EDUCATION AND EXAMINATION COMMITTEE OF THE SOCIETY OF ACTUARIES (SOA)

SPRING 2007
EXAM MLC

ACTUARIAL MODELS—LIFE CONTINGENCIES SEGMENT

INTRODUCTORY STUDY NOTE

1. The Actuarial Models–Life Contingencies Segment examination for Spring 2007 will be given on **Thursday, May 17, from 8:30 a.m. to 11:30 a.m.** The examination will consist of 30 multiple-choice questions.

   The score for the examination is determined solely on the basis of correct answers Therefore, candidates should answer every question to maximize their scores.

2. Any changes in the Course of Reading for this exam since the publication of the *Spring 2007 Basic Education Catalog* of the SOA are reflected in this Introductory Study Note and will also be posted on our Web site. **If any difference exists between information contained in this Introductory Study Note and that contained in the *Spring 2007 Basic Education Catalog*, this Introductory Study Note will govern**

3. The following list contains all study notes for this exam in Spring 2007. Candidates who have ordered the complete set of study notes, should verify immediately that they have copies of the listed items. Items marked with a # are new/updated for this session.

MLC-05-07#	Introductory Study Note (this study note)
	Notational differences between *Actuarial Mathematics (AM)* and *Models for Quantifying (MQR)* for candidates taking Exam MLC
	Sample Problem Mapping for Exam MLC
	Past exams www.soa.org/STATIC/examinations.html
MLC-24-05	Multi-State Transition Models with Actuarial Applications
MLC-25-05	Section 8.5 from the second printing of Actuarial Mathematics, Second Edition (only the variance recursion given by Equation 8.5.16 with i=1)
MLC-27-07#	Selected Sections of Chapter 5 from *Introduction to Probability Models*

4. The study notes for this exam include sample questions and solutions. The sample questions provide the candidate with the opportunity to practice on the types of questions that are likely to appear on the examination. New sample examinations will be released periodically or whenever the nature of the examination changes substantially.

5. Enclosed are errata for the following texts:

Candidates using the First Edition of *Models for Quantifying Risk* will need to supplement the text with the Errata Package available on the Actex web site www.actexmadriver.com

Actuarial Mathematics, (Second Edition) 1997, first printing, by Bowers, Hickman, Gerber, Jones & Nesbitt. These corrections have been made to the second printing of this edition

6. The Illustrative Life Table, the Illustrative Service Table, and a set of values from the standards normal distribution will be available for use on Exam M Actuarial Models–Life Contingencies Segment (Exam MLC). A copy of these is included with this note. Note that candidates will not be allowed to bring copies of the tables into the examination room.

7. A survey for examinations FM, MLC, MFE and C will be available on the SOA and CAS web sites after the examinations have been administered. Candidates are encouraged to provide feedback on the course readings and the examinations that they have taken.

8. Several book distributors carry some or all of the textbooks for the Society of Actuaries exams. A list of distributors appears in the *Spring 2007 Basic Education Catalog*. A set of order forms from these distributors is included with this study note package.

The order forms contain information about prices, shipping charges, mailing policy and credit card acceptance. Any book distributor who carries books for SOA exams may have their order form included in this set unless the SOA office receives substantial complaints about service. Candidates should notify the Publication Orders Department of the SOA in writing if they encounter serious problems with any distributor.

9. The examination questions for this exam will be based on the required readings for this exam. If a conflict exists (in definitions, terminology, etc.) between the readings for this exam and the readings for other exams, the questions should be answered on the basis of the readings for this exam

10. Candidates may use the battery or solar-powered Texas Instruments BA-35 model calculator, the BA II Plus*, the BA II Plus Professional* or TI-30X or TI-30Xa or TI-30X II* (IIS solar or IIB battery). Candidates may use more than one of the approved calculators during the examinations.

Calculator instructions cannot be brought into the exam room. During the exam, the calculator must be removed from its carrying case so the supervisor can confirm it is an approved model. Candidates using a calculator other than the approved models will have their examinations disqualified.

Candidates can purchase calculators directly from: Texas Instruments, Attn: Order Entry, PO Box 650311, Mail Station 3962, Dallas, TX 75265, phone 800/842-2737 or http://epsstore.ti.com

The memory of* ***TI-30X II, BA II Plus* and ***BA II Plus Professional*** *will need to be cleared by the examination supervisor upon the candidates' entrance to the examination room.*

11. Order forms for various seminars/workshops and study manuals are included with this set of study notes. These seminars/workshops and study manuals do not reflect any official interpretation, opinion, or endorsement of the Society of Actuaries.

12. A candidate planning to seek admission to the SOA should submit the Application for Admission as

Associate *before* completing the education requirements for Associateship as detailed in the *Spring 2007 Basic Education Catalog*.

13 In addition to the examination requirements, all prospective SOA Associates will be required to attend and successfully complete a seminar on professionalism prior to admission as a member. See the *SOA Spring 2007 Basic Education Catalog* for more information

14. The Society of Actuaries provides study notes to persons preparing for this examination. They are intended to acquaint candidates with some of the theoretical and practical considerations involved in the various subjects. While varying opinions are presented where appropriate, limits on the length of the material and other considerations sometimes prevent the inclusion of all possible opinions. These study notes do not, however, represent any official opinion, interpretation or endorsement of the Society of Actuaries. The SOA is grateful to the authors for their contributions in preparing study notes.

The American Academy of Actuaries (AAA) and the Conference of Consulting Actuaries (CCA) jointly sponsor the Associateship and Fellowship examinations with the SOA.

Notational differences between *Actuarial Mathematics* (*AM*) and *Models for Quantifying Risk* (*MQR*) for candidates taking Exam M

There are several notational differences between the two texts The ones listed in this note are the ones that may appear on the examination. Examination questions will be written so that candidates will not be disadvantaged by the text they used.

1. Force of mortality

MQR uses μ_x or μ_{x+t}. The subscript is always the current age. *AM* uses $\mu(x)$, $\mu(x+t)$, or $\mu_x(t)$. The first two refer to attained ages while the third one is reserved for indicating selection at age x and attained age $x+t$. Any of these symbols may appear on the exam.

2. Survival function

AM uses $s(x)$ while *MQR* uses $S(x)$. Either symbol may appear on the exam.

3. Future Complete lifetime

MQR uses T_x while *AM* uses $T(x)$ for the future complete lifetime of (x). Either symbol may appear on the exam. The same comment applies for joint life, T_{xy} or $T(xy)$, and last survivor statuses, $T_{\overline{xy}}$ or $T(\overline{xy})$.

4. Future Curtate Lifetime

In *AM* the random variable $K(x)$ is always the full number of years lived by (x) prior to death. *MQR* uses the same variable in the same way, but also uses $K_x = K(x) + 1$. In these situations the text of the question will define any notation used.

5. Present value of future losses

AM uses L or ${}_0L$ for loss at issue and ${}_tL$ for loss from t years after issue. *MQR* at times adds other pieces to the symbol to describe the policy The exam will use the generic notation from *AM* and describe the policy in the text of the problem.

6. Duration subscripts

At various times the texts count duration differently Something happening in the first year (between ages x and $x+1$) may be identified with a 0 or 1. For example, *AM* would refer to a premium for year 1, payable at time 0, with subscript 0 *MQR* would use the subscript 1 for such a premium. In these situations the text of the question will define any notation used.

7 Life table symbols

MQR uses ℓ_x while *AM* uses l_x. Either symbol may appear on the exam

8 Types of insurance

AM defines a "fully discrete" insurance as one in which both premiums and benefits are paid only at discrete time points: premiums at the start of the year; death benefits at the end of the year of death. *MQR* does not define such a policy. The terminology will be used in the exam. Both texts define "semi-continuous" (continuous benefits, discrete premiums) and "fully continuous" (both continuous), although *MQR* does so only in a footnote. These terms will be used on the exam.

9. Premiums determined by the equivalence principle

The term "benefit premiums" is used in both texts. In *MQR* it is not explicitly stated that such premiums are determined by the equivalence principle ($E(L) = 0$) For the exam, the term "benefit premium" implies that the premium is determined by the equivalence principle. *MQR* also uses "net premium" in this context. The term "net premium" will not be used in the exam. Likewise, the term "benefit reserves" on the exam will imply that the equivalence principle was used.

10 Premium that includes expenses

AM uses the term "expense-loaded premium" to denote premium that includes expenses in addition to the benefit premium. *MQR* uses the terms "gross premium" and "expense-augmented premium" to denote premium that includes expenses in addition to the benefit premium. Both texts use the symbol G to denote this premium. The term "expense-loaded premium" will be used on the exam. The related term "expense-augmented loss random variable", comparable to the loss at issue random variable but including expenses, will also be used.

Both texts also use the symbol G to denote the level contract premium or level gross premium, the premium paid for the insurance. When the symbol G is used on the exam the meaning of the symbol will be defined in the question.

11. Special insurance policies

This applies to neither text, but is a convention used in the exam. In some problems, reference is made to a "special insurance policy." Such policies have non-level benefits, premiums, or both, which are then described in the question. If an insurance policy is not defined as "special" then premiums and benefits are assumed to be level, unless there is explicit information to the contrary.

NORMAL DISTRIBUTION TABLE

Entries represent the area under the standardized normal distribution from $-\infty$ to z, Pr(Z<z)
The value of z to the first decimal is given in the left column. The second decimal place is given in the top row.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Values of z for selected values of Pr(Z<z)							
z	0.842	1.036	1.282	1.645	1.960	2.326	2.576
Pr(Z<z)	0.800	0.850	0.900	0.950	0.975	0.990	0.995

Illustrative Life Table: Basic Functions and Single Benefit Premiums at $i = 0.06$

x	l_x	$1000q_x$	$\ddot{a}_x$	$1000A_x$	$1000(^2A_x)$	$1000\,{}_5E_x$	$1000\,{}_{10}E_x$	$1000\,{}_{20}E_x$	x
0	10,000,000	20.42	16.8010	49.00	25.92	728.54	541.95	299.89	0
5	9,749,503	0.98	17.0379	35.59	8.45	743.89	553.48	305.90	5
10	9,705,588	0.85	16.9119	42.72	9.37	744.04	553.34	305.24	10
15	9,663,731	0.91	16.7384	52.55	11.33	743.71	552.69	303.96	15
20	9,617,802	1.03	16.5133	65.28	14.30	743.16	551.64	301.93	20
21	9,607,896	1.06	16.4611	68.24	15.06	743.01	551.36	301.40	21
22	9,597,695	1.10	16.4061	71.35	15.87	742.86	551.06	300.82	22
23	9,587,169	1.13	16.3484	74.62	16.76	742.68	550.73	300.19	23
24	9,576,288	1.18	16.2878	78.05	17.71	742.49	550.36	299.49	24
25	9,565,017	1.22	16.2242	81.65	18.75	742.29	549.97	298.73	25
26	9,553,319	1.27	16.1574	85.43	19.87	742.06	549.53	297.90	26
27	9,541,153	1.33	16.0873	89.40	21.07	741.81	549.05	297.00	27
28	9,528,475	1.39	16.0139	93.56	22.38	741.54	548.53	296.01	28
29	9,515,235	1.46	15.9368	97.92	23.79	741.24	547.96	294.92	29
30	9,501,381	1.53	15.8561	102.48	25.31	740.91	547.33	293.74	30
31	9,486,854	1.61	15.7716	107.27	26.95	740.55	546.65	292.45	31
32	9,471,591	1.70	15.6831	112.28	28.72	740.16	545.90	291.04	32
33	9,455,522	1.79	15.5906	117.51	30.63	739.72	545.07	289.50	33
34	9,438,571	1.90	15.4938	122.99	32.68	739.25	544.17	287.82	34
35	9,420,657	2.01	15.3926	128.72	34.88	738.73	543.18	286.00	35
36	9,401,688	2.14	15.2870	134.70	37.26	738.16	542.11	284.00	36
37	9,381,566	2.28	15.1767	140.94	39.81	737.54	540.92	281.84	37
38	9,360,184	2.43	15.0616	147.46	42.55	736.86	539.63	279.48	38
39	9,337,427	2.60	14.9416	154.25	45.48	736.11	538.22	276.92	39
40	9,313,166	2.78	14.8166	161.32	48.63	735.29	536.67	274.14	40
41	9,287,264	2.98	14.6864	168.69	52.01	734.40	534.99	271.12	41
42	9,259,571	3.20	14.5510	176.36	55.62	733.42	533.14	267.85	42
43	9,229,925	3.44	14.4102	184.33	59.48	732.34	531.12	264.31	43
44	9,198,149	3.71	14.2639	192.61	63.61	731.17	528.92	260.48	44
45	9,164,051	4.00	14.1121	201.20	68.02	729.88	526.52	256.34	45
46	9,127,426	4.31	13.9546	210.12	72.72	728.47	523.89	251.88	46
47	9,088,049	4.66	13.7914	219.36	77.73	726.93	521.03	247.08	47
48	9,045,679	5.04	13.6224	228.92	83.06	725.24	517.91	241.93	48
49	9,000,057	5.46	13.4475	238.82	88.73	723.39	514.51	236.39	49
50	8,950,901	5.92	13.2668	249.05	94.76	721.37	510.81	230.47	50
51	8,897,913	6.42	13.0803	259.61	101.15	719.17	506.78	224.15	51
52	8,840,770	6.97	12.8879	270.50	107.92	716.76	502.40	217.42	52
53	8,779,128	7.58	12.6896	281.72	115.09	714.12	497.64	210.27	53
54	8,712,621	8.24	12.4856	293.27	122.67	711.24	492.47	202.70	54
55	8,640,861	8.96	12.2758	305.14	130.67	708.10	486.86	194.72	55
56	8,563,435	9.75	12.0604	317.33	139.11	704.67	480.79	186.32	56
57	8,479,908	10.62	11.8395	329.84	147.99	700.93	474.22	177.53	57
58	8,389,826	11.58	11.6133	342.65	157.33	696.85	467.12	168.37	58
59	8,292,713	12.62	11.3818	355.75	167.13	692.41	459.46	158.87	59
60	8,188,074	13.76	11.1454	369.13	177.41	687.56	451.20	149.06	60
61	8,075,403	15.01	10.9041	382.79	188.17	682.29	442.31	139.00	61
62	7,954,179	16.38	10.6584	396.70	199.41	676.56	432.77	128.75	62
63	7,823,879	17.88	10.4084	410.85	211.13	670.33	422.54	118.38	63
64	7,683,979	19.52	10.1544	425.22	223.34	663.56	411.61	107.97	64
65	7,533,964	21.32	9.8969	439.80	236.03	656.23	399.94	97.60	65

Illustrative Life Table: Basic Functions and Single Benefit Premiums at $i = 0.06$

x	l_x	$1000q_x$	$\ddot{a}_x$	$1000A_x$	$1000(^2A_x)$	$1000\,_5E_x$	$1000\,_{10}E_x$	$1000\,_{20}E_x$	x
66	7,373,338	23.29	9.6362	454.56	249.20	648.27	387.53	87.37	66
67	7,201,635	25.44	9.3726	469.47	262.83	639.66	374.36	77.38	67
68	7,018,432	27.79	9.1066	484.53	276.92	630.35	360.44	67.74	68
69	6,823,367	30.37	8.8387	499.70	291.46	620.30	345.77	58.54	69
70	6,616,155	33.18	8.5693	514.95	306.42	609.46	330.37	49.88	70
71	6,396,609	36.26	8.2988	530.26	321.78	597.79	314.27	41.86	71
72	6,164,663	39.62	8.0278	545.60	337.54	585.25	297.51	34.53	72
73	5,920,394	43.30	7.7568	560.93	353.64	571.81	280.17	27.96	73
74	5,664,051	47.31	7.4864	576.24	370.08	557.43	262.31	22.19	74
75	5,396,081	51.69	7.2170	591.49	386.81	542.07	244.03	17.22	75
76	5,117,152	56.47	6.9493	606.65	403.80	525.71	225.46	13.04	76
77	4,828,182	61.68	6.6836	621.68	421.02	508.35	206.71	9.61	77
78	4,530,360	67.37	6.4207	636.56	438.42	489.97	187.94	6.88	78
79	4,225,163	73.56	6.1610	651.26	455.95	470.57	169.31	4.77	79
80	3,914,365	80.30	5.9050	665.75	473.59	450.19	151.00	3.19	80
81	3,600,038	87.64	5.6533	680.00	491.27	428.86	133.19	2.05	81
82	3,284,542	95.61	5.4063	693.98	508.96	406.62	116.06	1.27	82
83	2,970,496	104.28	5.1645	707.67	526.60	383.57	99.81	0.75	83
84	2,660,734	113.69	4.9282	721.04	544.15	359.79	84.59	0.42	84
85	2,358,246	123.89	4.6980	734.07	561.57	335.40	70.56	0.22	85
86	2,066,090	134.94	4.4742	746.74	578.80	310.56	57.83	0.11	86
87	1,787,299	146.89	4.2571	759.03	595.79	285.44	46.50	0.05	87
88	1,524,758	159.81	4.0470	770.92	612.51	260.21	36.61	0.02	88
89	1,281,083	173.75	3.8442	782.41	628.92	235.11	28.17	0.01	89
90	1,058,491	188.77	3.6488	793.46	644.96	210.36	21.13	0.00	90
91	858,676	204.93	3.4611	804.09	660.61	186.21	15.41	0.00	91
92	682,707	222.27	3.2812	814.27	675.83	162.90	10.91	0.00	92
93	530,959	240.86	3.1091	824.01	690.59	140.69	7.47	0.00	93
94	403,072	260.73	2.9450	833.30	704.86	119.79	4.93	0.00	94
95	297,981	281.91	2.7888	842.14	718.61	100.43	3.13	0.00	95
96	213,977	304.45	2.6406	850.53	731.83	82.78	1.90	0.00	96
97	148,832	328.34	2.5002	858.48	744.50	66.97	1.10	0.00	97
98	99,965	353.60	2.3676	865.99	756.60	53.09	0.60	0.00	98
99	64,617	380.20	2.2426	873.06	768.13	41.14	0.31	0.00	99
100	40,049	408.12	2.1252	879.70	779.08	31.12	0.15	0.00	100
101	23,705	437.28	2.0152	885.93	789.44	22.91	0.07	0.00	101
102	13,339	467.61	1.9123	891.76	799.21	16.37	0.03	0.00	102
103	7,101	498.99	1.8164	897.19	808.41	11.33	0.01	0.00	103
104	3,558	531.28	1.7273	902.23	817.02	7.56	0.00	0.00	104
105	1,668	564.29	1.6447	906.90	825.06	4.86	0.00	0.00	105
106	727	597.83	1.5685	911.22	832.53	2.99	0.00	0.00	106
107	292	631.64	1.4984	915.19	839.46	1.76	0.00	0.00	107
108	108	665.45	1.4341	918.82	845.84	0.98	0.00	0.00	108
109	36	698.97	1.3755	922.14	851.69	0.52	0.00	0.00	109
110	11	731.87	1.3223	925.15	857.04	0.26	0.00	0.00	110

Illustrative Life Table: Basic Functions and Single Benefit Premiums at $i = 0.06$

Lives are independent.

x	$\ddot{a}_{xx}$	$1000A_{xx}$	$1000(^2A_{xx})$	$\ddot{a}_{x:x+10}$	$1000A_{x:x+10}$	$1000(^2A_{x:x+10})$	x
0	16.1345	86.73	50.89	16.2844	78.24	34.71	0
5	16.6432	57.93	16.51	16.4093	71.17	19.17	5
10	16.4660	67.96	18.13	16.1541	85.62	22.70	10
15	16.2187	81.96	21.67	15.8187	104.60	28.49	15
20	15.9005	99.97	27.00	15.3934	128.67	37.00	20
21	15.8272	104.12	28.33	15.2962	134.18	39.11	21
22	15.7502	108.48	29.77	15.1945	139.94	41.39	22
23	15.6696	113.04	31.33	15.0883	145.95	43.83	23
24	15.5851	117.82	33.01	14.9774	152.22	46.46	24
25	15.4967	122.83	34.82	14.8617	158.77	49.28	25
26	15.4041	128.07	36.77	14.7411	165.60	52.31	26
27	15.3073	133.55	38.87	14.6154	172.71	55.56	27
28	15.2062	139.27	41.12	14.4845	180.12	59.03	28
29	15.1005	145.26	43.55	14.3484	187.83	62.75	29
30	14.9901	151.50	46.16	14.2068	195.84	66.72	30
31	14.8750	158.02	48.96	14.0598	204.16	70.97	31
32	14.7549	164.82	51.96	13.9071	212.80	75.50	32
33	14.6298	171.90	55.18	13.7488	221.76	80.34	33
34	14.4995	179.27	58.63	13.5848	231.05	85.48	34
35	14.3640	186.94	62.32	13.4150	240.66	90.96	35
36	14.2230	194.92	66.26	13.2393	250.60	96.78	36
37	14.0766	203.21	70.48	13.0579	260.88	102.96	37
38	13.9246	211.81	74.98	12.8705	271.48	109.52	38
39	13.7670	220.74	79.77	12.6774	282.41	116.46	39
40	13.6036	229.99	84.89	12.4784	293.68	123.80	40
41	13.4344	239.56	90.32	12.2737	305.26	131.56	41
42	13.2594	249.47	96.11	12.0633	317.17	139.75	42
43	13.0786	259.70	102.25	11.8474	329.39	148.38	43
44	12.8919	270.27	108.76	11.6260	341.92	157.46	44
45	12.6994	281.16	115.66	11.3994	354.75	166.99	45
46	12.5011	292.39	122.95	11.1677	367.87	177.00	46
47	12.2971	303.94	130.67	10.9310	381.26	187.48	47
48	12.0873	315.81	138.80	10.6898	394.92	198.44	48
49	11.8720	328.00	147.38	10.4441	408.82	209.88	49
50	11.6513	340.49	156.41	10.1944	422.96	221.81	50
51	11.4252	353.29	165.89	9.9409	437.31	234.22	51
52	11.1941	366.37	175.85	9.6840	451.85	247.10	52
53	10.9580	379.74	186.28	9.4240	466.57	260.46	53
54	10.7172	393.37	197.18	9.1614	481.43	274.27	54
55	10.4720	407.24	208.57	8.8966	496.42	288.54	55
56	10.2227	421.35	220.44	8.6301	511.50	303.24	56
57	9.9696	435.68	232.79	8.3623	526.66	318.35	57
58	9.7131	450.20	245.62	8.0938	541.86	333.85	58
59	9.4535	464.90	258.93	7.8249	557.08	349.73	59
60	9.1911	479.75	272.69	7.5563	572.28	365.94	60
61	8.9266	494.72	286.91	7.2885	587.44	382.46	61
62	8.6602	509.80	301.56	7.0221	602.53	399.26	62
63	8.3926	524.95	316.62	6.7574	617.50	416.30	63
64	8.1241	540.15	332.09	6.4952	632.34	433.53	64
65	7.8552	555.36	347.92	6.2360	647.02	450.93	65

Illustrative Life Table: Basic Functions and Single Benefit Premiums at $i = 0.06$

Lives are independent.

x	$\ddot{a}_{xx}$	$1000A_{xx}$	$1000(^2A_{xx})$	$\ddot{a}_{x:x+10}$	$1000A_{x:x+10}$	$1000(^2A_{x:x+10})$	x
66	7.5866	570.57	364.09	5.9802	661.50	468.44	66
67	7.3187	585.74	380.58	5.7283	675.76	486.02	67
68	7.0520	600.83	397.35	5.4809	689.76	503.62	68
69	6.7872	615.82	414.36	5.2385	703.48	521.21	69
70	6.5247	630.68	431.58	5.0014	716.90	538.72	70
71	6.2650	645.37	448.96	4.7701	730.00	556.11	71
72	6.0088	659.88	466.46	4.5450	742.74	573.34	72
73	5.7565	674.16	484.03	4.3263	755.11	590.36	73
74	5.5086	688.19	501.64	4.1146	767.10	607.12	74
75	5.2655	701.95	519.23	3.9099	778.69	623.59	75
76	5.0278	715.41	536.75	3.7125	789.86	639.71	76
77	4.7959	728.54	554.16	3.5227	800.60	655.46	77
78	4.5700	741.32	571.41	3.3406	810.91	670.79	78
79	4.3507	753.74	588.45	3.1663	820.78	685.67	79
80	4.1381	765.77	605.25	2.9998	830.20	700.08	80
81	3.9326	777.40	621.75	2.8412	839.18	713.99	81
82	3.7344	788.62	637.91	2.6905	847.71	727.37	82
83	3.5438	799.41	653.70	2.5476	855.80	740.21	83
84	3.3607	809.77	669.08	2.4125	863.44	752.49	84
85	3.1855	819.69	684.02	2.2851	870.66	764.20	85
86	3.0181	829.16	698.48	2.1652	877.44	775.34	86
87	2.8587	838.19	712.45	2.0527	883.81	785.89	87
88	2.7071	846.77	725.89	1.9475	889.77	795.86	88
89	2.5633	854.91	738.79	1.8493	895.33	805.25	89
90	2.4274	862.60	751.14	1.7579	900.50	814.05	90
91	2.2991	869.86	762.91	1.6731	905.30	822.29	91
92	2.1784	876.70	774.11	1.5947	909.73	829.96	92
93	2.0651	883.11	784.73	1.5225	913.82	837.07	93
94	1.9590	889.11	794.77	1.4563	917.57	843.64	94
95	1.8600	894.72	804.22	1.3957	921.00	849.67	95
96	1.7678	899.93	813.09	1.3407	924.11	855.20	96
97	1.6823	904.77	821.39	1.2908	926.93	860.21	97
98	1.6032	909.25	829.12	1.2460	929.47	864.75	98
99	1.5304	913.38	836.29	1.2060	931.73	868.81	99
100	1.4634	917.16	842.92	1.1706	933.74	872.43	100
101	1.4023	920.63	849.02	1.1395	935.50	875.61	101
102	1.3466	923.78	854.60	1.1124	937.03	878.39	102
103	1.2962	926.63	859.67	1.0892	938.35	880.78	103
104	1.2509	929.20	864.26	1.0695	939.46	882.81	104
105	1.2103	931.49	868.38	1.0531	940.39	884.50	105
106	1.1744	933.53	872.04	1.0397	941.15	885.89	106
107	1.1428	935.32	875.27	1.0289	941.76	887.00	107
108	1.1153	936.87	878.10	1.0205	942.24	887.87	108
109	1.0916	938.21	880.53	1.0141	942.60	888.54	109
110	1.0715	939.35	882.60	1.0093	942.87	889.03	110

Illustrative Service Table

x	$l_x^{(\tau)}$	$d_x^{(d)}$	$d_x^{(w)}$	$d_x^{(i)}$	$d_x^{(r)}$
30	100,000	100	19,990	0	0
31	79,910	80	14,376	0	0
32	65,454	72	9,858	0	0
33	55,524	61	5,702	0	0
34	49,761	60	3,971	0	0
35	45,730	64	2,693	46	0
36	42,927	64	1,927	43	0
37	40,893	65	1,431	45	0
38	39,352	71	1,181	47	0
39	38,053	72	989	49	0
40	36,943	78	813	52	0
41	36,000	83	720	54	0
42	35,143	91	633	56	0
43	34,363	96	550	58	0
44	33,659	104	505	61	0
45	32,989	112	462	66	0
46	32,349	123	421	71	0
47	31,734	133	413	79	0
48	31,109	143	373	87	0
49	30,506	156	336	95	0
50	29,919	168	299	102	0
51	29,350	182	293	112	0
52	28,763	198	259	121	0
53	28,185	209	251	132	0
54	27,593	226	218	143	0
55	27,006	240	213	157	0
56	26,396	259	182	169	0
57	25,786	276	178	183	0
58	25,149	297	148	199	0
59	24,505	316	120	213	0
60	23,856	313	0	0	3,552
61	19,991	298	0	0	1,587
62	18,106	284	0	0	2,692
63	15,130	271	0	0	1,350
64	13,509	257	0	0	2,006
65	11,246	204	0	0	4,448
66	6,594	147	0	0	1,302
67	5,145	119	0	0	1,522
68	3,504	83	0	0	1,381
69	2,040	49	0	0	1,004
70	987	17	0	0	970

Interest Functions

Interest Functions at $i = 0.06$

m	$i^{(m)}$	$d^{(m)}$	$i/i^{(m)}$	$d/d^{(m)}$	$\alpha(m)$	$\beta(m)$
1	0.06000	0.05660	1.00000	1.00000	1.00000	0.00000
2	0.05913	0.05743	1.01478	0.98564	1.00021	0.25739
4	0.05870	0.05785	1.02223	0.97852	1.00027	0.38424
12	0.05841	0.05813	1.02721	0.97378	1.00028	0.46812
∞	0.05827	0.05827	1.02971	0.97142	1.00028	0.50985

Special Note: Unless specified, the force of interest is constant in each question .

where $\alpha(m) = \dfrac{id}{i^{(m)}d^{(m)}}$ and

$$\beta(m) = \frac{i - i^{(m)}}{i^{(m)}d^{(m)}}$$

Errata for Actuarial Mathematics, 2nd Edition as of 05/30/1998

Page 68: In the line below (3.5.1), change "is equivalent to" to "implies".

Page 69: In the lines below (3.5.6) and (3.5.8), change "is equivalent to" to "implies".

Page 79: At the end of the first paragraph of Section 3.8, add "When the select status can be inferred from the force of mortality, the bracket notation will be suppressed to reduce the clutter of the symbols."

Page 89: In Exercise 3.38, in the displayed equation the subscript on the lefthand side should be "[x]".

Page 115: In equation (4.3.13), delete the "bar" on A.

Page 129: In Exercise 4.21 c., the final A symbol should be for a pure endowment.

Page 129: In first expression of Exercise 4.22 a., delete the first left parenthesis in the integrand and enclose it in brackets.

Page 142: In the fourth "Additional relation" at the bottom of Table 5.2.1, the exponent in the integrand should be "n-t".

Page 155: In the first two lines of display (5.5.3), the annuities-due should be paid mthly.

Page 159: In Exercise 5.9, Exercise 5.8 is a better reference than Exercise 5.7.

Page 162: In Exercise 5.29, the second superscript should be "(m)".

Page 166: In Exercise 5.54 delete "-due" and following "annum" insert "payable continuously and".

Page 166: In Exercise 5.59 the determination should be for a temporary complete annuity-immediate and the answer should be 10.41532.

Page 171: In the line following the solution to Example 6.2.1, "(6.2.6)" should be "(6.2.7)."

Page 173: In the fifth line of the Solution, "(5.2.15)" is better reference.

Page 177: In the display of the d.f. in the Solution for part a.: In the first line, the "less than or equal" should be "less than" and in the second line the "less than" should be "less than or equal". Apply the same changes to the d.f's in parts b. and c. on page 179.

Page 198: In Exercise 6.14, in the first line insert "either" following "under" and in the second line insert "or (5.4.10)" after "age".

Page 216: In the first line of Example 7.4.1, "6.4.1" should be "6.3.1".

Page 234: In the line following (8.3.1), ">" should be "> or =".

Page 235: In the fourth line below formula (8.3.9), the reference should be to "Exercise 8.7".

Page 243: To (8.5.9) add the restriction, AK(x) greater or equal to h≅. Delete the lines following (8.5.9).

Page 244: Delete parts b) & c) of Theorem 8.5.1 and delete Theorem 8.5.2. Change the line immediately before the corollary to read: ATheorem 8.5.1 a. Leads to the following corollary:≅

Page 245: Change the proofs of the b) & c) parts fo the Corollary to: Ab) and c) follow from an application of a)≅.

Page 253: In Exercise 8.18, Theorem 8.2 should be Theorem 8.5.2.

Page 261: In Figure 9.2.2, the vertical dotted line is located at "s" on the T(x) axis.

Page 268: In the first line of the third paragraph of Section 9.4, there should be a bar over xy in T(xy).

Page 269: The subscript on the second "F(t)" on the right-hand side of the first equation below (9.4.7) should be "T(y)".

Page 269: The first "of" in the first line of Example 9.4.1, should be "and".

Page 271: In the two line equation below (9.4.16), the subscripts on the 4th "q" in the first line and the last "q" in the second line should be "y+k".

Page 276: On the right-hand side of (9.6.4b) the subscript should be "T*(y)".

Page 277: In the solution for Example 9.6.2 b), T(y)'s survival function is valued at (t).

Page 278: In (9.6.10), the upper 2 is missing from the symbol for the second mixed partial derivative and in the numerator of the right-hand side the subscript on the second "f" should be "T(y)".

Page 292: In display (9.9.4), the "1" over the "x" in the subscript should be over the "y".

Page 298: In Exercise 9.1.c, the second T(x) should be T(y) and "coefficients" should be singular.

Page 299: In Exercise 9.8, "distributions" should be "distribution≅.

Page 299: In Exercise 9.11.b, add "for n>3".

Page 300: In Exercise 9.13, parts c. and d. should ask for "complete" expectations.
Page 301: Exercise 9.30.a. depends on material in Section 9.9.

Page 314: In line 7 from the bottom of the page, the "exp" is missing its closing bracket following the limits of the integration.

Page 349: In Example 11.4.1, assume that there are no withdrawals after the commencement of the annuity payments.

Page 358: In formula (11.6.1), the annuity subscript should start "[x+t]+m/12" instead of "[x+t]".

Page 376: In the final line of Table 12.3.1, the minus sign before the alpha parameter should be deleted.

Page 394: In the third line of Exercise 12.11, "lambda*beta" should be "lambda/beta".

Page 432: In the last line of Exercise 13.11, the "lambda/c" immediately before the integral should be deleted.

Page 434: In Exercise 13.24 item 5., "deficit" should be "negative surplus". The following should be added at the end of the exercise: [Hint: U(T-) and |U(T)| are dentically distributed in this exercise. To verify this fact, see Gerber and Shiu, North American Actuarial Journal, Vol. 2 (1998), No. 1., formula (3.7).]

Page 463: Each sentence in Exercises 4.16 and 4.17 request an answer.

Page 464: In Exercise 14.22 b., "12.16" should be "12.20".

Page 487: The "w" in the first sentence below (15.6.5) and the two "w's" in (15.6.6) should be "omegas".

Page 491: In the solution of Example 15.7.1, the derivative is with respect to "delta". Remember that the premium was set at time 0 and is not a function of the valuation interest rate.

Page 508: In (16.4.1), the first right (closing) parenthesis that follows "e-sub-(h-1)" should follow "c-sub-(h-1)".

Page 722: The answers for Exercise 3.36 should be: a) 0.000877 and b) 0.999189.

Page 722: The answers for Exercise 3.37 should be 0.4076 and 0.1786.

Page 723: The answers indicated for Exercises 4.11, 4.13, 4.14 and 4.15 should be relabeled for Exercises 4.14, 4.16, 4.17 and 4.19 respectively. The following answers should be inserted:

4.11 a) 0.2*z^(-0.8), 0<z<1 c) 1/6, 25/396

4.12 0.0 z<v^n
1.0-FsubT(logz/logv) v^n{<or=}z<1
1.0 1{<or=}z

4.13 a) 0.0 z<e^(-1)
z^0.2 e^(-1){<or=}z<1
1.0 1{<or=}z

Page 726: The answers for Exercise 6.1 should be: -0.43202 and 0.39760.

Page 728: The "infinity" in the second line should be "as".

Page 732: For both parts a. and c. of Exercise 9.11, the two "n-3" exponents should be "n-1".

Page 736: In the answers for parts a. and b. of Exercise 11.8, "480" should be "640", "240" should be "360", and "16,800" should be "22,400".

Page 740: The answer to Exercise 13.24 should be "10/3".

Page 741: For Exercises 4.16 and 4.17 place "alpha =" before the first expressions. Also in each exercise, the final inequality is the answer to the "Develop...." request.

ERRATA PACKAGE
for the First Edition of the textbook
Models for Quantifying Risk
(as of 7/15/06)

Type A Errata – Misprints or incorrect numbers:

Page	Change
12	In Equation (1.39), $(Da)_{\overline{n}\rvert}$ should be $(Ds)_{\overline{n}\rvert}$.
22	In Equation (2.6), $\frac{d^k}{dx^k}$ should be $\frac{d^k}{dt^k}$.
73	In Equation (3.34), and also in the preceding line, λ_i should be written as $\lambda_i(t)$, since it will vary with t in this case.
80	In the second line of Exercise 3-19, the words "are still exponentially distributed but not identically so" should be replaced by "are no longer identically exponentially distributed"
84	In Figure 4.1, $F(x)$ should be $f(x)$
97	In the second line after Equation (4.24), close the parenthesis after (4.24.
99	In the denominator of Equation (4.28b), $F_X(x)$ should be $F_X(d)$.
99	In the line preceding Equation (4.29), the reference to Equation (4.26a) should be (4.26b) and the reference to Equation (4.26b) should be (4.28b)
104	In the first line of Section 4.3.5, 100t% should be 100r%.
105	In the line before Equation (4.44b), the reference to Equation (4.41a) should be (4.44a).
154	In the first line after Equation (6.19), T_x should be $-T_x$.
186	In Equation (7.2), the upper limit should be m, where m is the total number of states.
211	In the third line of part (a) of Exercise 7-4, "in Year 2" should be "at the start of Year 3."
271	In four places (the third line following Equation (9.55), Equation (9.56), Equation (9.58), and Equation (9.60)), the symbol set $\lceil\ \rceil$ should instead be the set $\lfloor\ \rfloor$.
283	In part (b) of Exercise 9-24, delete the open parenthesis on (A_{35}.

311	In Equation (10.47), K_x under the angle should be k.
323	In the solution to Example 10 12, in the fourth line of the seven-line stack equation, 60 should be ln 60.
325	In the next-to-last line on the page, 60 should be .60.
357	In the last full line of Example 11 8, $A = .24905$ should be $A_x = .24905$
385	In Example 12 4, and again in the solution, .004 should be .00386.
469	Example 5.3 should be Example 14.3
516	In both Equation (15.31b) and Equation (15.31c), $J = j$ should be $J_x = j$
520	In the sixth line from the bottom of the page, $\overline{4\|}$ should be $\overline{5\|}$.
522	In the third line after the table, Section 12 1.5 should be Section 12.2.1.
524	In the table at the end of the solution to Example 15 15, $U_4 = 317.20$ should be $U_4 = 312\ 20$.
525	In part (b) of Exercise 15-4, $f_{J\|T}(j\|t)$ should be $f_{J\|T}(1\|t)$
527	In Exercise 15-13(b), (0) should be (1), (1) should be (2), and (2) should be (3).
543	In the first line of Example 16.5, the references to Examples 6.3 and 6.4 should be 16.3 and 16 4.
553	In the second line of Exercise 16-1, the value 30 should be .30.
554	In the fourth line of part (b) of Exercise 16-3, insert the words "of 100 members" between the word "group" and the word "for".
556	In the first line of Exercise 16-10, the word "On" should be "One".
579	In the second line, $M_L(r)$ should be $M_X(r)$ in two places.
579	In the last line of Example 17.6, $\psi(3\ 2)$ should be $\psi(3,2)$.
607	In the third line of part (c) of Exercise 18-6, the first i_2 should be i_1.
644	In the first line of the solution to Example B.6 6, [0, 30) should be [0, .30). In the centered expression, .60 should be .69.
645	In the first line following the centered expression in the solution to Example B.6.7, $x_1 290\ 57$ should be $x_1 = 290.57$.
648	The correct answer to Exercise 4-10(b) is 3129 75, not 5109.75.

648	The correct answer to Exercise 4-11(b) is 550 00, not 500 00.
648	The correct answer to Exercise 4-14 is .20000, not 200.00.
648	The correct answer to Exercise 4-15 is 8%, not 6%
649	In the answer to Exercise 5-1(b), the letter *t* should be *x*, in three places.
652	The correct answer to Exercise 6-27 is 1.4547, not 1.4745
654	The correct answer to Exercise 7-4(b) is 594,040, not 594,030
658	The correct answer to Exercise 10-26 is .26039, not .29911.
662	The correct answer to Exercise 13-15 is .18000, not .20226.
665	The correct answer to Exercise 15-7 is .19020, not .17520.
665	In the answers to Exercise 15-12, 085 should be .0875, .255 should be .2375, and .095 should be 090.
665	The correct answers to Exercise 15-13(a) are 29108 and .39788, not .283 and 414, respectively.
665	The correct answers to Exercise 15-13(b) are .32500, 24300, and .12096, not .340, 238, and 120, respectively.
666	The correct answers to Exercise 15-14 are 67857 and .30160, not .66694 and 30166, respectively.
666	The correct answers to Exercise 15-17(a) are 67, 1.11, and 18.45, not .61, 1.01, and 18.28, respectively.
666	The correct answers to Exercise 15-17(b) are 2 00 and 3 33, not 1.82 and 3.03, respectively.
666	The correct answer to Exercise 15-17(c) is .67, not .30
667	The correct answer to Exercise 16-1 is 2.35, not .3791
668	The correct answer to Exercise 16-7(d) is 1.839, not 1.829
668	The correct answer to Exercise 16-8 is .4375, not .5625
668	The correct answer to Exercise 16-10 is 10, not .02
669	The correct answer to Exercise 16-15 is 5 225, not .1914.
671	In the table of answers for Exercise 18-2, the value $-29\ 15$ should be $-29\ 25$.

Type B Errata – Clarifications of passages:

Page	Clarification
29	In the middle of the fourth line from the end of the page, change "an integer" to "a nonnegative integer".
30	In the line after Equation (2.34), append the phrase "where $\lambda > 0$".
32	The third line of Section 2.3.2 should read "... based on the two parameters μ and σ, where $\sigma > 0$, which are also ...".
51	In both the line after Equation (3.3a), and also the line after Equation (3.3b), append the phrase "for $x = 1,2,\ldots$"
59	Before ending the last sentence on the page, append the phrase "and are the only distributions contained in that class."
69	In the fifth line of Section 3.3, replace "*unit time*" with "*one unit of time*".
86	Equation (4.6) presumes that the transformation is increasing; if it is decreasing, then $F_Y(y) = 1 - F_X[g^{-1}(y)]$.
98	At the end of the paragraph three lines before Equation (4.27a), append the comment "In some texts, the expected payment per payment event is called the *mean excess loss*."
107	At the end of the sentence just before the word "This" in the fourth line of Section 4.4, append the comment "The implication of this is that the mean of X is then taken as the average of the data values."
112	In Exercise 4-11, "mean excess loss function" is understood to mean "expected payment per payment event".
113	In Exercise 4-17, "mean excess loss" is understood to mean "expected payment per payment event."
136	In the fourth line, following "and k," append the phrase "including the special case of $T_x = k$,".
166	In the second line of Example 6.8, before "Show that", append the phrase "where $r > 0$, $h > 0$, and $r + h < 1$".
170-171	Section 6.5.3 has been expanded to describe the hyperbolic assumption in greater detail. Table 6.3 in Section 6.5.4 has been similarly expanded. The reader should replace these two sections in the text with the new sections 6.5.3 – 6.5.4 attached to this errata package.
305	At the end of the sentence two lines after Equation (10.34), append the comment "Note, however, that $Y_{x:\overline{n}\rvert} = \ddot{Y}_{x:\overline{n+1}\rvert} - 1$, so $Var(Y_{x:\overline{n}\rvert}) = Var(\ddot{Y}_{x:\overline{n+1}\rvert})$. Therefore Equation (10.24b) is the same as Equation (10.34) with n replaced by $n+1$."
314	At the end of the fifth line, append the comment "The symbol ${}^2\bar{a}_x$ denotes the APV calculated at double the force of interest. Thus ${}^2\bar{a}_x$ is related to $\bar{a}_x$ in a similar manner as ${}^2\bar{A}_x$ is related to $\bar{A}_x$. Note, however, that ${}^2\bar{A}_x$ is also the second moment of $\bar{Z}_x$ but ${}^2\bar{a}_x$ is *not* the second moment of $\bar{Y}_x$.

344	At the end of the sentence in the line before Equation (11.12), append the comment "The present value of loss random variable for whole life insurance, L_x, is not to be confused with the L_x function defined by Equation (6.37b), although we use the same symbol in both cases."
351	Change the equation reference number (11.24) to (11.24a). Then in the next line append the sentence "Then using the result of Exercise 10-10 we also have $$Var[L(\bar{A}_x)] = ({}^2\bar{A}_x - \bar{A}_x^2)/(1-\bar{A}_x)^2."$$ Identify the new equation as (11.24b).
364	Exercise 11-3 should read "A 10-pay limited-payment whole life contract issued to (30) pays … ."
365	Exercise 11-5 should read "A 40-year-old home buyer … ."
379	In footnote 4, append the comment "Note also that ${}_tL_x$ is not to be confused with the function ${}_nL_x$ defined by Equation (6.37a)."
381	In the third line, immediately after the period, append the parenthetical sentence "(Note that ${}_2L$ is defined conditional on $K_x > 2$)". In the fourth line, change "$E[{}_2L]$" to "$E[{}_2L \mid K_x > 2]$"; in the sixth line, change "$E[{}_2L^2]$" to "$E[{}_2L^2 \mid K_x > 2]$"; in the eighth line, change "$Var({}_2L)$" to "$Var({}_2L \mid K_x > 2)$".
387-393	There are several errors in Sections 12.2.2-12.2.3 that cannot easily be corrected by simple changes. The reader should replace these two sections in the text with the new Sections 12.2.2-12.2.3 attached to this errata package.
397	Change formula reference number (12.59) to (12.59a). Then add a second line showing that the variance is also equal to $({}^2\bar{A}_{x+t} - \bar{A}_{x+t}^2)/(1-\bar{A}_x)^2$. Use formula reference number (12.59b) for the second expression.
415	In the second line of Exercise 12-10, change "$(x, x+1)$" to "$(x+j, x+j+1)$, for $j = 0,1,\ldots,t-1$"
428	At the end of the line before Section 13.2.1, append the comment "Note that the n-year certain and continuous annuity, defined in Example 10.14, is a special case of a last-survivor status, since the annuity pays until the second failure out of (40) and $\overline{10}\rceil$
530	In the eighth line of Exercise 15-22, delete the word "still"; in the second line of part (b) of this exercise, after the comma, insert the phrase "paid at the beginning of the year,"; amend part (c) of the exercise to read "Find the aggregate terminal benefit reserve fund (in thousands) at the end of each of the first ten years."
553	At the end of item (ii) of Exercise 16-2, append the words "as is the event of rain itself."
554	At the end of item (ii) of Exercise 16-4, append the words "per ship".
610	In the solution to Example A.1, delete the word "of".

Type C Errata – Replacement Sections:

6.5.3 HYPERBOLIC FORM FOR ℓ_{x+t}

Historically, textbooks on actuarial mathematics have included a third method for non-integral ages, namely the assumption that ℓ_{x+t} is a hyperbolic function between x and $x+1$. A hyperbolic function is a reciprocal linear function of the form $\ell_{x+t} = (a+bt)^{-1}$, so we have $\ell_x = \frac{1}{a}$ and $\ell_{x+1} = \frac{1}{a+b}$. From this we find

$$\frac{1}{\ell_{x+t}} = t \cdot \frac{1}{\ell_{x+1}} + (1-t) \cdot \frac{1}{\ell_x}, \qquad (6.61a)$$

showing that we can find values of ℓ_{x+t} by linear interpolation between the reciprocals of ℓ_x and ℓ_{x+1}. (Linear interpolation on the reciprocal of a function is called *harmonic interpolation* on the function itself.) From Equation (6.61a) we can find

$$({}_tp_x)^{-1} = \frac{\ell_x}{\ell_{x+t}} = t \cdot \frac{\ell_x}{\ell_{x+1}} + (1-t) \cdot \frac{\ell_x}{\ell_x}$$
$$= \frac{t}{p_x} + (1-t) = \frac{t+(1-t) \cdot p_x}{p_x},$$

so that

$${}_tp_x = \frac{p_x}{t+(1-t) \cdot p_x} = \frac{1-q_x}{1-(1-t) \cdot q_x} \qquad (6.61b)$$

and therefore

$${}_tq_x = 1 - {}_tp_x = \frac{t \cdot q_x}{1-(1-t) \cdot q_x}. \qquad (6.61c)$$

Using Equation (6.61b) we can then find

$$\mu_{x+t} = \frac{-\frac{d}{dt}\, {}_tp_x}{{}_tp_x} = \frac{(1-q_x) \cdot q_x}{[1-(1-t) \cdot q_x]^2} \div \frac{1-q_x}{1-(1-t) \cdot q_x} = \frac{q_x}{1-(1-t) \cdot q_x}, \qquad (6.61d)$$

for $0 < t < 1$, which is a decreasing function of t. It can also be shown that

$${}_{1-t}q_{x+t} = (1-t) \cdot q_x \qquad (6.61e)$$

In the past, the traditional actuarial approach to survival model estimation utilized a method that frequently involved functions of the form ${}_{1-t}q_{x+t}$, which could be simplified to $(1-t) \cdot q_x$ under the hyperbolic assumption. This approach to estimation work is no longer commonly used, so this major use of the hyperbolic assumption no longer exists. Today we might view it as being primarily of historical interest.

The Italian actuary Gaetano Balducci made major use of the hyperbolic distribution in several of his writings, such as [1] and [2]. Although he did not originate the use of this assumption, it has come to be called the *Balducci assumption*, or Balducci distribution.

EXAMPLE 6.9

In a certain life table, $\ell_x = 1000$ and $\ell_{x+1} = 900$. Evaluate m_x under each of the UDD, constant force, and hyperbolic assumptions

SOLUTION

From the given values we find $d_x = 100$ and $\mu = -\ln p_x =$ 10536. Under UDD, $L_x = \ell_x - \frac{1}{2}d_x = 950$, so we obtain $m_x = \frac{d_x}{L_x} =$ 10526 Under constant force, we directly have $m_x = \mu =$ 10536. Under hyperbolic, we first obtain

$$L_x = \int_0^1 \ell_{x+t}\, dt = \int_0^1 \left[\frac{1}{\ell_x} + t\left(\frac{1}{\ell_{x+1}} - \frac{1}{\ell_x}\right)\right]^{-1} dt$$

$$= \left. \frac{\ln\left[\frac{1}{\ell_x} + t\left(\frac{1}{\ell_{x+1}} - \frac{1}{\ell_x}\right)\right]}{\frac{1}{\ell_{x+1}} - \frac{1}{\ell_x}} \right|_0^1$$

$$= \frac{\ln\left(\frac{1}{\ell_{x+1}}\right) - \ln\left(\frac{1}{\ell_x}\right)}{\frac{1}{\ell_{x+1}} - \frac{1}{\ell_x}} = 948.2456.$$

Then $m_x = \frac{100}{948.2456} =$ 10546 □

6.5.4 SUMMARY

Table 6 3 summarizes most of the results developed in this section. Further analysis of these assumptions is given by Batten [3], and Mereu [23] gives a presentation of the use of these assumptions in actuarial calculations.

TABLE 6.3

Function	Linear (UDD)	Exponential (Constant Force)	Hyperbolic (Balducci)
ℓ_{x+t}	$t \cdot \ell_{x+1} + (1-t) \cdot \ell_x$	$(\ell_{x+1})^t \ (\ell_x)^{1-t}$	$\left(\frac{t}{\ell_{x+1}} + \frac{1-t}{\ell_x}\right)^{-1}$
${}_t p_x$	$1 - t \ q_x$	$(p_x)^t = e^{-\mu t}$	$\frac{1-q_x}{1-(1-t) \cdot q_x}$
${}_t q_x$	$t \cdot q_x$	$1-(1-q_x)^t$	$\frac{t \cdot q_x}{1-(1-t) \cdot q_x}$
μ_{x+t}	$\frac{q_x}{1-t \cdot q_x}$	$\mu = -\ln p_x$	$\frac{q_x}{1-(1-t) \cdot q_x}$
${}_t p_x \mu_{x+t}$	q_x	$\mu \cdot e^{-ut}$	$\frac{q_x(1-q_x)}{[1-(1-t) \ q_x]^2}$
L_x	$\ell_x - \frac{1}{2} d_x$	$\frac{d_x}{\mu}$	$-\ell_{x+1}\left(\frac{\ln p_x}{q_x}\right)$
m_x	$\frac{q_x}{1-\frac{1}{2} q_x}$	μ	$\frac{(q_x)^2}{-p_x \ln p_x}$

12.2.2 RANDOM VARIABLE ANALYSIS – CASH BASIS

Equation (12.30c) can also be derived by use of random variables. Given that $K_x > t$ (i.e., the contingent contract has not yet failed as of time t), then in each year starting with the $(t+1)^{st}$ year there is a cash income of P at the beginning of the year and a cash outgo of either 1 or 0 at the end of the year, depending on whether or not failure occurs in that year. The cash loss in each year, considering only income and outgo in that year, is therefore $1-P$ if failure occurs or $-P$ if failure does not occur. (If there is no failure in that year, then the cash "loss" for that year is actually a gain of P) But the outgo in the year of failure is at year-end and the income is at year-beginning, so we must consider $v-P$ as the present value (at the beginning of the year) of the cash loss in the year of failure. (There is also a third case to consider: if failure has occurred *prior to* that year, then there is neither income nor outgo and the present value of cash loss for that year is zero.) Let C_j denote the random variable for the present value of cash loss in year j, for $j = t+1, t+2, \ldots,$ where each C_j random variable is conditional only on survival to age $x+t$. This is illustrated in the following diagram.

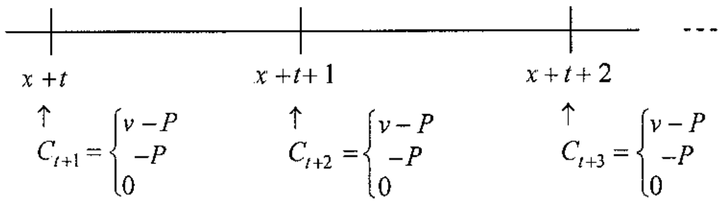

FIGURE 12.6

Note that the index j is measured from age x, so year j is the year from age $x+j-1$ to age $x+j$. But the discussion here is conditional on $K_x > t$, so year j can also be identified as the year from age $x+t+j-t-1$ to age $x+t+j-t$, for $j = t+1, t+2, \ldots$. (This orientation will be helpful for correctly identifying the probability values associated with the different possible values of each C_j random variable.)

Then the definition of the random variable C_j is

$$C_j = \begin{cases} v-P & \text{for} \quad K_x = j \\ -P & \text{for} \quad K_x > j \\ 0 & \text{for} \quad K_x < j \end{cases} \tag{12.31}$$

The probability of the event $K_x = j$ is ${}_{j-t-1}p_{x+t} \; q_{x+j-1} = {}_{j-t-1|}q_{x+t}$, the probability of the event $K_x > j$ is ${}_{j-t-1}p_{x+t} \; p_{x+j-1} = {}_{j-t}p_{x+t}$, and the probability of the event $K_x < j$ is ${}_{j-t-1}q_{x+t}$. (Note that, for $j = t+1$, ${}_{j-t-1}q_{x+t} = {}_{0}q_{x+t} = 0$.) Note also that these three probability values sum to one, as required. Then the conditional expected value of C_j, given $K_x > t$, is

$$
\begin{aligned}
E[C_j \mid K_x > t] &= (v-P) \; {}_{j-t-1}p_{x+t} \; q_{x+j-1} \\
&\quad + (-P) \; {}_{j-t-1}p_{x+t} \; p_{x+j-1} \\
&\quad + (0) \; {}_{j-t-1}q_{x+t} \\
&= v \; {}_{j-t-1|}q_{x+t} - P \; {}_{j-t-1}p_{x+t},
\end{aligned}
\qquad (12\,32a)
$$

since $q_{x+j-1} + p_{x+j-1} = 1$. In particular, when $j = t+1$ we have

$$
E[C_{t+1} \mid K_x > t] = v \cdot q_{x+t} - P, \qquad (12.32b)
$$

since ${}_0p_{x+t} = 1$

Recall that ${}_tL$ is the present value of loss at time t, given $K_x > t$, considering all future years, whereas C_j is the present value of loss for year j only By discounting the loss in year j back to time t, for all future years j, we have the present value of all future loss. That is,

$$
{}_tL = C_{t+1} + v \; C_{t+2} + v^2 \cdot C_{t+3} + \cdots \qquad (12\,33a)
$$

We can write Equation (12 33a) as

$$
\begin{aligned}
{}_tL &= C_{t+1} + v\left(C_{t+2} + v \; C_{t+3} + \cdots\right) \\
&= C_{t+1} + v \; {}_{t+1}L.
\end{aligned}
\qquad (12.33b)
$$

It is important to note that ${}_tL$ and C_{t+1} are conditional on $K_x > t$, whereas ${}_{t+1}L$ is conditional on $K_x > t+1$. Now we take the conditional expectation of Equation (12.33b), given $K_x > t$, obtaining

$$
E[{}_tL \mid K_x > t] = E[C_{t+1} \mid K_x > t] + v \cdot E[{}_{t+1}L \mid K_x > t]. \qquad (12.34)
$$

We note in Equation (12 34) that $E[{}_tL \mid K_x > t] = {}_tV$, as established in Section 12.1.4, and $E[C_{t+1} \mid K_x > t] = v \; q_{x+t} - P$, as established by Equation (12.32b). However it is *not* true that $E[{}_{t+1}L \mid K_x > t] = {}_{t+1}V$, but rather that $E[{}_{t+1}L \mid K_x > t+1] = {}_{t+1}V$. How can we evaluate $E[{}_{t+1}L \mid K_x > t]$?

Consider the *unconditional* expectation of ${}_{t+1}L$. We can write

$$E[{}_{t+1}L] = E[{}_{t+1}L \mid K_x > t] \cdot Pr(K_x > t) \qquad (12.35a)$$

and also

$$\begin{aligned} E[{}_{t+1}L] &= E[{}_{t+1}L \mid K_x > t+1] \; Pr(K_x > t+1) \\ &= {}_{t+1}V \; Pr(K_x > t+1). \end{aligned} \qquad (12.35b)$$

By equating the two expressions for $E[{}_{t+1}L]$ we find

$$\begin{aligned} E[{}_{t+1}L \mid K_x > t] &= E[{}_{t+1}L \mid K_x > t+1] \; \frac{Pr(K_x > t+1)}{Pr(K_x > t)} \\ &= {}_{t+1}V \cdot \frac{{}_{t+1}p_x}{{}_t p_x} \\ &= {}_{t+1}V \; p_{x+t}. \end{aligned} \qquad (12.36)$$

Then substituting for the three expectations in Equation (12.34) we have

$${}_tV = v \; q_{x+t} - P + v \cdot {}_{t+1}V \; p_{x+t}$$

or

$${}_tV + P = v \cdot q_{x+t} + v \; p_{x+t} \; {}_{t+1}V,$$

as already established by Equation (12.30c).

12.2.3 Random Variable Analysis – Accrued Basis

The expression for ${}_tL$ given by Equation (12.33b) expresses the random variable for the present value of future loss at duration t as the discounted sum of the random variables for cash loss in each contract year after duration t. An expression for the variance of ${}_tL$ would involve a covariance term for each C_i, C_j pair in the sum Therefore Equation (12.33b) is not a convenient approach to considering the variance of ${}_tL$ allocated to future contract years.

In this section we return to the notion of loss to the insurer in year j, for $j = t+1, t+2, \ldots$, but this time as the *accrued loss*, rather than the *cash loss*, as was the case in Section 12.2.2. Analogous to the definition of C_j, given by Equation (12.31), we now define the random variable for the present value (at the beginning of the year) of the accrued loss in year j by

$$\Lambda_j = \begin{cases} 0 & \text{for} \quad K_x < j \\ v - {}_{j-1}V - P & \text{for} \quad K_x = j \\ v \cdot {}_jV - {}_{j-1}V - P & \text{for} \quad K_x > j \end{cases} \tag{12.37}$$

As in Section 12.2.2, each Λ_j random variable is conditional on survival to age $x+t$, which is denoted by $K_x > t$

The conditional expected value of Λ_j, given survival to age $x+t$, is

$$E[\Lambda_j \mid K_x > t] = 0 \tag{12.38}$$

(see Exercise 12-18), and the conditional variance is given by

$$Var(\Lambda_j \mid K_x > t) = {}_{j-t-1}p_{x+t} \cdot v^2(1-{}_jV)^2 \cdot p_{x+j-1} \cdot q_{x+j-1} \tag{12.39}$$

(see Exercise 12-19). (Note that since the conditional expected value is zero, then the conditional variance is the same as the conditional second moment.)

Next we want to relate the random variable Λ_j to the random variable C_j defined in Section 12.2.2 First we modify the definition of Λ_j given by Equation (12.37) to read

$$\Lambda_j = \begin{cases} 0 & \text{for} \quad K_x < j \\ v - {}_{j-1}V - P = [v-P] + (0 - {}_{j-1}V) & \text{for} \quad K_x = j, \\ v \cdot {}_jV - {}_{j-1}V - P = [-P] + (v \cdot {}_jV - {}_{j-1}V) & \text{for} \quad K_x > j \end{cases} \tag{12.40}$$

and we note that the terms in brackets represent the definition of the random variable C_j. In each line of Equation (12.40) the term in parentheses represents the present value (at the beginning of year j) of the reserve increase over that year. If survival occurs, the reserve increases from ${}_{j-1}V$ to ${}_jV$, so the present value of the increase is $v \cdot {}_jV - {}_{j-1}V$. If failure occurs, the reserve goes from ${}_{j-1}V$ to zero, so the present value of the increase is $v \cdot 0 - {}_{j-1}V = -{}_{j-1}V$. (If $K_x < j$, the reserve increase in year j is zero.)

Let ΔV_j denote the present value of reserve increase in this j^{th} year. Then all three lines of Equation (12.40) satisfy the relationship

$$\Lambda_j = C_j + \Delta V_j \tag{12.41}$$

so

$$C_j = \Lambda_j - \Delta V_j, \tag{12.42}$$

for $j = t+1, t+2, \ldots$.

Now we use Equation (12.42) to substitute for each C_j term in Equation (12.33a), obtaining

$$\begin{aligned} {}_tL &= C_{t+1} + v \cdot C_{t+2} + v^2 \cdot C_{t+3} + \cdots \\ &= (\Lambda_{t+1} - \Delta V_{t+1}) + v(\Lambda_{t+2} - \Delta V_{t+2}) + v^2(\Lambda_{t+3} - \Delta V_{t+3}) + \cdots \\ &= \Lambda_{t+1} + v \cdot \Lambda_{t+2} + v^2 \cdot \Lambda_{t+3} + \cdots \\ &\qquad - (\Delta V_{t+1} + v \cdot \Delta V_{t+2} + v^2 \cdot \Delta V_{t+3} + \cdots) \end{aligned} \tag{12.43}$$

The term in parentheses in Equation (12.43) represents the present value at age $x+t$ of all future reserve increases, so it is the excess of the ultimate reserve over the current reserve of ${}_tV$. But the ultimate reserve is zero, since the contingent contract must ultimately be fulfilled, so the sum of the terms in parentheses must be $0 - {}_tV$. Thus we conclude that

$${}_tL = \Lambda_{t+1} + v \cdot \Lambda_{t+2} + v^2 \cdot \Lambda_{t+3} + \cdots + {}_tV \tag{12.44}$$

Note that since the conditional expected value of each Λ term is zero, and the conditional expected value of ${}_tL$ is ${}_tV$, then taking the expectation on both sides of Equation (12.44) produces the expected result ${}_tV = {}_tV$.

Of greater interest to us is the variance of ${}_tL$. It can be shown (see Example 12.6) that the several Λ_j are conditionally uncorrelated, so that $Cov(\Lambda_j, \Lambda_k \mid K_x > t) = 0$. Along with the fact that $Var({}_tV) = 0$, taking the conditional variance on both sides of Equation (12.44) then gives us

$$\begin{aligned} Var({}_tL \mid K_x > t) &= Var(\Lambda_{t+1} \mid K_x > t) \\ &\quad + v^2 \; Var(\Lambda_{t+2} \mid K_x > t) \\ &\quad + v^4 \; Var(\Lambda_{t+3} \mid K_x > t) + \cdots \end{aligned} \tag{12.45}$$

Then substituting Equation (12.39) for each conditional variance term in Equation (12.45) we obtain

$$\begin{aligned} Var({}_tL \mid K_x > t) &= v^2 (1 - {}_{t+1}V)^2 \cdot p_{x+t} \cdot q_{x+t} \\ &\quad + v^2 \; p_{x+t} \left[v^2 (1 - {}_{t+2}V)^2 \right] p_{x+t+1} \; q_{x+t+1} \\ &\quad + v^4 \; {}_2p_{x+t} \left[v^2 (1 - {}_{t+3}V)^2 \right] p_{x+t+2} \cdot q_{x+t+2} + \cdots \end{aligned} \tag{12.46}$$

Finally, we want to develop a recursion formula for $Var({}_tL \mid K_x > t)$. Starting with Equation (12.44) we have

$$\begin{aligned} {}_tL &= \Lambda_{t+1} + v \; \Lambda_{t+2} + v^2 \; \Lambda_{t+3} + \cdots + {}_tV \\ &= \Lambda_{t+1} + v(\Lambda_{t+2} + v \; \Lambda_{t+3} + \cdots + {}_{t+1}V) + {}_tV - v \; {}_{t+1}V \\ &= \Lambda_{t+1} + v \; {}_{t+1}L + {}_tV - v \cdot {}_{t+1}V. \end{aligned} \tag{12.47}$$

When we take the conditional variance on the right side of Equation (12.47), we see that the variance of both ${}_tV$ and ${}_{t+1}V$ is zero and, since ${}_{t+1}L$ is a linear combination of uncorrelated Λ_j terms, then Λ_{t+1} and ${}_{t+1}L$ are uncorrelated as well. Then we have

$$Var({}_tL \mid K_x > t) = Var(\Lambda_{t+1} \mid K_x > t) + v^2 \; Var({}_{t+1}L \mid K_x > t)$$

$$= v^2(1 - {}_{t+1}V)^2 \; p_{x+t} \; q_{x+t}$$

$$+ v^2 \; p_{x+t} \cdot Var({}_{t+1}L \mid K_x > t+1). \qquad (12.48)$$

The results we have developed here for $Var({}_tL \mid K_x > t)$ are part of the Hattendorf Theorem. An alternative, but equivalent, derivation of these results is given by Bowers, et al. [4] in a more general setting. Note that our Equation (12.45) corresponds to their Equation (8.5.14), our Equation (12.46) corresponds to their Equation (8.5.15), and our Equation (12.48) corresponds to their Equation (8.5.16), with $i = 1$.

EXAMPLE 12.6

Show that $Cov(\Lambda_j, \Lambda_k \mid K_x > t) = 0$, where $k > j$.

SOLUTION

We have

$$Cov(\Lambda_j, \Lambda_k \mid K_x > t) = E[\Lambda_j \cdot \Lambda_k \mid K_x > t]$$

$$- E[\Lambda_j \mid K_x > t] \cdot E[\Lambda_k \mid K_x > t].$$

By Equation (12.38), both expected value terms to the right of the minus sign are zero. For the conditional expected value of the product $\Lambda_j \cdot \Lambda_k$, if failure occurs prior to the k^{th} year, then $\Lambda_k = 0$ so $\Lambda_j \cdot \Lambda_k = 0$ and the expected value is zero. But if failure has not occurred prior to the k^{th} year, then Λ_j is the constant $c = v \cdot {}_jV - {}_{j-1}V - P$, since survival through year j is given. This makes $\Lambda_j \cdot \Lambda_k = c \cdot \Lambda_k$, with expected value $c \cdot E[\Lambda_k \mid K_x > t] = 0$. Therefore the expected value of the product $\Lambda_j \cdot \Lambda_k$ is zero in any case, so the covariance is zero as well. □