%$

$\sigma_p = 13.24\%$

Compare



Q is better than P because

1. Q has higher return $\quad 10\% > 9.24\%$
2. Q has smaller risk, $\quad \sigma_Q = 12\% < \sigma_p = 13.24\%$

$\Rightarrow$ Q is more efficient than P.

11. i. 1. modified and Macaulay duration –matching – sensitivity to changes in interest rates, assuming cash flows do not change
2. effective duration – matching – sensitivity in changes in interest rates, taking into account that cash flows may change
3. convexity – matching – second derivative of price yield relationship
4. D3 matching and interest rate techniques – match 3rd, 4th, etc. derivatives of cash flows
5. partial duration matching – match duration of several key rates
6. cash flow matching – make sure cumulative amount of CF received is more than CF have to pay out
7. Horizon matching – cash flow match for first years, immunize after
8. non-parallel interest rate analysis – test several non-parallel shifts
9. scenario – based dollar quantification – test several scenarios and see how much need to fund cash outflows
10. maturity matching – match maturities of assets and liabilities

ii. 1. –only good for small movements of interest rates
–doesn't account for embedded options
–only good for parallel rate changes and flat curve
2. –not good once interest rates have moved
–still exposed to non-parallel shifts
3. -still exposed to big non-parallel shifts
4. -may not be option-adjusted
-hard to tell what still exposed to
-manager may not understand
-corrective actions may not be easy to apply
-assumptions may be wrong
5. -only good for key rate changes
-so many key rates, can be confusing
-doesn't account for embedded options
6. -can never do exactly, so still at risk
-does not account for embedded options
-does not quantify dollar amount at risk
7. -still subject to non-parallel after CF match period
-still only good for small interest rate changes after match period
-doesn't account for embedded options
8. -may not test enough scenarios
-may not include embedded options
-may not provide actionable results
9. -does not provide concise, actionable results
10. -does not include embedded option
-no good for assets/liabilities with no maturity date
-does not quantify dollar amount at risk

12. (a) Taxation – regulations can be different for foreign investments
Exchange Rate – risk that the dollar will depreciate relative to foreign currency
Country-specific risks – political, economic or financial environment in foreign country
Credit Risk – for foreign security

Exchange rate risk – must have that

$$\left(1+r_{US}\right)=\left(1+r_{foreign}\right)\left(F\left(1\right)/E_0\right)$$

where F is the forward exchange rate @ time 1
E is the current exchange rate

(to avoid arbitrage)

(b) Index Fund Return = $\sum$ (Benchmark weight)(Index Return + Currency Appreciation)

$$= \left(0.4\right)\left(0.1+0.2\right)+0.25\left(0.05-0.1\right)+0.35\left(0.07+0.25\right)$$
$$= 21.95\%$$

Manager Return = $\sum$ (Mgr Weight)(Mgr Ret + Currency Appreciation)
since = to Index Return

$$21.95\% = \left(0.35\right)\left(0.12+0.2\right)+x\left(0.07-0.1\right)+y\left(0.2+0.25\right)$$
$$= 0.112-0.03x+0.45y$$

where x equals amount invested in European
y equals amount invested in Australian

And we know (2) $x+y+0.35=1$ since $\sum w_i = 1$

So $0.2195 = 0.112-0.03x+0.45y$
(1) $0.1075 = -0.03x+1.5y$

(1) / 0.3 $3.5833 = -x+15y$
(2) $+\ \ 0.65 = x+y$

$$4.233 = 16y$$
$$y = 0.26458$$

$$x = 0.3854$$

$\Rightarrow$ amount invested in European = 0.3854 = 38.54%
Australian = 26.458%

12. Continued

(c) i. Currency Selection = $\sum$ (Currency Appreciation)(Mgr Wgt – Index Wgt)

= - 4.4895%

ii. Country Selection = $\sum$ (Index Return)(Mgr Wgt – Index Wgt)

= - 0.42094%

iii. Stock Selection = $\sum$ (Mgr Weight)(Manager Ret – Index Ret)

= + 4.91034%

13. (a) redirect principal

–sequential pay: principal pay to first tranche, second tranche get payment when first tranche pay off

–planned amortized class – get scheduled payment when prepayment is within prepayment bands
– volatility absorbed by companion
– type II and III PAC has narrower bands

–target amortization class
–lower band same as PAC pricing spread
–higher return than PAC as there is extension risk

–companion – absorb volatility for PAC

–performance based on: –class it supports
–collateral backing CMO
–prepayment rate

redirect principal and interest

–Z bond – do not get principal or interest in the beginning, get all principal and interest when other class pay off.
– interest is calculated in accretion class

–accretion class (VADM) – get interest from Z class

–principal only PO or interest only IO strips
–pay principal only: – very positive duration, benefit from fast prepayment
–interest only – very negative duration benefit from slow prepayment

(b)

Year	tranche 1	tranche 2	tranche 3
0	20,000	35,000	65,000
1	20,000-12,124.31-1,078.76=6,796.93	35,000	65,000
2	0	27002.82	65000
3	0	10852.72	65000
4	0	0	58636.39
5	0	0	40688.75
6	0	0	22615.47
7	0	0	4462.16
8	0	0	0

13. Continued

(c)

Year	tranche 1	tranche 2	tranche 3
1	$7200 \times \frac{20}{120} = 1200$	2100	3900
2	$6467.81 \times \frac{6796.93}{106796.93} = 407.82$	2100	3900
3	0	$5520.17 \times \frac{27002.82}{92002.82} = 1620.17$	3900

14. (a) -reasonable and long enough to meet New York 126 requirements
-variaties of interest curve shape
-long term volatility less than short term
-annual change should be reasonable
-upper and lower bands for political reasons (126 requirements says 4% and 25%)

(b) i. -continuous version of lattice tree
-initial rate = current spot rate
$i_{t+1} = i_t \times e^{VF \times Z}$
VF is volatility factor
$Z \sim N(0,1)$ standard normal distribution

ii. mean reversion
-pull back into normal level using a correction factor
-bands can cause stickiness without mean reversion

correction factor
-can be applied before or after
before = $i_{t+1} = (i_t + cf)e^{VF \cdot Z}$
after = i_{t+1} = tentative $i_{t+1} + C$ (goal rate – tentative interest rate)

secondary rate determination
-usually generate 2 rates
-the second one is generated with correlation to the first one
-use deterministic procedure to finish the curve

(c) i. shapes and level of interest rates are determined separately
ii. shapes of interest curve
1. steep upwards sloping
2. early peak
3. oscillating, starting up
4. level
5. oscillating, starting down
6. early valley
7. steep inverted

use probability matrix to determine movement between shapes
-can not move more than 2
-deliberately move towards shape 1

iii. constraints
-use log normal to determine tentative curve
-adjust so that change less than *x%* from prior year
-adjust so that all rates change less than *y*% from prior year
-overall minimum and maximum applied. After adjustment shape will be different from original 7 shapes

15. (a) Advantages:

1. less computations if use multiple factors
2. allows cash flows to be path dependent
3. Interest rates jumps are more realistic (similar to lattice)
4. do not have to worry about tree rejoining.

Disadvantages:

difficult to use for derivatives / options that can be exercised early e.g. American put options

(b) 1. Antithetic Variable Technique

-average a pair of unbiased estimators that are highly negatively correlated to each other to reduce variance

-can determine by using u_i to calculate f_1 and $1-u_i$; to calculate f_2 where $u_i \sim UNIF(0, 1)$

-may not always reduce variance unless the model chosen is monotonic with respect to the underlying numbers.

2. Control Variate Technique

-the idea here is replace the security under consideration with a similar but simpler one with a simple and known closed form solution – improves the accuracy of the results

the control variate chosen should be highly correlated with the security under consideration.

3. Sampling Approach

-divide sampling region of random variate into sub-regions

-use simulation for each sub-region

-ensures getting more simulation in more important regions rather than sampling over the whole distribution.

4. Low-Discrepancy Method

-much like Monte Carlo simulation

-chooses points that are as uniformly close as possible.

-converges to an accurate and stable value faster than Monte Carlo.

5. Path dependency

-need way to decide whether should exercise early

-path bundling techniques developed to value American-style derivatives using simulation.