Exam MLC Spring 2007

FINAL ANSWER KEY

Question #	*Answer*		*Question #*	*Answer*
1	**E**		**16**	**B**
2	**B**		**17**	**D**
3	**D**		**18**	**C**
4	**E**		**19**	**D**
5	**C**		**20**	**C**
6	**A**		**21**	**B**
7	**E**		**22**	**C**
8	**E**		**23**	**B**
9	**E**		**24**	**A**
10	**C**		**25**	**B**
11	**A**		**26**	**A**
12	**D**		**27**	**A**
13	**C**		**28**	**C**
14	*****		**29**	**A**
15	**D**		**30**	**D**

* The exam problem was defective. Mechanically, all candidates received credit. The problem now shows the exam version and the intended meaning. The answer to the intended wording is E.

****BEGINNING OF EXAMINATION****

1. You are given:

(i) ${}_{3}p_{70} = 0.95$

(ii) ${}_{2}p_{71} = 0.96$

(iii) $\int_{71}^{75} \mu_x dx = 0.107$

Calculate ${}_{5}p_{70}$.

(A) 0.85

(B) 0.86

(C) 0.87

(D) 0.88

(E) 0.89

2. You are given:

(i) $\mu_x(t) = c, \qquad t \geq 0$

(ii) $\delta = 0.08$

(iii) $\bar{A}_x = 0.3443$

(iv) $T(x)$ is the future lifetime random variable for (x).

Calculate $\mathrm{Var}\left(\bar{a}_{\overline{T(x)}|}\right)$.

(A) 12

(B) 14

(C) 16

(D) 18

(E) 20

3. For a fully discrete whole life insurance of 1000 on the select life $[60]$:

(i) Ultimate mortality follows the Illustrative Life Table.

(ii) The select period is 3 years.

(iii) $i = 0.06$

(iv) $1000\ A_{[60]} = 359.00$

Calculate $1000\ {}_5V_{[60]}$, the benefit reserve at the end of year 5 for this insurance.

(A) 112

(B) 116

(C) 121

(D) 126

(E) 130

4. For a fully discrete whole life insurance of 150,000 on (x), you are given:

(i) $^{2}A_x = 0.0143$

(ii) $A_x = 0.0653$

(iii) The annual premium is determined using the equivalence principle.

(iv) L is the loss-at-issue random variable.

Calculate the standard deviation of L.

(A) 14,000

(B) 14,500

(C) 15,100

(D) 15,600

(E) 16,100

5. Heart/Lung transplant claims in 2007 have interarrival times that are independent with a common distribution which is exponential with mean one month. As of the end of January, 2007 no transplant claims have arrived.

Calculate the probability that at least three Heart/Lung transplant claims will have arrived by the end of March, 2007.

(A) 0.18

(B) 0.25

(C) 0.32

(D) 0.39

(E) 0.45

6. People arrive at a food bank at a Poisson rate of 10 per day. 80% of them donate non-perishable units of food and 20% withdraw units of food. Individual food donations are distributed with mean 15 and variance 75 and individual food withdrawals are distributed with mean 40 and variance 533. The number arriving and the amounts of donations and withdrawals are independent.

Using the normal approximation, calculate the probability that the amount of food units at the end of seven days will be at least 600 more than at the beginning of the week.

(A) 0.07

(B) 0.09

(C) 0.11

(D) 0.13

(E) 0.15

7. For five special fully continuous whole life insurances on (35):

(i) The death benefit on each insurance is 1000.

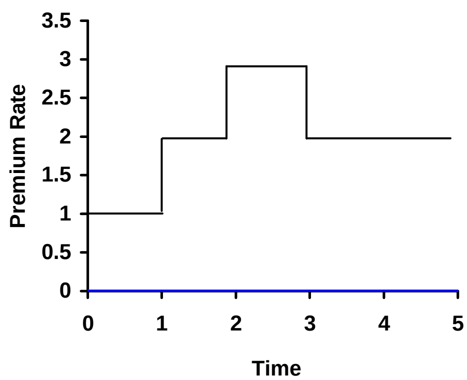
(ii) Benefit premium rates for the first five years are shown in the graphs below.

(iii) Benefit premium rates after five years for each insurance are level, not necessarily at the same rate as during year 5.

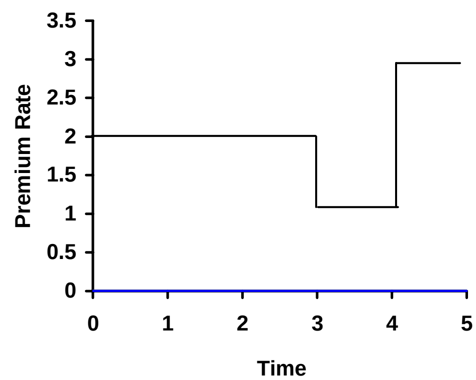
(iv) $\delta = 0.05$

Which of these benefit premium patterns results in the largest benefit reserve at the end of year 3?

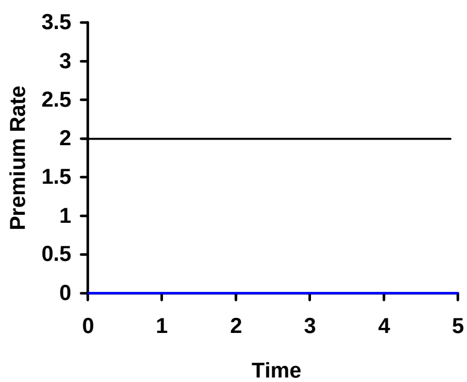
(A)



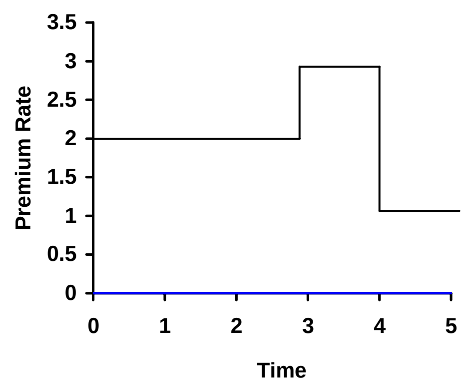
(B)



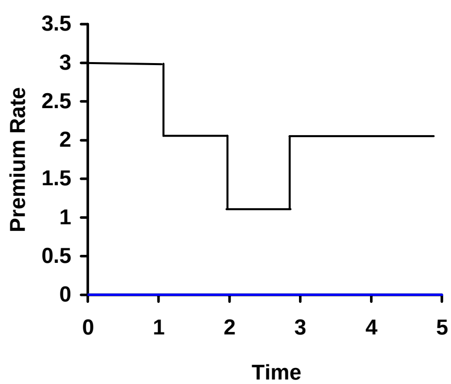
(C)



(D)



(E)



8. Kevin and Kira excel at the newest video game at the local arcade, “Reversion”. The arcade has only one station for it. Kevin is playing. Kira is next in line. You are given:

(i) Kevin will play until his parents call him to come home.

(ii) Kira will leave when her parents call her. She will start playing as soon as Kevin leaves if he is called first.

(iii) Each child is subject to a constant force of being called: 0.7 per hour for Kevin; 0.6 per hour for Kira.

(iv) Calls are independent.

(v) If Kira gets to play, she will score points at a rate of 100,000 per hour.

Calculate the expected number of points Kira will score before she leaves.

(A) 77,000

(B) 80,000

(C) 84,000

(D) 87,000

(E) 90,000

9. For a double decrement table, you are given:

(i) In the single decrement table associated with cause (1), decrements are uniformly distributed over the year.

(ii) In the single decrement table associated with cause (2), decrements occur at only two points during the year. Three-quarters of the decrements occur at time 1/5 and the remaining one-quarter occur at time 3/5.

(iii) $q'^{(1)}_{25} = 0.10$ and $q'^{(2)}_{25} = 0.12$

Calculate $q^{(2)}_{25}$.

(A) 0.108

(B) 0.110

(C) 0.112

(D) 0.114

(E) 0.116

10. For whole life insurances of 1000 on (65) and (66):

(i) Death benefits are payable at the end of the year of death.

(ii) The interest rate is 0.10 for 2008 and 0.06 for 2009 and thereafter.

(iii) $q_{65} = 0.010$ and $q_{66} = 0.012$

(iv) The actuarial present value on December 31st 2007 of the insurance on (66) is 300.

Calculate the actuarial present value on December 31st 2007 of the insurance on (65).

(A) 279

(B) 284

(C) 289

(D) 293

(E) 298

11. For a 10-payment, fully discrete, 20-year term insurance of 1000 on (40), you are given:

(i) $i = 0.06$

(ii) Mortality follows the Illustrative Life Table.

(iii) The following expenses, which are incurred at the beginning of each policy year:

	Year 1		Years 2+	
	% of premium	Constant	% of premium	Constant
Taxes	4%	0	4%	0
Sales Commission	25%	0	5%	0
Policy maintenance	0%	10	0%	5

Calculate the expense-loaded premium using the equivalence principle.

(A) 18.21

(B) 18.35

(C) 18.53

(D) 18.71

(E) 18.95

12. For a special fully discrete 3-year term insurance on (55), whose mortality follows a double decrement model:

(i) Decrement 1 is accidental death; decrement 2 is all other causes of death.

(ii)

x	$q_x^{(1)}$	$q_x^{(2)}$
55	0.002	0.020
56	0.005	0.040
57	0.008	0.060

(iii) $i = 0.06$

(iv) The death benefit is 2000 for accidental deaths and 1000 for deaths from all other causes.

(v) The level annual contract premium is 50.

(vi) ${}_1L$ is the prospective loss random variable at time 1, based on the contract premium.

(vii) $K(55)$ is the curtate future lifetime of (55).

Calculate the smallest value of z such that $\Pr\left[{}_1L \le z \middle| K(55) \ge 1\right] \ge 0.95$.

(A) 743

(B) 793

(C) 843

(D) 893

(E) 943

13. For a fully discrete 3-year term insurance of 1000 on (x), you are given:

(i) $i = 0.10$

(ii) The mortality rates and terminal reserves are given by:

h	q_{x+h}	$1000_{h+1}V$
0	0.3	95.833
1	0.4	120.833
2	0.5	0

(iii) ${}_1L$ is the prospective loss random variable at time 1, based on the benefit premium.

(iv) $K(x)$ is the full number of years lived by (x) prior to death; i.e. the curtate future lifetime random variable for (x).

Calculate $\text{Var}\left({}_1L \middle| K(x) \geq 1\right)$.

(A) 238,000

(B) 247,000

(C) 256,000

(D) 265,000

(E) 274,000

14. The probability that (30) will survive five years is a. The probability that (30) will die before (35) is b. The future lifetimes of (30) and (35) are independent and identically distributed.

Determine the probability that (30) will die second and within five years of the death of (35).

(A) $1 - a - \frac{1}{2}b$

(B) $1 - \frac{1}{2}a - \frac{1}{2}b$

(C) $\frac{1}{2} - a - \frac{1}{2}b$

(D) $\frac{1}{2} - \frac{1}{2}a - b$

(E) $1 - \frac{1}{2}a - b$

Errata: The above is the problem as it appeared on the exam. The last sentence of the first paragraph should have said "The future lifetimes of (30) and (35) are independent and follow the same mortality table."

15. Residents in a Continuing Care Retirement Community (CCRC) can be in one of three statuses: Independent Living (#1), Health Center (#2) and Dead (#3).

(i) $$Q_0 = \begin{bmatrix} 0.6 & 0.3 & 0.1 \\ 0.2 & 0.5 & 0.3 \\ 0 & 0 & 1 \end{bmatrix}, \quad Q_1 = \begin{bmatrix} 0.4 & 0.4 & 0.2 \\ 0 & 0.4 & 0.6 \\ 0 & 0 & 1 \end{bmatrix},$$

$$Q_2 = \begin{bmatrix} 0.2 & 0.1 & 0.7 \\ 0 & 0.1 & 0.9 \\ 0 & 0 & 1 \end{bmatrix}, \quad Q_3 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

(ii) Transitions occur at the end of each year.

(iii) The CCRC incurs a cost of 1000 at the end of year k for transition from Independent Living at the start of that year to Health Center at the start of the next year, for all k.

(iv) The CCRC wishes to charge a fee P at the start of each of the first 3 years for each resident then in Independent Living.

(v) Nathan enters Independent Living at time 0.

(vi) $i = 0.25$

Calculate P for Nathan using the equivalence principle.

(A) 239

(B) 242

(C) 245

(D) 248

(E) 251

16. The number of coins Lucky Tom finds in successive blocks as he walks to work follows a homogeneous Markov model:

(i) States 0, 1, 2 correspond to 0, 1, or 2 coins found in a block.

(ii) The transition matrix is:

$$Q = \begin{pmatrix} 0.2 & 0.5 & 0.3 \\ 0.1 & 0.6 & 0.3 \\ 0.1 & 0.5 & 0.4 \end{pmatrix}$$

(iii) Tom found one coin in the first block today.

Calculate the probability that Tom will find at least 3 more coins in the next two blocks today.

(A) 0.43

(B) 0.45

(C) 0.47

(D) 0.49

(E) 0.51

17. Your son has been driving for one year without an accident. To encourage continued safe driving you offer him a choice now of one of the following:

(i) For the next three years, 100 at the end of each year in which he has no accidents.

(ii) R at the end of Year 3 if he has no accidents for the next three years.

You assume that the probability of an accident-free year is 0.8 if the previous year was accident-free and 0.7 if there was an accident in the previous year.

Using 4% annual interest, calculate R so that the two choices are actuarially equivalent.

(A) 315

(B) 378

(C) 426

(D) 479

(E) 513

18. You are given the following extract from a 2-year select-and-ultimate mortality table:

$[x]$	$l_{[x]}$	$l_{[x]+1}$	l_{x+2}	$x+2$
65	-	-	8200	67
66	-	-	8000	68
67	-	-	7700	69

The following relationships hold for all x:

(i) $3\, q_{[x]+1} = 4\, q_{[x+1]}$

(ii) $4\, q_{x+2} = 5\, q_{[x+1]+1}$

Calculate $l_{[67]}$.

(A) 7940

(B) 8000

(C) 8060

(D) 8130

(E) 8200

19. For a whole life insurance of 1 on (40), you are given:

(i) $i = 0.06$

(ii) $p_{50} = p_{51} = p_{52}$

(iii) ${}_{10}V_{40} = {}_{13}V_{40}$

(iv) $\ddot{a}_{50} = 10.0$

Calculate p_{50}.

(A) 0.942

(B) 0.946

(C) 0.950

(D) 0.954

(E) 0.958

20. For professional athletes Derek and A-Rod:

(i) Professional athletes are subject to a constant total force of mortality $\mu^{(\tau)} = 0.001$.

(ii) Professional athletes are subject to a constant force of mortality due to crashes of the team airplane $\mu^{(1)} = 0.0002$.

(iii) Mortality of athletes on the same team follows a common shock model, where all team members die if the team plane crashes.

(iv) Future lifetimes of athletes on different teams are independent.

(v) Derek and A-Rod are on the same team now, but after one year will play for different teams.

Calculate the probability that both Derek and A-Rod survive two years.

(A) 0.9958

(B) 0.9960

(C) 0.9962

(D) 0.9964

(E) 0.9966

21. You are given the following information about a new model for buildings with limiting age ω.

(i) The expected number of buildings surviving at age x will be $l_x = (\omega - x)^{\alpha}$, $x < \omega$.

(ii) The new model predicts a $33\frac{1}{3}\%$ higher complete life expectancy (over the previous DeMoivre model with the same ω) for buildings aged 30.

(iii) The complete life expectancy for buildings aged 60 under the new model is 20 years.

Calculate the complete life expectancy under the previous DeMoivre model for buildings aged 70.

(A) 8

(B) 10

(C) 12

(D) 14

(E) 16

22. For a special whole life insurance on (40), you are given:

(i) The death benefit is 1000 for the first 10 years and 2500 thereafter.

(ii) Death benefits are payable at the moment of death.

(iii) Z is the present-value random variable.

(iv) Mortality follows DeMoivre's law with $\omega = 100$.

(v) $\delta = 0.10$

Calculate $\Pr(Z > 700)$.

(A) 0.059

(B) 0.079

(C) 0.105

(D) 0.169

(E) 0.212

23. Assuming constant forces of decrement in each year of age, you are given:

(i) $q_x'^{(1)} = 0.10$

(ii) $q_{x+1}'^{(2)} = 0.25$

(iii) $\mu_x^{(2)} = 0.20$

(iv) $\mu_{x+1}^{(1)} = 0.15$

Calculate ${}_{1|1}q_x^{(2)}$ for a double decrement table.

(A) 0.169

(B) 0.172

(C) 0.175

(D) 0.178

(E) 0.181

24. For a three-year temporary life annuity due of 100 on (75), you are given:

(i) $\int_0^x \mu(t)\,dt = 0.01x^{1.2}, \quad x > 0$

(ii) $i = 0.11$

Calculate the actuarial present value of this annuity.

(A) 264

(B) 266

(C) 268

(D) 270

(E) 272

25. Subway trains arrive at a certain station according to a nonhomogenous Poisson process.

$\lambda(t)$, the intensity function (trains per minute), varies with *t*, the time in minutes after 7:00 AM:

(i) $\lambda(t) = 0.05,\qquad 0 \le t < 10$

(ii) $\lambda(t) = t/200,\qquad 10 \le t < 20$

(iii) $\lambda(t) = 0.10,\qquad 20 \le t$

Calculate the probability that exactly four trains arrive between 7:00 AM and 7:25 AM.

(A) 0.05

(B) 0.07

(C) 0.09

(D) 0.11

(E) 0.13

26. A certain scientific theory supposes that mistakes in cell division occur according to a Poisson process with rate 4 per day, and that a specimen fails at the time of the 289th such mistake. This theory explains the only cause of failure.

T is the time-of-failure random variable in days for a newborn specimen.

Using the normal approximation, calculate the probability that $T > 68$.

(A) 0.84

(B) 0.86

(C) 0.88

(D) 0.90

(E) 0.92

27. For a special whole life insurance, you are given:

(i) $b_t = e^{-t}$, $t > 0$

(ii) μ is constant.

(iii) $\delta = 0.06$

(iv) $Z = e^{-T} v^T$, where T is the future lifetime random variable.

(v) $E[Z] = 0.03636$

Calculate Var[Z].

(A) 0.017

(B) 0.021

(C) 0.025

(D) 0.029

(E) 0.033

28. You are the pricing actuary reviewing cash values on fully discrete whole life insurances of 10,000 on (40). A desired asset share pattern has been chosen. You are to determine cash values that will produce those asset shares.

You are given:

(i) The gross or contract premium is 90.

(ii) Renewal expenses, payable at the start of the year, are 5% of premium.

(iii) $q_{55}^{(\text{death})} = 0.004$

(iv) $q_{55}^{(\text{withdrawal})} = 0.050$

(v) $i = 0.08$

(vi) ${}_{15}AS = 1150$ and ${}_{16}AS = 1320$ are the asset shares at the ends of years 15 and 16.

Calculate ${}_{16}CV$, the cash value payable upon withdrawal at the end of year 16.

(A) 810

(B) 860

(C) 910

(D) 960

(E) 1010

29. For a special fully discrete, 30-year deferred, annual life annuity-due of 200 on (30), you are given:

(i) The single benefit premium is refunded without interest at the end of the year of death if death occurs during the deferral period.

(ii) Mortality follows the Illustrative Life Table.

(iii) $i = 0.06$

Calculate the single benefit premium for this annuity.

(A) 350

(B) 360

(C) 370

(D) 380

(E) 390

30. For a special fully continuous whole life insurance of 1 on (x), you are given:

(i) Mortality follows a double decrement model.

(ii) The death benefit for death due to cause 1 is 3.

(iii) The death benefit for death due to cause 2 is 1.

(iv) $\mu_x^{(1)}(t) = 0.02, \qquad t \geq 0$

(v) $\mu_x^{(2)}(t) = 0.04, \qquad t \geq 0$

(vi) The force of interest, δ, is a positive constant.

Calculate the benefit premium for this insurance.

(A) 0.07

(B) 0.08

(C) 0.09

(D) 0.10

(E) 0.11

****END OF EXAMINATION****