Exam MFE Spring 2007

FINAL ANSWER KEY

Question #	*Answer*	
1	B	
2	A	
3	C	
4	E	
5	D	
6	C	
7	E	
8	C	
9	A	
10	B	
11	D	
12	A	
13	E	
14	E	
15	C	
16	D	
17	B	
18	A	
19	D	

****BEGINNING OF EXAMINATION****
ACTUARIAL MODELS – FINANCIAL ECONOMICS SEGMENT

1. On April 30, 2007, a common stock is priced at $52.00. You are given the following:

(i) Dividends of equal amounts will be paid on June 30, 2007 and September 30, 2007.

(ii) A European call option on the stock with strike price of $50.00 expiring in six months sells for $4.50.

(iii) A European put option on the stock with strike price of $50.00 expiring in six months sells for $2.45.

(iv) The continuously compounded risk-free interest rate is 6%.

Calculate the amount of each dividend.

(A) $0.51

(B) $0.73

(C) $1.01

(D) $1.23

(E) $1.45

2. For a one-period binomial model for the price of a stock, you are given:

(i) The period is one year.

(ii) The stock pays no dividends.

(iii) $u = 1.433$, where u is one plus the rate of capital gain on the stock if the price goes up.

(iv) $d = 0.756$, where d is one plus the rate of capital loss on the stock if the price goes down.

(v) The continuously compounded annual expected return on the stock is 10%.

Calculate the true probability of the stock price going up.

(A) 0.52

(B) 0.57

(C) 0.62

(D) 0.67

(E) 0.72

3. You are asked to determine the price of a European put option on a stock. Assuming the Black-Scholes framework holds, you are given:

(i) The stock price is \$100.

(ii) The put option will expire in 6 months.

(iii) The strike price is \$98.

(iv) The continuously compounded risk-free interest rate is $r = 0.055$.

(v) $\delta = 0.01$

(vi) $\sigma = 0.50$

Calculate the price of this put option.

(A) \$3.50

(B) \$8.60

(C) \$11.90

(D) \$16.00

(E) \$20.40

4. For a stock, you are given:.

(i) The current stock price is $50.00.

(ii) $\delta = 0.08$

(iii) The continuously compounded risk-free interest rate is $r = 0.04$.

(iv) The prices for one-year European calls (C) under various strike prices (K) are shown below:

K	C
$40	$ 9.12
$50	$ 4.91
$60	$ 0.71
$70	$ 0.00

You own four special put options each with one of the strike prices listed in (iv). Each of these put options can only be exercised immediately or one year from now.

Determine the lowest strike price for which it is optimal to exercise these special put option(s) immediately.

(A) $40

(B) $50

(C) $60

(D) $70

(E) It is not optimal to exercise any of these put options.

5. For a European call option on a stock within the Black-Scholes framework, you are given:

(i) The stock price is $85.

(ii) The strike price is $80.

(iii) The call option will expire in one year.

(iv) The continuously compound risk-free interest rate is 5.5%.

(v) $\sigma = 0.50$

(vi) The stock pays no dividends.

Calculate the volatility of this call option.

(A) 50%

(B) 69%

(C) 123%

(D) 139%

(E) 278%

6. Consider a model with two stocks. Each stock pays dividends continuously at a rate proportional to its price.

$S_j(t)$ denotes the price of one share of stock *j* at time *t*.

Consider a claim maturing at time 3. The payoff of the claim is

$$\text{Maximum}\left(S_1(3), S_2(3)\right).$$

You are given:

(i) $S_1(0) = \$100$

(ii) $S_2(0) = \$200$

(iii) Stock 1 pays dividends of amount $(0.05)S_1(t)\,dt$ between time *t* and time $t + dt$.

(iv) Stock 2 pays dividends of amount $(0.1)S_2(t)\,dt$ between time *t* and time $t + dt$.

(v) The price of a European option to exchange Stock 2 for Stock 1 at time 3 is $10.

Calculate the price of the claim.

(A) $96

(B) $145

(C) $158

(D) $200

(E) $234

7. You are given the following information:

Bond maturity (years)	1	2
Zero-coupon bond price	0.9434	0.8817

A European call option, that expires in 1 year, gives you the right to purchase a 1-year bond for 0.9259.

The bond forward price is lognormally distributed with volatility $\sigma = 0.05$.

Using the Black formula, calculate the price of the call option.

(A) 0.011

(B) 0.014

(C) 0.017

(D) 0.020

(E) 0.022

8. Let $S(t)$ denote the price at time t of a stock that pays no dividends. The Black-Scholes framework holds. Consider a European call option with exercise date T, $T > 0$, and exercise price $S(0)e^{rT}$, where r is the continuously compounded risk-free interest rate.

You are given:

(i) $S(0) = \$100$

(ii) $T = 10$

(iii) $\text{Var}\left[\ln S(t)\right] = 0.4t$, $t > 0$.

Determine the price of the call option.

(A) \$7.96

(B) \$24.82

(C) \$68.26

(D) \$95.44

(E) There is not enough information to solve the problem.

9. You use a binomial interest rate model to evaluate a 7.5% interest rate cap on a $100 three-year loan.

You are given:

(i) The interest rates for the binomial tree are as follows:

$$r_0 = 6.000\%$$
$$r_u = 7.704\%$$
$$r_d = 4.673\%$$
$$r_{uu} = 9.892\%$$
$$r_{ud} = r_{du} = 6.000\%$$
$$r_{dd} = 3.639\%$$

(ii) All interest rates are annual effective rates.

(iii) The risk-neutral probability that the annual effective interest rate moves up or down is $\frac{1}{2}$.

(iv) The loan interest payments are made annually.

Using the binomial interest rate model, calculate the value of this interest rate cap.

(A) $0.57

(B) $0.96

(C) $1.45

(D) $1.98

(E) $2.18

10. For two European call options, Call-I and Call-II, on a stock, you are given:

Greek	Call-I	Call-II
Delta	0.5825	0.7773
Gamma	0.0651	0.0746
Vega	0.0781	0.0596

Suppose you just sold 1000 units of Call-I.

Determine the numbers of units of Call-II and stock you should buy or sell in order to both delta-hedge and gamma-hedge your position in Call-I.

(A) buy 95.8 units of stock and sell 872.7 units of Call-II

(B) sell 95.8 units of stock and buy 872.7 units of Call-II

(C) buy 793.1 units of stock and sell 692.2 units of Call-II

(D) sell 793.1 units of stock and buy 692.2 units of Call-II

(E) sell 11.2 units of stock and buy 763.9 units of Call-II

11. For a two-period binomial model for stock prices, you are given:

(i) Each period is 6 months.

(ii) The current price for a nondividend-paying stock is $70.00.

(iii) $u = 1.181$, where u is one plus the rate of capital gain on the stock per period if the price goes up.

(iv) $d = 0.890$, where d is one plus the rate of capital loss on the stock per period if the price goes down.

(v) The continuously compounded risk-free interest rate is 5%.

Calculate the current price of a one-year American put option on the stock with a strike price of $80.00.

(A) $9.75

(B) $10.15

(C) $10.35

(D) $10.75

(E) $11.05

12. You are given the following information:

(i) $S(t)$ is the value of one British pound in U.S. dollars at time *t*.

(ii) $\frac{dS(t)}{S(t)} = 0.1dt + 0.4dZ(t)$.

(iii) The continuously compounded risk-free interest rate in the U.S. is $r = 0.08$.

(iv) The continuously compounded risk-free interest rate in Great Britain is $r^* = 0.10$.

(v) $G(t) = S(t)e^{(r-r^*)(T-t)}$ is the forward price in U.S. dollars per British pound, and *T* is the maturity time of the currency forward contract.

Based on Itô's Lemma, which of the following stochastic differential equations is satisfied by $G(t)$?

(A) $dG(t) = G(t)\left[0.12dt + 0.4dZ(t)\right]$

(B) $dG(t) = G(t)\left[0.10dt + 0.4dZ(t)\right]$

(C) $dG(t) = G(t)\left[0.08dt + 0.4dZ(t)\right]$

(D) $dG(t) = G(t)\left[0.12dt + 0.16dZ(t)\right]$

(E) $dG(t) = G(t)\left[0.10dt + 0.16dZ(t)\right]$

13. Let $P(r, t, T)$ denote the price at time t of \$1 to be paid with certainty at time T, $t \le T$, if the short rate at time t is equal to r.

For a Vasicek model you are given:

$$P(0.04, 0, 2) = 0.9445$$
$$P(0.05, 1, 3) = 0.9321$$
$$P(r^*, 2, 4) = 0.8960$$

Calculate r^*.

(A) 0.04

(B) 0.05

(C) 0.06

(D) 0.07

(E) 0.08

14. For a one-year straddle on a nondividend-paying stock, you are given:

(i) The straddle can only be exercised at the end of one year.

(ii) The payoff of the straddle is the absolute value of the difference between the strike price and the stock price at expiration date.

(iii) The stock currently sells for \$60.00.

(iv) The continuously compounded risk-free interest rate is 8%.

(v) In one year, the stock will either sell for \$70.00 or \$45.00.

(vi) The option has a strike price of \$50.00.

Calculate the current price of the straddle.

(A) \$0.90

(B) \$4.80

(C) \$9.30

(D) \$14.80

(E) \$15.70

15. For a six-month European put option on a stock, you are given:

(i) The strike price is $50.00.

(ii) The current stock price is $50.00.

(iii) The only dividend during this time period is $1.50 to be paid in four months.

(iv) $\sigma = 0.30$

(v) The continuously compounded risk-free interest rate is 5%.

Under the Black-Scholes framework, calculate the price of the put option.

(A) $3.50

(B) $3.95

(C) $4.19

(D) $4.73

(E) $4.93

16. For an American perpetual option within the Black-Scholes framework, you are given:

(i) $h_1 + h_2 = 7/9$

(ii) The continuously compounded risk-free interest rate is 5%.

(iii) $\sigma = 0.30$

Calculate h_1.

(A) 0.73

(B) 1.12

(C) 1.46

(D) 1.51

(E) 2.09

17. Let $S(t)$ denote the price at time t of a stock that pays dividends continuously at a rate proportional to its price. Consider a European gap option with expiration date T, $T > 0$.

If the stock price at time T is greater than $100, the payoff is

$$S(T) - 90;$$

otherwise, the payoff is zero.

You are given:

(i) $S(0) = \$80$

(ii) The price of a European call option with expiration date T and strike price $100 is $4.

(iii) The delta of the call option in (ii) is 0.2.

Calculate the price of the gap option.

(A) $3.60

(B) $5.20

(C) $6.40

(D) $10.80

(E) There is not enough information to solve the problem.

18. Consider two nondividend-paying assets *X* and *Y*, whose prices are driven by the same Brownian motion Z. You are given that the assets *X* and *Y* satisfy the stochastic differential equations:

$$\frac{dX(t)}{X(t)} = 0.07\,dt + 0.12\,dZ(t)$$

$$\frac{dY(t)}{Y(t)} = G\,dt + H\,dZ(t),$$

where *G* and *H* are constants.

You are also given:

(i) $d\left(\ln\left[Y(t)\right]\right) = 0.06\,dt + \sigma dZ(t)$

(ii) The continuously compounded risk-free interest rate is 4%.

(iii) $\sigma < 0.25$

Determine *G*.

(A) 0.065

(B) 0.070

(C) 0.075

(D) 0.100

(E) 0.120

19. Assume that the Black-Scholes framework holds. The price of a nondividend-paying stock is \$30.00. The price of a put option on this stock is \$4.00.

You are given:

(i) $\Delta = -0.28$

(ii) $\Gamma = 0.10$

Using the delta-gamma approximation, determine the price of the put option if the stock price changes to \$31.50.

(A) \$3.40

(B) \$3.50

(C) \$3.60

(D) \$3.70

(E) \$3.80

****END OF EXAMINATION****