Starting with the Fall 2009 session the following questions from Spring 2007 no longer partain to the current syllabus for Exam C: # 4, 19, 22, 23, 27 and 34.

SPRING 2007
EXAM C SOLUTIONS

Question #1
Key: D

The data are already shifted (have had the policy limit and the deductible of 50 applied). The two 350 payments are censored. Thus the likelihood function is

$$L = \frac{f(100)f(200)f(250)[1-F(400)]^2}{[1-F(50)]^5}$$
$$= \frac{\theta^{-1}e^{-100/\theta}\theta^{-1}e^{-200/\theta}\theta^{-1}e^{-250/\theta}\left(e^{-400/\theta}\right)^2}{\left(e^{-50/\theta}\right)^5}$$
$$= \theta^{-3}e^{-1100/\theta}.$$

Question #2
Key: E

The Bühlmann estimates are always linear, which rules out answer (D).
The Bühlmann estimate is the least-squares approximation to the Bayesian estimate. In (A) it entirely below the Bayesian estimate and in (B) it is entirely above. There are straight lines with smaller errors.
The Bayesian estimate cannot produce an answer that is outside the range of the prior distribution. All estimates must be between 0.1 and 0.6, which is not true for (C).
Only (E) does not violate these requirements, and therefore is a feasible solution.

Question #3
Key: B

S_{101} is binomial with parameters $m = 101$ and Q. Then,

$$Var(S_{101}) = E[Var(S_{101} \mid Q)] + Var[E(S_{101} \mid Q)]$$
$$= E[101Q(1-Q)] + Var(101Q)$$
$$= 101E(Q) - 101E(Q^2) + 101^2 Var(Q)$$
$$= 101E(Q) - 101E(Q^2) + 10201E(Q^2) - 10201E(Q)^2$$
$$= 101\frac{1}{100} + 10100\frac{1(2)}{100(101)} - 10201\left(\frac{1}{100}\right)^2$$
$$= 1.9899.$$

Question #4
Key: A

The random variable $\ln(S_{0.5}/0.25)$ has a normal distribution with parameters

$\mu = (0.015 - 0.35^2/2)(0.5) = 0.044375$

$\sigma^2 = 0.35^2(0.5) = 0.6125.$

The upper limit for the normal random variable is

$0.044375 + 1.645\sqrt{0.6125} = 0.45149$.

The upper limit for the stock price is

$0.25e^{0.45149} = 0.39266$.

Question # 5
Key: E

$$\chi^2 = \sum_{j=1}^{20} \frac{(O_j - 50)^2}{50} = 0.02\left[\sum_{j=1}^{20} O_j^2 - 100\sum_{j=1}^{20} O_j + 20(50)^2\right]$$
$$= 0.02[51,850 - 100(1,000) + 50,000] = 37$$

With 19 degrees of freedom, the critical value at a 0.01 significance level is 36.191 and therefore the null hypothesis is rejected at this level.

Question # 6
Key: A

$\mu(I) = v(I) = 0.5$

$\mu(II) = v(II) = 1.5$

$\mu = 0.5\theta + 1.5(1-\theta) = 1.5 - \theta$

$v = 0.5\theta + 1.5(1-\theta) = 1.5 - \theta$

$a = (0.5)^2\theta + (1.5)^2(1-\theta) - (1.5-\theta)^2 = \theta - \theta^2$

$$Z = \frac{1}{1 + \dfrac{1.5-\theta}{\theta - \theta^2}} = \frac{\theta - \theta^2}{1.5 - \theta^2}$$

Question # 7
Key: E

The second moment is

$$\int_0^{50} x^2 \frac{30}{100(50)}dx + \int_{50}^{100} x^2 \frac{36}{100(50)}dx + \int_{100}^{200} x^2 \frac{18}{100(100)}dx$$
$$+ \int_{200}^{350} x^2 \frac{16}{100(200)}dx + \int_{350}^{400} 350^2 \frac{16}{100(200)}dx$$
$$= \frac{30}{5,000}\frac{x^3}{3}\bigg|_0^{50} + \frac{36}{5,000}\frac{x^3}{3}\bigg|_{50}^{100} + \frac{18}{10,000}\frac{x^3}{3}\bigg|_{100}^{200} + \frac{16}{20,000}\frac{x^3}{3}\bigg|_{200}^{350} + \frac{16}{20,000}350^2 x\bigg|_{350}^{400}$$
$$= 250 + 2,100 + 4,200 + 9,300 + 4,900 = 20,750.$$

Question # 8
Key: B

Using the recursive formula

$$\Pr(S=0) = e^{-3} = 0.0498$$
$$\Pr(S=1) = \frac{3}{1}(0.4)(0.0498) = 0.0598$$
$$\Pr(S=2) = \frac{3}{2}[(0.4)(0.0598) + 2(0.3)(0.0498)] = 0.0807$$
$$\Pr(S=3) = \frac{3}{3}[(0.4)(0.0807) + 2(0.3)(0.0598) + 3(0.2)(0.0498)] = 0.0980.$$

Then,

$$\Pr(S \le 3) = 0.0498 + 0.0598 + 0.0807 + 0.0980 = 0.2883.$$

Question # 9
Key: B

Because 0.9610 < 0.9810 < 0.9827 there are 4 simulated claims.

To simulate a claim, solve $u = 1 - \left(\frac{36}{36+x}\right)^{2.8}$ for $x = 36[(1-u)^{-1/2.8} - 1]$.

For the first four uniform random numbers, the simulated losses are 12.704, 58.029, 4.953, and 9.193. With the deductible of 5 and maximum of 30, the payments are 7.704, 30, 0, and 4.193. The total of these four payments is 41.897.

Question # 10
Key: E

Shifting adds δ to the mean and median. The median of the unshifted exponential distribution is the solution to $0.5 = S(m) = e^{-m/\theta}$ for $m = \theta\ln(2)$. The equations to solve are

$$300 = \theta + \delta$$

$$240 = \theta\ln(2) + \delta.$$

Subtracting the second equation from the first gives

$$60 = \theta[1 - \ln(2)], \quad \theta = 195.53.$$

From the first equation, $\delta = 300 - 195.53 = 104.47$.

Question # 11
Key: D

$$\hat{v} = \frac{1+3+1}{3} = \frac{5}{3}$$

$$\hat{a} = \frac{1}{3-1}\left[\left(5 - \frac{20}{3}\right)^2 + \left(9 - \frac{20}{3}\right)^2 + \left(6 - \frac{20}{3}\right)^2\right] - \frac{5/3}{3} = 34/9$$

$$Z = \frac{3}{3 + \dfrac{5/3}{34/9}} = \frac{306}{351} = 0.8718$$

Question # 12
Key: A

$$\hat{S}(9) = 0.16 \Rightarrow r_{10} = 0.16(200) = 32$$

$$0.04045 - 0.02625 = \frac{s_{10}}{r_{10}(r_{10} - s_{10})}$$

$$0.0142 = \frac{s_{10}}{32(32 - s_{10})} \Rightarrow s_{10} = 10.$$

Question # 13
Key: C

$$E(Y) = \int_{30,000}^{\infty} (x - 30,000)(10,000)^{-1} e^{-x/10,000} dx$$
$$= \int_0^{\infty} y(10,000)^{-1} e^{-(y+30,000)/10,000} dx$$
$$= e^{-3}(10,000) = 497.87$$
$$E(Y^2) = \int_{30,000}^{\infty} (x - 30,000)^2 (10,000)^{-1} e^{-x/10,000} dx$$
$$= \int_0^{\infty} y^2 (10,000)^{-1} e^{-(y+30,000)/10,000} dx$$
$$= e^{-3}(2)(10,000)^2 = 9,957,413.67$$
$$Var(Y) = 9,957,413.67 - 497.87^2 = 9,709,539.14$$
$$SD(Y) = 3,116.01$$
$$CV = 3,116.01 / 497.87 = 6.259.$$

Question # 14
Key: E

$$f(x) = \frac{\alpha\theta^{\alpha}}{(x+\theta)^{\alpha+1}}, \quad L(\alpha,\theta) = \frac{\alpha^{20}\theta^{20\alpha}}{\prod_{j=1}^{20}(x_j+\theta)^{\alpha+1}}$$
$$\ln L(\alpha,\theta) = 20\ln(\alpha) + 20\alpha\ln(\theta) - (\alpha+1)\sum_{j=1}^{20}\ln(x_j+\theta).$$

Under the null hypotheses, $\alpha = 2$ and $\theta = 3.1$. The loglikelihood value is
$20\ln(2) + 20(2)\ln(3.1) - 3(39.30) = -58.7810.$

Under the alternative hypotheses, $\alpha = 2$ and $\theta = 7.0$. The loglikelihood value is
$20\ln(2) + 20(2)\ln(7.0) - 3(49.01) = -55.3307.$

Twice the difference of the loglikelihood values is 6.901.
There is one degree of freedom (no estimated parameters in the null hypothesis versus one estimated parameter in the alternative hypothesis).
At the 0.01 significance level the critical value is 6.635 and so the null hypothesis is rejected.

Question # 15
Key: D

For one year,
$\pi(q \mid x_1) \propto q^{x_1}(1-q)^{8-x_1} q^{a-1}(1-q)^8 = q^{x_1+a-1}(1-q)^{16-x_1}$.
This is a beta distribution with parameters $x_1 + a$ and $17 - x_1$. The mean is the Bayesian credibility estimate of q and 8 times that value is the estimate of the expected number of claims in year 2. Then, with $x_1 = 2$,
$2.5455 = 8\frac{2+a}{17+a}$ which implies $a = 5$.
For two years,
$\pi(q \mid x_1, x_2) \propto q^{x_1}(1-q)^{8-x_1} q^{x_2}(1-q)^{8-x_2} q^{a-1}(1-q)^8 = q^{x_1+x_2+a-1}(1-q)^{24-x_1-x_2}$.
This is a beta distribution with parameters $x_1 + x_2 + a$ and $25 - x_1 - x_2$. The mean is the Bayesian credibility estimate of q and 8 times that value is the estimate of the expected number of claims in year 2. Then, with $x_1 = 2$, $x_2 = k$, and $a = 5$,
$3.73333 = 8\frac{7+k}{30}$ which implies $k = 7$.

Question # 16
Key: C

Two of the five observations are less than or equal to 150 and so the empirical estimate is 2/5 = 0.4.
For the kernel density, analyze the contribution of each data point as follows:
82 is uniformly spread from 32 to 132. With the entire range below 150, it contributes 0.2 to the answer.
126 is uniformly spread from 76 to 176. The contribution is 0.2(150 – 76)/100 = 0.148.
161 is uniformly spread from 111 to 211. The contribution is 0.2(150 – 111)/100 = 0.078.
The last two points are more than 50 above 150 and so contribute nothing. The total is
0.2 + 0.148 + 0.078 = 0.426.

Question # 17
Key: A

$$E(S) = 100(20,000) = 2,000,000$$
$$Var(S) = 100(5,000)^2 + 25^2(20,000)^2 = 252,500,000,000$$
$$SD(S) = 502,494$$
$$\Pr[S > 1.5(2,000,000)] = \Pr\left(Z > \frac{3,000,000 - 2,000,000}{502,494}\right)$$
$$= \Pr(Z > 1.99) = 0.0233.$$

Question # 18
Key: C

The maximum likelihood estimate of a Poisson mean is the sample mean. Here it is

$$\frac{1000(0)+1200(1)+600(2)+200(3)}{3000}=1.$$

The variance of the sample mean is the variance of a single observation divided by the sample size. For the Poisson distribution, the variance equals the mean, so the mle of the variance is the sample mean. The estimated variance of the sample mean is 1/3000 and the lower end point of the confidence interval is $1-1.645\sqrt{1/3000}=0.970$.

Question # 19
Key: C

The random variable $\ln(S_2/50)$ has a normal distribution with mean $(0.15-0.3^2/2)(2)=0.21$ and variance $(0.3^2)(2) = 0.18$, and thus a standard deviation of 0.4243.
The three uniform random numbers become the following three values from the standard normal: 2.12, -1.77, 0.77. Upon multiplying each by the standard deviation of 0.4243 and adding the mean of 0.21, the resulting normal values are 1.109, -0.541, and 0.537. The simulated stock prices are obtained by exponentiating these numbers and multiplying by 50. This yields, 151.57, 29.11, and 85.54. The average of these three numbers is 88.74.

Question # 20
Key: D

In each row of the table, take the largest absolute difference between F^* and the two empirical values. By row, the results are 0.079, 0.089, 0.098, 0.101, 0.111, and 0.108. The overall maximum is 0.111. To complete the test, multiply by the square root of the sample size (200) to obtain 1.570. Because this number is between 1.52 and 1.63, reject at a 0.02 significance level but do not reject at 0.01.

Question # 21
Key: A

$$\mu(\alpha)=\alpha\theta, \quad v(\alpha)=\alpha\theta^2$$

$$v=E(\alpha\theta^2)=50\theta^2, \quad a=Var(\alpha\theta)=\theta^2 Var(\alpha)$$

$$0.25=\frac{2}{2+\dfrac{50\theta^2}{\theta^2 Var(\alpha)}} \Rightarrow Var(\alpha)=25/3=8.33$$

Question # 22
Key: D

Group 1 has the baseline distribution with $S_0(x) = (200/x)^\alpha$, $f_0(x) = \alpha 200^\alpha x^{-\alpha-1}$. Group 2 has the following survival and density functions (where *b* is used for e^β):
$S_1(x) = [S_0(x)]^b = (200/x)^{\alpha b}$, $f_1(x) = \alpha b 200^{\alpha b} x^{-\alpha b-1}$.
The loglikelihood function for the six observations is

$$\begin{aligned}&\ln(\alpha)+\alpha\ln(200)-(\alpha+1)\ln(275)+\ln(\alpha)+\alpha\ln(200)-(\alpha+1)\ln(325)\\ &\quad+\ln(\alpha)+\alpha\ln(200)-(\alpha+1)\ln(520)+\ln(\alpha b)+\alpha b\ln(200)-(\alpha b+1)\ln(215)\\ &\quad+\ln(\alpha b)+\alpha b\ln(200)-(\alpha b+1)\ln(250)+\ln(\alpha b)+\alpha b\ln(200)-(\alpha b+1)\ln(300).\end{aligned}$$

Differentiating with respect to the two variables and gathering terms gives the two equations:

$$\frac{3}{b}-0.701\alpha=0$$

$$\frac{6}{\alpha}-0.710b-1.759=0.$$

Then, b = 2.51 and its logarithm, 0.92 is the estimate of β.

The algebra can be made easier by setting $c = \alpha b$. Then the derivatives lead to two equations each in a single variable.

Question # 23
Key: C

At time 1 there are three possible outcomes: 3(1.1) – 0 = 3.3, 3.3 – 2 = 1.3, and 3.3 – 6 = -2.7 with probabilities 0.6, 0.3, and 0.1 respectively. The last case terminates the process with ruin and a probability of 0.1. In the other two cases the process continues.

At time 2 there are three possible outcomes for each of the two continuing cases. Two examples are (3.3 + 2)1.1 – 0 = 5.83 with probability 0.6(0.6) = 0.36 and (1.3 + 2)(1.1) – 6 = -2.37 with probability 0.3(0.1) = 0.03. The six ending values and their probabilities are: (5.83, 0.36), (3.83, 0.18), (3.63, 0.18), (1.63, 0.09), (-0.17, 0.06), and (-2.37, 0.03). Ruin takes place in the last two cases, adding 0.09 to the ruin probability for a total of 0.19 so far. The other four cases continue.

While there are now twelve outcomes at time 3, if the year starts at 5.83, 3.83, or 3.63 then adding the premium of 2 and earning 10% interest takes the surplus above 6 and so ruin is not possible. If the year starts at 1.63, surplus prior to claims is (1.63 + 2)(1.1) = 3.993 and only a claim of 6 will cause ruin. The probability is 0.09(0.1) = 0.009. Adding this in gives the answer of 0.199.

Question # 24
Key: E

For the 20th percentile, begin with 0.2(16 + 1) = 3.4 and interpolate for 0.6(75) + 0.4(81) = 77.4. For the 70th percentile, begin with 0.7(17) = 11.9 and interpolate for 0.1(122) + 0.9(125) = 124.7. The equations and their solution are:

$$0.2 = 1 - e^{-(77.4/\theta)^\tau}, \quad 0.7 = 1 - e^{-(124.7/\theta)^\tau}$$

$$0.22314 = (77.4/\theta)^\tau, \quad 1.20397 = (124.7/\theta)^\tau$$

$$5.39558 = (124.7/77.4)^\tau$$

$$\tau = \ln(5.29558)/\ln(124.7/77.4) = 3.53427$$

$$\theta = 77.4/0.22314^{1/3.53427} = 118.32$$

where the equation on the third line is obtained by dividing the second equation on the second line by the first equation on that line.

Question # 25
Key: A

$$\hat{\mu} = \hat{v} = \bar{x} = \frac{46(0)+34(1)+13(2)+5(3)+2(4)}{100} = 0.83$$

$$s^2 = \frac{46(-0.83)^2 + 34(0.17)^2 + 13(1.17)^2 + 5(2.17)^2 + 2(3.17)^2}{99} = 0.95061$$

$$\hat{a} = 0.95061 - 0.83 = 0.12061$$

$$Z = \frac{1}{1 + \frac{0.83}{0.12601}} = 0.12688$$

The estimated number of claims in 5 years for this policyholder is
0.12688(3) + 0.87312(0.83) = 1.10533. For one year the estimate is 1.10533/5 = 0.221.

Question # 26
Key: B

The formula for the number in the risk set is $r_j = P_j + d_j$ and so the risk set values are 6, 11, 16, 11, 3, 1, and 0. The number of claims are in the *x* column of the table. Then,

$$\hat{S}(500) = \frac{6-1}{6} = \frac{5}{6} = 0.83333$$

$$\hat{S}(6000) = \frac{5}{6}\frac{9}{11}\frac{12}{16}\frac{4}{11}\frac{2}{3} = 0.12397.$$

The answer is the ratio, 0.12397/0.83333 = 0.14876.

Question # 27
Key: D

The risk measure is

$$\int_0^\infty g[S(x)]dx = \int_0^\infty S(x)^{1/2}\,dx$$

$$= \int_0^\infty \left(\frac{1000}{1000+x}\right)^2 dx$$

$$= \left.\frac{1000^2(1000+x)^{-1}}{-1}\right|_0^\infty$$

$$= 1000.$$

Question # 28
Key: B

The empirical distribution indicates probabilites of 0.2 at 10, 0.4 at 100, and 0.4 at 1000 for a mean of 0.2(10) + 0.4(100) + 0.4(1000) = 442.

Two percentiles from the lognormal distribution can be inferred from the graph. The 20th percentile is at 10 and the 60th percentile is at 100. To obtain the parameters, two equations can be solved:

$$0.2 = F(10) = \Phi\left(\frac{\ln 10 - \mu}{\sigma}\right), \quad 0.6 = F(100) = \Phi\left(\frac{\ln 100 - \mu}{\sigma}\right)$$

$$\frac{\ln 10 - \mu}{\sigma} = -0.842, \quad \frac{\ln 100 - \mu}{\sigma} = 0.253$$

$$\ln 10 = \mu - 0.842\sigma, \quad \ln 100 = \mu + 0.253\sigma$$

$$\ln 100 - \ln 10 = 1.095\sigma$$

$$\sigma = 2.1028, \quad \mu = \ln 10 + 0.842\sigma = 4.0731$$

$$\text{mean} = e^{4.0731 + 2.1028^2/2} = 535.92.$$

The difference is 93.92.

Question # 29
Key: E

There are four possible bootstrap samples for the two fire losses and four for the wind losses. Thus there are 16 equally likely outcomes. Because some combinations produce the same squared error (for example – 3,3 for fire and 0,3 for wind and 3,3 for fire and 3,0 for wind) they are not repeated in the table below.

Fire losses	Wind losses	Total loss	Eliminated	Fraction	Square error	Number
3,3	0,0	6	0	0.0000	0.0400	1
3,3	0,3	9	2	0.2222	0.0005	2
3,3	3,3	12	4	0.3333	0.0178	1
3,4	0,0	7	0	0.0000	0.0400	2
3,4	0,3	10	2	0.2000	0.0000	4
3,4	3,3	13	4	0.3077	0.0116	2
4,4	0,0	8	0	0.0000	0.0400	1
4,4	0,3	11	2	0.1818	0.0003	2
4,4	3,3	14	4	0.2857	0.0073	1

The mean square error is $\frac{1(0.0400)+2(0.0005)+\cdots+1(0.0073)}{16}=0.0131.$

Question # 30
Key: C

$\pi(\beta \mid x) \propto f(x \mid \beta)\pi(\beta)=\beta^{-1}e^{-x/\beta}\beta^{-3}e^{-c/\beta}=\beta^{-4}e^{-(x+c)/\beta}$

There are two ways to proceed. One is to recognize that as a function of β this is an inverse gamma density with $\alpha=3$ and $\theta=x+c$. The mean of (x + c)/2 can be looked up.

The second method is to complete the integrals. Use fact (iii) to obtain

$\int_0^\infty \beta^{-4}e^{-(x+c)/\beta}d\beta=\frac{2}{(x+c)^3}$.

The complete posterior density is

$\pi(\beta \mid x)=\frac{(x+c)^3}{2}\beta^{-4}e^{-(x+c)/\beta}$

and fact (iii) can be used to obtain the mean.

$E(\beta \mid x)=\int_0^\infty \beta\frac{(x+c)^3}{2}\beta^{-4}e^{-(x+c)/\beta}d\beta=\frac{(x+c)^3}{2}\frac{1}{(x+c)^2}=\frac{x+c}{2}.$

Question # 31
Key: A

$$L(\alpha)=\prod_{j=1}^{7}\frac{f(x_j\,|\,\alpha)}{1-F(100\,|\,\alpha)}=\frac{\prod_{j=1}^{7}\dfrac{\alpha 400^{\alpha}}{(400+x_j)^{\alpha+1}}}{\left[\left(\dfrac{400}{400+100}\right)^{\alpha}\right]^{7}}$$

$$l(\alpha)=\ln L(\alpha)=7\ln\alpha+7\alpha\ln 400-(\alpha+1)\sum_{j=1}^{7}(400+x_j)-7\alpha\ln 0.8$$

$$=7\ln\alpha-3.79\alpha-47.29$$

$$l'(\alpha)=7\alpha^{-1}-3.79=0$$

$$\hat{\alpha}=7/3.79=1.847$$

$$E(X)=\frac{\theta}{\alpha-1}=\frac{400}{0.847}=472.26$$

Question # 32
Key: E

(A) is false. This is required for Bühlmann, but is not necessary for Bühlmann-Straub.
(B) is false. This method depends only the moments of the distributions. No particular distribution is required.
(C) is false. This number appears in limited fluctuation credibility.

Question # 33
Key: D

At time 8 (the first uncensored time) the risk set is 7 (original 8 less the one censored at 4) and there is 1 observation. At time 12 the risk set is 5 and there are 2 observations. Then,

$$\hat{H}(12)=\frac{1}{7}+\frac{2}{5}=0.5429$$

$$V\hat{a}r[\hat{H}(12)]=\frac{1}{7^2}+\frac{2}{5^2}=0.1004.$$

The upper limit is

$$0.5429+1.645\sqrt{0.1004}=1.064.$$

Question # 34
Key: E

The returns are ln(56/54) = 0.03637, ln(48/56) = -0.15415, ln(55/48) = 0.13613, ln(60/55) = 0.08701, ln(58/60) = -0.0339, ln(62/58) = 0.06669. The sample mean is 0.02302 and the sample variance is 0.01071 (McDonald divides by one less than the sample size when computing a sample variance). These are monthly values. The annual mean and variance are 0.02302(12) = 0.27624 and 0.01071(12) = 0.12852. The expected return is 0.27624 + 0.12852/2 = 0.3405.

Question # 35
Key: E

$$\Pr\left(G=1/3\,|\,D=0\right)=\frac{\Pr(D=0\,|\,G=1/3)\Pr(G=1/3)}{\Pr(D=0\,|\,G=1/3)\Pr(G=1/3)+\Pr(D=0\,|\,G=1/5)\Pr(G=1/5)}$$
$$=\frac{(1/3)(2/5)}{(1/3)(2/5)+(1/5)(3/5)}=\frac{10}{19}$$

Question # 36
Key: B

$$\mu=0.6(2000)+0.3(3000)+0.1(4000)=2500$$
$$v=1000^2$$
$$a=0.6(2000)^2+0.3(3000)^2+0.1(4000)^2-2500^2=450,000$$
$$Z=\frac{80}{80+\dfrac{1,000,000}{450,000}}=0.97297$$
$$\bar{x}=\frac{24,000+36,000+28,000}{80}=1100$$
estimate is $0.97297(1100)+0.02703(2500)=1137.84$

Question # 37
Key: E

$F(300)=1-e^{-300/100}=0.9502$. The variance of the estimate is 0.9502(0.0498)/n. The equation to solve is
$$0.9502(0.01)=2.576\sqrt{\frac{0.9502(0.0498)}{n}}.$$
The solution is n = 3477.81.

Question # 38
Key: B

$$\frac{0.5}{0.65} = \frac{r_4 - s_4}{r_4} = \frac{r_4 - 3}{r_4} \Rightarrow r_4 = 13.$$

The risk set at the 5th death time is the 13 from the previous time less the 3 who died and less the 6 who were censored. That leaves r_5 = 13 – 3 – 6 = 4.

$$\frac{0.25}{0.5} = \frac{r_5 - s_5}{r_5} = \frac{4 - s_5}{4} \Rightarrow s_5 = 2.$$

Question # 39
Key: C

Before the deductible is imposed, the expected number of losses is $r\beta$ = 15. From the Weibull distribution, $S(200) = e^{-(200/1000)^{0.3}} = 0.5395$. Thus 54% of losses will result in payments for an expected number of 0.5395(15) = 8.0925.

Question # 40
Key: D

The estimated probability of one or more claims is 400/2000 = 0.2. For this binomial distribution the variance of the estimator is estimated as 0.2(0.8)/2000 = 0.00008. The upper bound is $0.2 + 1.96\sqrt{0.00008} = 0.21753$.