Answer #1

a)

- Disaggregate performance using multiple pricing benchmarks
  - Liability performance
    - Product design, pricing improvements
  - Asset performance
    - Asset selection
    - Return due to taking on interest rate and credit risk
    - View assets and liabs. together in order to reduce risk and maximize return

b)

- An asset portfolio with the same interest rate risk as the liability (credit and other risks ignored)
  - Lehman index adjusted to duration 3.1

- An asset portfolio with the same interest rate risk as the actual assets
  - Lehman index adjusted to duration 3.3

c)

- Liability performance would be (in terms of yield)
  - Benchmark (1) — actual liability

Return due to the assumption of interest rate risk

- Benchmark (2) — Benchmark (1)

Asset performance would be (in terms of yield)

- Actual assets — Benchmark (2)

d)

- Liability performance

$= 6.30\% - 6.60\% = -0.30\%$

Return due to the assumption of interest rate risk

$= 6.40\% - 6.30\% = 0.10\%$

Asset performance would be (in terms of yield)

$= 6.38\% - 6.40\% = -0.02\%$

Answer #2

a)

- ALM will summarize the needs of diverse areas of the company and try to come up with a clear view of the more important aspects of the company regarding risk
- ALM can help make decisions like:
  - Entering a new line of business
  - Exiting a product market
  - Pricing, hedging
  - Allocation of capital
  - Amount and type of reinsurance
  - Investment considerations
- ALM can provide due diligence to help justify decisions that are controversial
- ALM can help to fulfill the requirements/needs of the different constituents of the company and outside parties: shareholders, regulators, rating agencies, employees, etc.
- ALM can help managers and other employees to see the firm from different perspectives, e.g.:
  - Time horizon: measure the results on a short and long-term basis (can have conflicting goals)
  - Accounting standards: statutory, GAAP, tax and economic basis
  - Future business: can incorporate the future business of the firm

b)

Initial Conditions:
- Assess the initial conditions of the firm and economy
- Should be in line with recent experience

Scenario Generator:
- Set of plausible scenarios for economic conditions, assets and liabilities
- Can be deterministic or randomly generated scenarios

Financial Calculator:
- Convert the scenarios into financial results using different measurement basis, e.g., economic, tax, regulatory and GAAP

Optimizer:
- If an objective function can be specified then optimal strategies can be proposed

Results:
- Give conclusions on the simulation performed: graphical output, distributions and output of the key variables

c)

- LifeCo is more concerned with interest rate risk
- DFA is concerned with all types of risk (interest rate, inflation, spread risk . . .)
- LifeCo is on a product line level
- DFA is on a total company level (holistic view)
- LifeCo uses measures of interest rate risk such as duration, key rate duration, etc.
- DAF uses results of simulation to determine risk levels
- LifeCo chooses asset portfolio to optimize risk/reward profile
- DFA uses optimizer to evaluate a set of strategies to maximize financial reward (doesn't just focus on assets)

d)

- Clarify the goals/objectives of the insurance company and the questions of most importance to them
- Assemble reliable data on the company's risks
- Determine technology platform
- Decide whether to perform the modeling tasks in the company or outside the company
- Confirm choice of methodology and risk measures

e)

- Secure senior management commitment by explaining the benefits of ALM, such as:
  - Risk reduction
  - Manage earnings volatility
  - Increased stock price
- Clarify the roles and responsibilities of each department in the company
- Leverage the cash-flow testing platform
- Choose appropriate metrics/methods: have metrics that are related to the real profitability of the firm
- Ensure an efficient and responsive mitigation process — once risks have been identified, they must be mitigated or managed

Answer #3

a)

Using standard deviation assumes returns are normally distributed, and this is not always true, especially in the short run

- It is a symmetric distribution, doesn't match investor's notion of loss aversion and utility
- Assumes a quadratic utility curve, which doesn't match most investor's notion of risk as well as LifeCo's.
  - LifeCo's objective is to minimize potential for significant loss while standard deviation is a measure of deviations from the mean

b)

An alternative is the lower partial moment framework

Where $LPM_n = \sum_{Rp=\infty}^{\tau} \rho(\tau - Rp)^n$

1). Setting $n = 2$ gives the target semi-variance this looks at the deviations below a target rate of return

This measure will allow LifeCo to meet the objectives of minimizing significant loss by selecting appropriate τ, as well as, maximizing economic value

2). Setting $n = 1$ gives the target shortfall It gives the expected loss. It will not meet LifeCo's objective of minimizing loss

c)

Using the target semi-variance $(n = 2)$ is the most appropriate

It meets the main objectives of LifeCo., which is to minimize the chance of significant loss and maximize economic value

d)

Minimize $LPM_2 = \sum_{Rp=-\infty}^{\tau} Pp(\tau - Rp)^n$

Such that $\sum \times_\partial E(R_\partial) = Rp^*$

and, $\sum \times_\partial = 1, \times_\partial > 0$

Where τ is the worst case return and Rp is the return of the portfolio

Answer #4

a)

Modified duration:

- Only considers small parallel shift: convexity and non-parallel shift not considered
- Sensitivity of cash flows to interest rate is not taken into account. Like embedded options in Universal Life and policy loans in Traditional life
- Generally speaking, not a good measure

b)

Effective duration:

i)

- Interest rate sensitivity of cash flows is considered
- Convexity and changing of yield curve shape are still not considered
- Better than modified duration

Effective key-rate duration:

ii)

- Measure sensitivity of price change to all the representative points on the yield curve
- Interest rate sensitive cash flows and yield curve shape change are considered
- Better measure than other two

c)

i)

- Constant growth model produces the modified duration

$$MV = d/(k-g)$$

- Where d = dividend, g = growth, k = cost of capital

Usually this model produces a much higher duration (28 years) than observed in practice

ii)

$$MV = TV + FV$$

- Tangible value (TV): value from existing business, with limited ability for inflation pass through as it might not be possible to reset prices. In the extreme case assume 0. Duration is medium-long
- Franchise value (FV): value from future business or growth. Assuming 100% inflation flow through as future prices can be reset, we get a duration of 0
- Duration is weighted average of two components described above. Final result is more reasonable

d)

i)

- Accrual of interest on principal until all bonds ahead are paid up

ii)

- When prepayments are within collar, interest accrues on Z for a fixed period

iii)

- May change to regular interest paying status under certain event
- May change back to accrual status

iv)

- If triggered by a certain cumulative event, such as prepayment, then may jump to receive principal ahead of other classes. Duration is volatile and can be very short
- Sticky means that the bond does not revert back to accrual status

e)

From the best to the worst:

1. Z-PAC: long duration and higher stability of cash flow
2. Z-bond: long duration with more prepayment risk
3. Tricky Z: volatility of cash flow
4. Jump Z: high volatility of duration (bond duration can shorten dramatically)

Because in this case we want to increase the asset duration to match the liability duration, assets with long duration and less volatility are preferred

Answer #5

a)

- Stated Maturity — longer maturities are more price and volatility sensitive just like traditional bonds underlying index
- Leverage — amount the underlying index affects the calculation of coupon or principal amounts

b)

- Difference between the at-the-market swap rate of 7.5% and the fixed coupon of $4\% = 3.5\%$
- Present value of 3.5% over 7 years at $7.5\% = 18.5\%$ of \$100 million = \$18.5 million

c)

i) Scenario 1

- Cumulative return of S&P $= \prod_{i=1}^{7}(1+r_i)-1$
  $= 22.73\%$

- S&P is at a 1.227 X purchase price at the end of 7 years. This less than the strike of 1.3 X purchase
  - Principal received = face amount of \$100 million
  - IRR = coupon yield because it was purchased at par, so IRR = 4%

ii) Scenario 2

Cumulative 7-year S&P return = 41.56%. Additional amount received at maturity
$= (1.4156 - 1.3) \times \$100 million$
$= \$11.56 million$
So IRR > 4%

- In addition to \$100 million face amount
  - 1.4156 = cumulative return
  - 1.3 = strike price
  - \$100 million = face

d)

- SCN maturity = 7 years remaining term to maturity on equity linked GICs $= 4.5 years$
- This makes the SCN inappropriate. Also the GICs are currently in the money. The SCN starts out of the money until it returns 30% — not appropriate

- Also the size is inappropriate, the GIC has $55 million in assets at 75% participation rate. The SCN has $100 million at 100% — not appropriate!!

Answer #6

a)

The answer will have the form $\frac{dF}{F} = udt + \sum_{i=1}^{3} \sigma_i dZ_i$

Where $u = R + \sum_{i=1}^{3} \lambda_i \sigma_i$ and $\lambda_i = (u - r) / R_i$

$\lambda_R = (.05 - .05)/.1 = 0$

$\lambda_S = (.1 - .05)/.2 = .25$

$\lambda_T = (.03 - .05)/.2 = -.1$

$So,\ \mu = .05 + .25\sigma_S - .1\sigma_T$

and, dz_1, dz_2, dz_3 have the same correlation as R, S and T

b)

1) Break the time to maturity, say H, into N times steps of $\Delta t = H / N$
2) Use the discrete formulas for R, S and T or the formula for F to model changes over time. For example: $R(t + \Delta t) - R(t) = R(t)(.05\Delta t + .1\varepsilon_1 \Delta t^{1/2})$
3) Need to draw a random sample (z_1, z_2, z_3) from a multivariate normal distribution and use Cholesky decomposition to get the correct correlation. Variance reduction techniques could be used in the process
4) Repeat steps 1 to 3 for the N time steps to determine the payoff of F at maturity
5) Discount the payoff at the risk free rates generated. This is one scenario
6) Repeat for a large number of scenarios and average the results
7) You can use standard error of estimates to determine a confidence interval for the results or to determine the number of scenarios

Answer #7

a)

The SOP plan has a two-year at-the-money call option embedded in it, since employees can buy the stock in two years for the price at issue. This option is European. The ESPP contains an embedded put option. If the price is below 80% of initial value, the employees get their money back (i.e., sell the sock for what they bought it for). This option is also European. Strike is 80% of purchase price

b)

Alternative hedges:

i) Naked and covered position: it's the worst strategy and the largest risk exposure

ii) Stop-loss hedges:
- Buy shares when stock price rises higher than strike price
- And sell share when stock price less than strike price. The cost is very high due to frequency trading

iii) Delta hedging;
- Calculate $\Delta = \frac{dc}{dS}$, for short a call and long Δ shares rebalance when Delta changes to maintain hedge

c)

Current stock price is 30. Therefore, ESPP options are in the money if price finishes below $80\%(30) = \$24$

Probability of exercise in risk-neutral would $= 1 - N(d_2) = N(-d_2)$

$$= 1 - N(.408) = \left[1 - \left(.6554 + .8(.6591 - .6554)\right)\right]$$

$$= 1 - .65796 = .34204$$

$$d_1 = \frac{\ln\left(\frac{S_0}{X}\right) + \left(r - q + \frac{\sigma^2}{2}\right)(T)}{\sigma\sqrt{T}}$$

$$d_1 = \frac{\ln\left(\frac{30}{24}\right) + \left(.05 - .03 + \frac{.402}{2}\right)(1)}{.40\sqrt{1}} = .80786$$

$$d_2 = d_1 - \sigma\sqrt{T} = .80786 - .40 = .408$$

d)

At-the-money call option (SOP) delta 1 options for year-end 2000 options

$$\Delta = e^{-qT} N(d_1)$$

$$d_1 = \frac{\ln\left(\frac{S_0}{X}\right) + \left(r - q + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$r = .05, q = .03, \sigma = .40, \mathrm{T} = 1, \times = 24, S_0 = 30$$

$$d_1 = \frac{\ln\left(\frac{30}{24}\right) + (.05 - .03 + .40^2/2)(1)}{.40\sqrt{1}} = .808$$

$$\Delta = e^{-.03(1)} N(.808) = e^{-.03}(.79042) = .767$$

Total delta for these options $= -10{,}000(.767) = -7{,}670.60$ since the company is short the options

Year-end 2001 options (SOP)

$$\Delta = e^{-qT} \mathrm{N}(d_1)$$

$$S_0 = 30, \quad X = 30, \quad r = .05, \quad q = .03, \quad \mathrm{T} = 2, \quad \sigma = .40$$

$$d_1 = \frac{\ln(1) + \left(.05 - .03 + \frac{.40^2}{2}\right)(2)}{.40\sqrt{2}} = .35$$

$$\Delta = e^{-.03(2)} \mathrm{N}(.35) = e^{-.06}(.6368) = .5997$$

Total delta for the options $= -15{,}000(.5997) = -8995.73$ since the company is short the options

ESPP options: one option =

$$\Delta = e^{qT}\left[N(d_1)-1\right]$$

$$d_1 = \frac{\ln\left(\frac{30}{24}\right)+\left(.05-.03+\frac{.40^2}{2}\right)(1)}{.40\sqrt{1}} = .808$$

$$N(d_1) = .79042$$

$$\Delta = e^{-.03(1)}\left[.79042-1\right] = -.203386$$

Δ of these options is = # options $\frac{360{,}000}{24} = 15{,}000$

$$\Delta = -15{,}000(-.203386) = 3{,}050.79$$

Since the company is short the options

Delta of whole portfolio:

$$= -7{,}670.60 - 8{,}995.73 + 3{,}050.79$$
$$= -13{,}615.54$$

e)

Year-end 2000 SOP options:

$$N(d_1) = N(.808) = .79042$$
$$N(d_2) = .65796$$
$$d_1 \frac{\ln\left(\frac{S_0}{X}\right)+\left(r-q+\frac{\sigma^2}{2}\right)(T)}{\sigma\sqrt{T}}$$
$$d_2 = d_1 - \sigma\sqrt{T}$$
$$= .808 - .40 = .408$$

$$c = S_0 e^{-q\tau} N(d_1) - X e^{-r\tau} N(d_2)$$
$$= 30e^{-.03(1)}(.79042) - 24e^{-.05(1)}(.65796)$$
$$= 7.99$$

Total value of these options = $10{,}000(7.99) = 79{,}909$

Year-end 2001 SOP options $d_1 = \dfrac{\ln\left(\dfrac{S_0}{\times}\right) + \left(r - q + \dfrac{\sigma^2}{2}\right)(T)}{\sigma\sqrt{T}}$

$d_1 = .35 \quad N(d_1) = .6368$

$d_2 = d_1 - \sigma\sqrt{T} = .35 - .40\sqrt{2} = -.22$

$N(d_2) = .4129$

$$c = Se^{-q\tau} N(d_1) - X e^{-r\tau} N(d_2)$$
$$= 30e^{-.03(2)}(.6368) - 30e^{-.05(2)}(.4129)$$
$$= 6.78$$

Total value of these options $= 15{,}000(6.78)$
$= 101{,}749$

For ESPP options:

$S = 30$
$X = .80(30) = 24$
$r = .05 \quad \sigma = .40$
$q = .03 \quad T = 1$

$$d_1 = \frac{\ln\left(\frac{S_0}{X}\right) + \left(r - q + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$d_2 = d_1 - \sigma\sqrt{T}$
$= .808 - .40\sqrt{1}$
$= .408$

$$d_1 = \frac{\ln\left(\frac{30}{24}\right) + \left(.05 - .03 + \frac{.40^2}{2}\right)(1)}{.40\sqrt{1}} = .808$$

$N(d_1) = N(.808) = .79042$ $\qquad$ $N(-d_1) = 1 - .79042 = .20958$

$N(d_2) = N(.408) = .65796$ $\qquad$ $N(-d2) = 1 - .65796 = .34204$

$$
\begin{aligned}
P &= Xe^{-rT}N(-d_2) - S_0e^{-qT}N(-d_1) \\
&= 24e^{-.05(1)}(.34204) - 30e^{-.03(1)}(.20958) \\
&= 1.707
\end{aligned}
$$

Total value of this group of options:

$$\#\ options = \frac{360{,}000}{24} = 15{,}000$$

$$Value = 15{,}000(1.707) = \$25{,}605$$

Total Value of options $= 79{,}909 + 101{,}749 + 25{,}605$
$= 207{,}263$

f)

- Gamma is a second-order measure that can be used to hedge large variations in value. It is the second derivative of portfolio value with respect to the asset price. It is also the rate of change of Δ with respect to the asset price:

$$\Gamma = \frac{d^2\pi}{dS^2} = \frac{d\Delta}{dS}$$

- Matching gammas corrects the error introduced by delta hedging, i.e., that changes are linear. Gamma is analogous to convexity in immunization

- It can be applied by matching gammas (i.e., making the portfolio gamma neutral) by maintaining a position of

$\frac{\Gamma}{\Gamma_T}$ in the traded option

Γ = portfolio gamma

Γ_T = gamma of traded option

After adjusting for gamma neutrality, must rebalance to make delta neutral

Answer #8

a)

- Need to use realistic, equilibrium model for asset adequacy testing
- Not practical if realistic and arb.-free
  - Term premium cannot be est. (confounding)
- HJM model is arb.-free
- HJM can not be realistic
- CIR model — equilibrium Model
- CIR can be a realistic model
- CIR is the only choice for realistic-equilibrium model
- Not pricing ⇒ arb.-free not needed ⇒ no need to calibrate exactly to current price
- Ease for application is a main criteria

b)

- Need to model assets and liability CFs, and
  - Payment function for MBS
  - Lapse and withdrawal for interest-sensitive annuities
  - Call behavior for callables
  - Default rates for bonds
  - All of the above are dependant on interest rate
- Parameters for pre-payments, defaults, policyholder behavior are usually estimated using regression on historical data

c)

- Interest rate model may be incorrect
  - Not have enough factors
  - Inappropriate for what is priced or current environment
- Model correct but data not good
- Model correct but solution is not
- Bugs in programming of model
- Data unstable

Answer #9

a)

i Rental income — if this is a high percentage of GDP this is a negative factor w.r.t. political stability — inhabitants don't need to be industrious or productive, just learn how to benefit from natural resources (as in the case of oil-rich countries)

ii Trauma — after sufficient time has passed, trauma leads to infrastructure investment and is a positive factor for political stability

iii GDP per capita is the most important factor — high per capita GDP leads to political stability and is of course an indicator of economic success already achieved — so GDP growth is an important measure for capacity

b)

- Highest: Country C has the strongest per capita GDP and the second lowest proportion of GDP from natural resources (rental income). Infant mortality is not far behind other countries (higher is negative as it implies inferior distribution of wealth) [proxy for distribution of income]
- Lowest: Country B has slightly better GDP than Country A but most of the difference comes from rental income (at least if oil reserves is only indicator) and inferior infant mortality rate
- Middle: Country A is ahead of B and behind C with best infant mortality rate and second lowest rental income/per capita GDP

c)

- Inflation — stability of inflation rate is positive factor
- Democracy — Is a legitimator of power as it represents choice of people. Other forms of government have potential to give rise to political unrest
- Competitiveness in international markets — ability to be economically viable if external growth in GDP (wealth)
- Human Capital — power if deployed in nation's economy
- Agriculture — like rental income, some agriculture is good, but too much is bad
- Distribution of Income — Wealth less effective if in the hands of few. Population should share benefits of GDP to be positive factor.
- Quality of life — similar to above, high quality for all = political stability

Answer #10

The exposure subject to first factor:

$$-3.5\times 0.44-1.4\times 0.44-2.8\times 0.44-2.5\times 0.44-0.7\times 0.44$$
$$=-4.796$$

The exposure subject to second factor:

$$-3.5\times(-0.8)-1.4\times(-0.25)-2.8\times 0.05-2.5\times 0.35-0.7\times 0.42$$
$$=1.841$$

The exposure subject to third factor:

$$-3.5\times 0.43-1.4\times(-0.69)-2.8\times(-0.22)-2.5\times 0.08-0.7\times 0.52$$
$$=-0.487$$

Let f_1, f_2, f_3 stand for principal components factor score

$$f=-4.796f_1+1.841f_2-0.487f_3$$

As they are independent from each other:

$$\sigma^2 f=\left((-4.796)^2\times 17^2+(1.841)^2\times 6^2+(-0.487)^2 3^2\right)$$
$$=6771.62$$
$$\sigma f=82.29$$
$$VAR=2.33\times 82.3=191.74$$

So 1 month 99% VAR is 191.74

Answer #11

a)

i)

- Increased refinancing
- Premium leads to higher
- Recent leads to higher

ii)

- Increased refinancing
- Discount leads to lower
- Seasoned leads to lower
- Difference depends on size of premium / discount
- Mortgages priced off shorter portion of yield curve often used to refinance longer priced mortgages

b)

- Prepayment increases cash inflows from mortgage investments
- Reinvestment only available at lower interest rates
- Gic's presumably based on interest at higher than current rates
- No incentive for Gic holders to surrender - so no increased cash out flows to liabilities
- Liabilities may grow longer
- Negative impact on income (asset returns - Gic interest)
- Less prepayment from vintaged discount pool
- More prepayment from recent premium pool
- Prepayments from discount pool at par = gains greater than expected
- Prepayments from premium pool at par = losses, greater than expected

c)

- Collateral
- Effective Collars
- Priorities
- Effective Duration
- Interest rate sensitivity

d)

- Cannot judge by collars on issue - need to calculate effective collars
- premiums more likely to refinance than discount.
- but given past interest rate history, burnt out.
- discounted more likely to constitute sophisticated borrowers, so likely to refinance at first occasion
- both have 5 years to run
- PAC offer better security against call and extension risk than pass through

older PAC has greater security than younger, all things being equal.

Answer #12

a)

Accounting Exposure:

- Transaction Exposure: the exposure arising from the changes in foreign currency and commodity prices that adversely impact the expenses and the revenue to which the firm has already committed
- Translation: change in foreign currency that will adversely impact the assets and liabilities in the local currency of the parent
- Future Exposure/Contingent Exposure: exposure due to changes of foreign currency, commodity prices (and interest rates) that will adversely impact the expenses and revenues to project that the firm is expected to do, but not yet booked
- Competitive Exposure: exposure due to change in the risk factors that will adversely impact the firm regarding its market share of sales and ultimately its earnings (net income)
- Strategic Exposure: is the sum of these exposures

b)

- Yes, the company should manage because risk management policies will have an impact on the cash flow of the firm. Main reasons:
  - Non-linearity of taxes
  - Taxes are non-linear because:
    - AMT
    - Losses that cannot be deducted because of lack of taxable income
  - Taxes normally have a convex structure
  - Taxes will affect the more the tax schedule is convex and the more the earnings are volatile
  - Taxes are not proportional to income
- Cost of financial distress: there are many cost associated with financial distress
  - Lack of confidence of the employees and other parties
  - Management diverts their attention from action that positively impacts the value of the firm
  - Cost associated with bankruptcy
  - Doubling the losses can more than double the cost of financial distress (because of non-explicit cost of financial distress)
- Improved financial and investment decisions
  - Can help to use more debt (may be beneficial)
  - Decrease the conflict of interest between the bondholder and the stockholder
  - Do not reject project with positive NPV (under-investment problem)

- Actions that ↑ cash flow volatility will transfer wealth from bondholder to stockholder

- Managerial Self-Interest: the managers are assumed to have a large part of their wealth associated to the success of the firm:
  - May have stock options
  - If company fails may lose job
  - Market perception of the ability of the managers to manage a company
  - Managers are assumed to have a concave objective function
  - The managers cannot diversify as the investors can do
- Capital Market Imperfections: must minimize earnings volatility

c)

- Losses due to human errors, management failure and system inadequacy
- Can be hedged or mitigated by:
  - Having an independent risk management unit that reports to the senior management
  - Independent audit to verify adherence to policies
  - Policies, standards, control and check
  - Good system

d)

- Risk that the contract cannot be enforced with given counter parties
- May be due to: lack of documentation, lack of regulation, insolvency, other party cannot contract, series of contract are considered illegal
- Netting and insolvency: netting is very good way to reduce risk with counter party, must verify when it applies
- Unenforceability and illegality
  - Parties may not be able to contract (ultra vires)

Answer #13

a)

No income means that in the no-default world this is a European call

$$c = P(0,T)[F_0 N(d_1) - XN(d_2)]$$

P(0,T)	Value at time 0 of the zero-coupon maturing at time T	=1
F_0	Value of the forward price at time 0	=1000
X	Strike price	=1000
T	Time to option Maturity	=1

$$d_1 = \frac{\ln(\frac{F_0}{X}) + \frac{\sigma^2 T}{2}}{\sigma\sqrt{T}} = \frac{\sigma\sqrt{T}}{2} = 0.1$$

$$d_2 = d_1 - \sigma\sqrt{T} = -0.1$$

c=1000(N(0.1)-N(-0.1))=1000(0.5398-0.4602)=79.6

b)

- American calls are worth more than European calls in the world with defaults because if the writer shows distress signs call may be exercised, hence

$$C_{American} \geq C_{European}$$

- This is an asset on your balance sheet, so let
  - f is the European call value with default
  - f* is the default free European call value
  - y(T) is the yield on the zero coupon bond P(T) issued by the option writer
  - y*(T) is the yield on the risk-free zero coupon bond P*(T). We get:

$$h(0,T) = \frac{f^* - f}{f^*} = \frac{P^*(T) - P(T)}{P^*(T)} = \frac{e^{-y^*(T)T} - e^{-y(T)T}}{e^{-y^*(T)T}}$$

- Because the proportional loss is the same for the bond and the derivative given the independence of default occurrence from other variables. Hence.

$$f = f^* \times e^{[y^*(T) - y(T)]T}$$

- Is the required lower bound, and:

$$C_{American} \geq f = f^* \times e^{[y^*(T) - y(T)]T} = 79.6 \times e^{-0.015} = 78.4$$

c)

- We need to hedge interest rate risk and the risk of default. Hence use Risk free zero-coupons
- Replicate option payoffs
- Solve for the necessary positions in each of zero-coupons

Answer #14

a)

Analogy between the call option and real option:

Call Option On Stock:	Real Option on Project:
- Current stock price	- Expected future cash flow
- Time to maturity	- Time until opportunity
- Exercise price	- Investment cost
- Stock price volatility	- Expected cash flow volatility
- Risk-free rate	- Risk-free rate

But this analogy is deficient as:

1. Unexclusiveness of the ownership and competitive interaction
   a. Traditional option exercise the option, have the exclusiveness of the option
   b. Real option exercise non-proprietary option will invoke competition respond by reducing the value of the underlying asset
2. Tradability and preemption
   a. Generally most real options are not tradable or are at high prices
   b. Early exercise motivated for the intent of preemption
3. Across time interaction and compoundness
   a. Generally real options are compound (option on an option)

b)

Exclusiveness of ownership and competition

1. Most real options can be replicated by competitors if it is a lower barrier to entry. Under these circumstances, the option is shared with the market and may lead to a competitive loss. Exclusiveness of the ownership can be characterized as proprietary or shared
2. Inter/Intra Project: most projects are compound options; stand alone investments are simple options
3. Urgency of Decision: Investment can be made now or deferrable

c)

- Proprietary — simple — expiring
- Shared — simple — expiring
- Proprietary — simple — deferrable
- Shared — simple — deferrable

d)

- DCF/DTA: Ignores or cannot properly capture management's flexibility to adopt decisions. Unable to capture the value of operating options properly
- CCA: Can quantify the flexibility of an option. Combines the best features of DTA and NPV without the drawbacks

e)

$V_0 = 50 \qquad V_1^+ 50 \times 1.7 = 85$

$I_0 = 52 \qquad V_1^- 50 \times .8 = 40$

$$Z^+ = \max\left(V_1^+ - I_1, 0\right)$$
$$= \max(85 - 52 \times 1.1, 0) = 27.8$$
$$Z^- = \max\left(V_1^- - I_1, 0\right)$$
$$= \max(40 - 52 \times 1.1, 0) = 0$$

$$Z_0 = \frac{.5 \times 27.8 + .5 \times 0}{1.825} = 11.12$$

DCF/DTA option $= 11.12 - (50 - 52) = 13.12$

$$CCA - p = \frac{1 + r - d}{u - d} = \frac{1}{3}$$

$$Z_0 = \frac{27.8 \times \frac{1}{3} + 0 \times \frac{2}{3}}{1 + 0.1} = 8.42$$

$$Option \quad = 8.42 - (50 - 52) = 10.42$$

f)

$S_0 = .1 \times 50 = 5$

$$N = \frac{27.8 - 0}{5 \times 1.7 - 5 \times .8} = 11.8 \ \ million\ shares\ with\ \ S_0 = 5$$

Total Cost is 30.8

Borrowing remainder at risk free rate $B = 22.48$

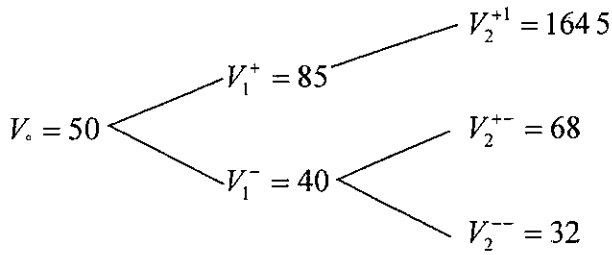
$E^+ = 6.18 \times 5 \times 1.7 - B \times 1.1 = 27.8$

$E^- = 6.18 \times 5 \times .8 - B \times 1.1 = 0$

So if the holder can pay 8.42 to get it why pay 11.12

So the CCA gives the correct value

g)



$D_2 = -25 \times (1.14)^2 = 32.69$

$I_0^D = 25 \quad I_0^E = 27$

$$Z_2^{++} = \max(V_2^{++} - D_2, 0) = 112.01$$

$$Z_2^{+-} = \max(V_2^{+-} - D_2, 0) = 35.51$$

$$Z_2^{--} = \max(V_2^{--} - D_2, 0) = 0$$

$$Z_1^{+} = \frac{PZ_2^{++} + (1-p)Z_2^{+}}{1+r} = \frac{112.1 \times \frac{1}{3} + 35.51 \times \frac{2}{3}}{1.1} = 55.66$$

$$Z_1^{-} = \frac{PZ_2^{+-} + (1-p)Z_2^{--}}{1+r} = \frac{35.51 \times \frac{1}{3} + 0 \times \frac{2}{3}}{1.1} = 10.76$$

$$Z_0 = \frac{PZ_1^{+} + (1-p)Z_1^{-}}{1+r} - Z_E = \frac{55.46 \times \frac{1}{3} + 10.76 \times \frac{2}{3}}{1.1} - 27 = -3.67$$

$-3.67 - (50 - 52) = -1.67$ as the flexible value of the option

Answer #15

a)

- An investor's utility is not based on expected return/expected variance
- An investor may compare performance against a benchmark and may feel uncomfortable with large deviation
- An investor may lack of confidence in the assumptions, thus doubt on the results
- MV optimizer tends to give extreme allocation results
- Results are very sensitive to the assumptions; small change in assumptions could lead to large change in allocation

b)

MVTE utility fraction for portfolio P

$$U(P) = \text{Exp Ret}(P) - \frac{(ExpRisk(P))^2}{rt} - \frac{(EspTe(P))^2}{tet}$$

Let w be the weight allocated to bonds

$$\text{Exp Ret}(P) = wx0.06 + (1-w) \times 0.07$$

$$= 0.07 - 0.01\,w$$

$$(ExpRisk(P))^2 = (0.05w)^2 + 2w(1-w)(0.8)(0.05)(0.1) + (0.1(1-w))^2$$

$$= \frac{45w^2 - 120w + 100}{10{,}000}$$

$$(ExpTe(P))^2 = (0.07 - 0.01w - 0.4 \times 0.06 - 0.6 \times 0.07)^2$$

$$= \frac{(0.4-w)^2}{10{,}000}$$

Hence

$$u(P) = 0.07 - 0.01w - \frac{45w2 - 120w + 100}{10{,}000 \times rt} - \frac{(0.4-w)^2}{10{,}000 \times tet}$$

Take derivative, and set to zero

$$\frac{du(P)}{dw} = -0.01 - \frac{90w - 120}{10{,}000 \times rt} - \frac{2(0.04 - w)(-1)}{10{,}000 \times tet}$$

$$= \frac{1}{10{,}000}\left[-\left(\frac{90}{rt} + \frac{2}{tet}\right)w - 100\frac{120}{rt} + \frac{0.8}{tet}\right]$$

$\frac{du^2(P)}{dw_2} < 0 \rightarrow$ maximum value when $\frac{du(P)}{dw} = 0$

$$\frac{du(P)}{dw} = 0 \rightarrow w = \frac{\left(\frac{120}{rt} + \frac{0.8}{tet} - 100\right)}{\left(\frac{90}{rt} + \frac{2}{tet}\right)}$$

Substitute $rt = 0.75$ $tet = 0.02$ to get

$$w = \frac{100}{220} = 0.4545$$

c)

i) As $tet = \infty, w \rightarrow \frac{60}{120} = 0.5$

Mean-Variance optimizer: 50% bonds, 50% equities

ii) As $tet \rightarrow 0$, no tracking error is tolerated. We obtain the target portfolio

$w = 0.4$ 40% bonds, 60% equities

Answer #16

a)

Five Steps:

1. Estimate the firm's cost of capital
   a. Weighted average cost of debt and cost of equity capital
   b. Assumptions from marketplace
2. Estimate the return on firm's asset portfolio
   a. Need to consider defaults
   b. Expressed as premium over risk-free rate
3. Assess degree of financial leverage

$$= DDE / \left[\left(1 - \mathrm{T} \times F_{\cdot}^{fvl^*}\right) \times FVL_{\cdot}^{*}\right]$$

   a. DDE = discounted distributable earnings
   b. FVL* = fair value of liability without tax
   c. F^{FVL^*} = proposition of liability that's taxable
4. Assess impact of non-investment- related risks
   a. Estimate using RBL on invested and non-invested assets
5. Estimate policyholders risk-sharing burden
   a. Transfer of risk through experience rating, dividends, etc.

(b)

t	Assets	Liability	Liability Cash flow
0	1080	1000	0
1	1080	1000	42
2	1080	1000	42
3	0	0	1042

$$FVL_t^* = \frac{FVL^*_{t-1} + L(t) + L(t)}{1 + r - \theta^D}$$

$$FVL_3^* = 0$$

$$FVL_2^* = \frac{0 + 1042}{1 + .03 + .015} = 997.13$$

$$FVL_1^* = \frac{997.13 + 42}{1.045} = 994.38$$

$$FVL_0^* = \frac{994.38 + 42}{1.045} = 991.75$$

Fair Value of Taxes $= \left(PVI^{MVA} - PVI^{FVL^*}\right)\frac{T}{1-T}$

$FVL = FVL^* + (FV\ taxes)$

$PVI_{t-1}^{MVA} = \frac{PVI_t^{MVA} + (r+\theta^D)TVA_{t-1}}{1+r+\theta^D}$ since no risk premium, $\theta^D = 0$

$PVI_2^{MVA} = \frac{0+(.03)(1080)}{1.03} = 31.46$

$PVI_1^{MVA} = \frac{31.46+.03(1080)}{1.03} = 62.00$

$PVI_0^{MVA} = \frac{62.00+.03(1080)}{1.03} = 91.65$

$PVI_{t-1}^{FVL^*} = \frac{PVI_t^{FVL+} + (r+\theta^D)(TVL_{t-1})}{1+r+\theta^D}\theta^D = 0$

$PVI_2^{FVL+} = \frac{0+.03(1000)}{1.03} = 29.13$

$PVI_1^{FVL+} = \frac{29.13+.03(1000)}{1.03} = 57.41$

$PVI_0^{FVL^*} = \frac{57.41+.03(1000)}{1.03} = 84.86$

t	FVL*	PVI^{MVA}	PVI^{FVL^*}	FV Taxes	FVL
0	991.75	91.65	84.86	3.66	$995.41
1	994.38	62.00	57.41	2.47	$996.85
2	997.13	31.46	29.13	1.25	$998.38
3	0	0	0	0	$0

Question #17

a)

- As interest rate increase the Market Value of the portfolio will decrease. The insurer needs to set competitive rates but also cover their expenses. Interest rate cap hedge interest rate increases and generates cash flows for adverse situation. Caps are based on CMT and CMS rates, they are cheaper, and track the new-money rates offered on competing annuities

b)

- Used if the primary concern is that flattening of the yield curve will cause corporate and swaps spreads to widen. The insurer select the desired term (tenor) of the swap, and the dealer sets the lock at the end of the term at a certain spread. If the spread is wider that the lock, the dealer pays the insurer, if the spread is narrower the insurer pays to the dealer. The amount owed is Nominal Amount* (spread-lock)* duration of the swap. As the portfolio has a large percentage of bonds a spread lock would be an appropriate strategy

c)

- If the interest rate increases a yield curve swap will help by shortening the duration. The insurer pays fixed and receives floating. The fixed rate is based on CMT or CMS rates of maturities of 5, 7,or 10 years. It allows the insurer to track the new money rates

d)

- Interest rate floors hedge the risk of falling interest rates. As there is MBS in the portfolio we also need to hedge the risk of prepayment in extreme rally of interest rate. It also hedges the risk of falling below the minimum guarantee rate of 4%. The floor pays if the yield is less than the strike. Long assets can be replaced with short assets + floors

e)

A prepayment cap hedges the risk of falling rates. It pays an amount designed to meet the hedging need at each level of prepayment. It is like synthetically selling the MBS to the dealer. The investor can choose products driven by interest rate or driven by experience in the MBS market. The decision depends on the investor's view whether the option is overvalued or undervalued compare to pure interest rate option

Answer #17

a)

- Cash flows are dependent on the interest rate path that is experienced
- Book value redemption/surrender, i.e., prepayment/surrender options where the cost of prepayment or surrender is not impacted by the current level of interest rates
- Interest rate options next exercised with full economic efficiency

b)

- No interest rate sensitive surrenders, i.e., no policyholder options
- Fixed credited rated, i.e., no insurance company options

c)

i Increase RSA with little or no impact on OAD and MTL
ii Little or no impact on RSA and OAD and reduce MTL
iii Increase RSA and reduce OAD and MTL
iv Increase RSA and reduce OAD and MTL

****END OF EXAMINATION****