## \*\* BEGINNING OF EXAMINATION \*\* MORNING SESSION

**1.** (6 points) LifeCo's CEO has asked you to analyze the profitability of the Institutional Pensions – GIC line of business and the drivers of its profitability.

Asset yields are effective annual, earned, net of investment expenses.	
The average industry required return on capital:	12.0%
The Lehman aggregate bond index, 1999:	6.7%
The Lehman aggregate bond index, 1999, adjusted to duration of 3.3:	6.4%
The Lehman aggregate bond index, 1999, adjusted to duration of 3.1:	6.3%

- (a) List the benefits of an integrated asset/liability performance attribution process.
- (b) Explain the two benchmarks you will use to construct a performance attribution system using the above indices.
- (c) Define each of your performance attribution components and explain their function.
- (d) Calculate the return of each performance attribution component using the summary information from the balance sheet for this line of business.

- 2. (11 points) LifeCo is considering purchasing a P&C company. You are a consulting actuary that has been retained by LifeCo to help analyze the implications of such an acquisition on LifeCo's ALM process.
  - (a) Explain how ALM can improve decision-making and enhance the ability to view LifeCo's businesses from multiple perspectives.
  - (b) Describe the main components of a DFA as typically used by a P&C insurer.
  - (c) Compare LifeCo's ALM process to DFA.
  - (d) Detail the tasks that LifeCo management must perform in implementing a new ALM process.
  - (e) Recommend the best practices that LifeCo should consider adopting in its new ALM process.

**3.** (5 points) You are in charge of setting an investment strategy for the surplus account of LifeCo. The primary investment objective of the surplus account is to safeguard principal while seeking to maximize the total rate of return over time. One of the tasks is to determine the asset allocation among various asset classes.

As a member of LifeCo's Portfolio Rebalance Subcommittee you have just completed a full evaluation of capital market information in terms of expected returns, standard deviations and correlations among asset classes. Next, you want to decide on a risk measure to be used to construct an efficient portfolio.

One of the Subcommittee members has suggested selecting portfolio standard deviation (as used by Harry Markowitz in his quadratic optimization) as the risk measure.

- (a) Evaluate the appropriateness of using standard deviation as the risk measure for the surplus portfolio.
- (b) Describe and compare two other alternatives to using standard deviation as the risk measure, and evaluate how the use of each would impact the financial objectives of LifeCo.
- (c) Propose the most suitable risk measure for LifeCo and justify your choice.
- (d) Formulate a mathematical optimization program to construct an efficient portfolio using the risk measure chosen in (c).

- **4.** (*10 points*) The management of LifeCo is concerned about the duration mismatch reported in the December 31, 1999 ALM report for its traditional and non-traditional life products segments. It has directed the Corporate Actuarial Department and the Investment Department to investigate the mismatch and to evaluate the possibility of using Z-bonds to reduce it.
  - (a) Criticize the use of modified duration as a measure of interest rate sensitivity for the two liability segments and the underlying invested assets.
  - (b) Compare the following interest rate sensitivity measures with modified duration:
    - (i) effective duration
    - (ii) effective key-rate duration
  - (c) For equities:
    - (i) interpret the reported modified duration
    - (ii) describe how the Franchise Factor Model can be used to model the interest rate sensitivity of equities
  - (d) Compare the payment profile of the following types of accrual bonds:
    - (i) Z-bonds
    - (ii) Z-PAC
    - (iii) Tricky Z
    - (iv) Jump-Z with cumulative sticky trigger
  - (e) Rank the accrual bonds presented in (d) according to their suitability to help reduce the duration mismatch of the traditional life product segment. Justify your answer.

**5.** (*5 points*) You are a research analyst supporting the corporate bond desk at LifeCo and have been asked to consider investing in synthetic convertible notes (SCN) as a new asset class for the Equity Linked GIC portfolio.

You are given the following information for an SCN:

Face amount:	\$100 million
Maturity:	7 years
Coupon:	4% paid annually
Strike:	$1.3 \times \text{purchase price}$
Purchase price:	par
Index:	S&P 500
At-the-market swap rate on LIBOR:	7.5%

- (a) List and briefly describe the 3 factors that impact price volatility and performance sensitivity of structured notes.
- (b) Calculate the up-front payment the swap dealer receives on the hedge.
- (c) Calculate the annual internal rate of return (IRR) over the term of the SCN for the following 2 scenarios of annual returns of the S&P 500 index:

Scenario	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7
1	-5%	-7%	12%	0%	16%	8%	-1%
2	8%	15%	3%	9%	-6%	20%	-10%

(d) Analyze the suitability of this SCN for the asset portfolio backing LifeCo's Equity Linked GICs.

**6.** (*7 points*) A risk-free investment, R, and investments S and T have the following Ito processes:

$$dR = 0.05Rdt + 0.10Rdz_1$$
$$dS = 0.10Sdt + 0.20Sdz_2$$
$$dT = 0.03Tdt + 0.20Tdz_3$$

The correlation matrix for R, S and T is known.

The payoff of a European derivative, F, is determined from the values of R, S and T on F's exercise date.

- (a) Formulate an expression for the process followed by F using constants where possible.
- (b) Describe the steps required to value F using a Monte Carlo simulation.

**7.** (*11 points*) A regulatory proposal would require companies to disclose the value of options granted to their employees. Your CFO is concerned that under this proposed regulation, changes in your firm's stock price will create earnings volatility, and has identified two options programs which would impact earnings under this proposal.

	Price at purchase	Purchasing rights
Stock Option	market price that	employees are given the right to purchase
Plan (SOP)	prevailed when the option was granted	shares of the firm's stock at the end of two years
Employee Stock	80% of the market	stock is delivered to employees at the end of
Purchase Plan	price at the beginning	the year if the stock price is above the
(ESPP)	of the year	discounted level at that time, however, if the
		price is below this level, the employee's
		money is returned and no stock is delivered

- The options granted under the SOP from year-end 1999 have expired.
- At year-end 2000, options on 10,000 shares were granted under the SOP when the stock price was \$24.
- At year-end 2001, options on 15,000 shares were granted under the SOP when the stock price was \$30.
- Under the ESPP, employees have subscribed to purchase \$360,000 worth of stock in 2002.

It is January 1, 2002. The firm pays a continuous annual dividend of 3.0%, the stock's annual volatility is 40%, and the risk free rate is 5.0%. There are no exchange-traded options available on this stock.

- (a) List the options embedded in these two programs.
- (b) Propose alternatives to hedge against changes in the stock price.
- (c) Determine the probability that the options granted under the ESPP will finish inthe-money.
- (d) Calculate the delta of the portfolio of options under the SOP and ESPP programs and interpret the sign of the delta.
- (e) Determine the total value of the options granted under these two programs.
- (f) Select a second order measure that can be used to hedge variations in value and describe how it would be applied.

- 8. (5 points) You have been asked to evaluate the interest rate generator that your company uses for scenario testing. The current model is the CIR model and you are considering building the HJM model. The model would be used to simulate results for a new portfolio that includes both callable and non-callable corporate bonds, and mortgage-backed securities on the asset side and interest-sensitive annuities on the liability side. The actuaries need to test the adequacy of your asset-liability matching. They will do so by generating stochastic interest rates to ensure that assets are sufficient to cover liabilities under each stochastic interest rate scenario.
  - (a) Evaluate which interest rate generator would be most appropriate.
  - (b) Assess the key aspects of the assets and liabilities needed to be modeled, given an interest rate generator.
  - (c) Highlight the points where you should pay more attention to avoid model risk, given the scope of the work.

## \*\*END OF EXAMINATION\*\* MORNING SESSION

#### \*\*BEGINNING OF EXAMINATION\*\* AFTERNOON SESSION Beginning With Question 9

**9.** (*3 points*) As a research analyst, you have been asked to investigate the political risk factors that arise when investing in international equities.

Using only the model developed by Marvin Zonis:

- (a) Describe how the following factors affect political stability and the capacity for economic success:
- (i) rental income
- (ii) trauma
- (iii) Gross Domestic Product (GDP) per capita.
- (b) Rank the following countries in terms of political stability and the capacity for economic success from highest to lowest. Justify your answer.

Factor	Infant Mortality (per 1000)	GDP (per capita)	Oil Revenues (billions)
Country A	1.0	750	70
Country B	1.1	800	100
Country C	1.2	1200	80

(c) Describe the other factors identified in the model developed by Marvin Zonis.

**10.** (*3 points*) You are given the following information for a bond portfolio with a current market value of \$5 billion.

1-Year Rate	2-Year Rate	3-Year Rate	4-Year Rate	5-Year Rate
-\$3.5 million	-\$1.4 million	-\$2.8 million	-\$2.5 million	-\$0.7 million

Change in Market Value for a 1 Basis Point Rate Increase

Factor Loadings for U.S. Treasury Principal Components (PC)

	1-Year	2-Year	3-Year	4-Year	5-Year
PC 1	0.44	0.44	0.44	0.44	0.44
PC 2	-0.80	-0.25	0.05	0.35	0.42
PC 3	0.43	-0.69	-0.22	0.08	0.52

#### 1-Month Standard Deviation of Factor Scores

PC 1	PC 2	PC 3
17 basis points	6 basis points	3 basis points

Historical data has shown that factor scores for each principal component are independent from each other.

Calculate the 1-month 99% VaR using "Principal Components Analysis" and the above data.

- **11.** (5 points) Your life insurance company client currently uses passthrough mortgage securities to back its GIC portfolio. There are two main types of mortgage securities available:
  - (i) recently issued premium coupon mortgages
  - (ii) seasoned discount coupon mortgages

You have been asked to evaluate the use of PACs as an alternative investment strategy.

Your client has been offered:

- 10 year PACs issued in 1997 based on 8% mortgages with 85% to 300% PSA collars
- 5 year PACs issued in 2002 based on 6% mortgages with 85% to 300% PSA collars
- (a) Assess the likely impact of a decrease in short term interest rates of 100 basis points on prepayment rates for each of the two types of passthrough mortgage securities.
- (b) Predict the likely impact of the prepayment rate change described in (a) on your client's cash flows and net income.
- (c) List and explain the items you would consider in evaluating the suitability of any given PAC.
- (d) Recommend the PAC that will provide the better match for a 5 year GIC issued in 2002.

- **12.** (*7 points*) A multinational non-financial company is exposed to foreign currency and commodity price risks. You have been asked by the company to help them understand and manage these risks.
  - (a) Describe the different kinds of exposure faced by a non-financial company using an example of a negative impact for each one.
  - (b) Recommend whether or not the company should manage these strategic risks.
  - (c) Describe the following risks associated with the use of derivatives and propose ways to mitigate them:
    - (i) operational risk
    - (ii) legal risk
- **13.** (8 *points*) You are buying an over-the-counter one year American call option on a zero-coupon, default risk-free, bond with a one year maturity.

You are given:

- the risk free interest rate is 0% at every duration
- the default risk premium on a one year zero-coupon bond issued by the option writer that ranks equal to the option in the event of default is 1.5%
- the strike price and the forward value are \$1000
- the volatility is 20%
- (a) (2 points) Calculate the price of this option in the no-default world using Black's model.
- (b) (4 points) Demonstrate how the answer in (a), appropriately adjusted for the presence of risk of default by the option writer, can be used as the lower bound for the price of this option. Calculate this lower bound.
- (c) (2 points) Explain how the over-the-counter dealer can create a replicating portfolio to delta-hedge a similar European call option on this bond against movements in interest rates and credit rating.

**14.** (*13 points*) You are an investment actuary in charge of the derivatives trading and research department for an investment banking firm. A Wall Street venture capitalist has approached you regarding a potential acquisition of an industrial company.

You have been asked to study the following acquisition project and explain the use of contingent-claims analysis in capital budgeting:

- After one year the project is assumed to generate expected values of subsequent cash flows which follow a multiplicative binomial process.
- For each annual period, the value either increases to u = 1.7 or decreases to d = 0.8 of its earlier value.
- The gross value of the project's expected cash flows is currently valued at \$50 million.
- The probability that governs state transitions is equal to 0.5.
- The buyer is considering the opportunity to invest a required immediate outlay of \$52 million (all equity).

Use the following assumptions:

- A twin security for this transaction exists in the financial markets, and its payoffs are 10% of the gross project values.
- The expected rate of return (or risk-adjusted discount rate) is 25% per annum.
- The risk-free rate is 10% per annum.
- (a) Explain how real options on projects and call options on stocks are analogous and record the major areas where this analogy is deficient.
- (b) Describe the key strategic questions that the management of the acquiring company must address when evaluating an investment.
- (c) Review the possible option classifications and the different components of the option value to help management recognize and understand the potential combinations of real options available in an investment opportunity.
- (d) Compare and contrast the use of the traditional (passive) discounted cash flow / decision-tree analysis (DCF/DTA) approach versus the options-based approach of contingent-claims analysis to valuing real investments when operating and financial flexibility options are present.

# 14. Continued

(e) Assume the firm has a one-year deferral agreement granting it the exclusive right, but not the obligation, to make the investment by next year if the project value next year turns out to exceed the necessary investment at that time.

Determine the value of the option to wait provided by the agreement using both the contingent-analysis and traditional DCF/DTA approaches.

- (f) Select the approach in (e) that produces the correct value for this investment opportunity. Justify your choice by describing how the firm would arrange for the least amount of funds needed to purchase the project.
- (g) Assume that the firm has the option to finance \$25 million out of the required investment outlay of \$52 million by borrowing it against the project's expected future cash flows. The amount is to be repaid with interest in two years at an annual interest rate of 14%. The balance of \$27 million is to be supplied today by the firm's equity holders.

Determine the value of the financial flexibility provided in this debt financing arrangement.

**15.** (*4 points*) A pension fund has an asset allocation target of 40% bonds and 60% equities. You are given the following assumptions:

	Expected return	Standard deviation
Bonds	6%	5%
Equities	7%	10%

Return correlation	= 0.80
Risk tolerance	= 0.75
Tracking-error tolerance	= 0.02

- (a) Explain why the optimal asset allocation based on the mean-variance (MV) optimizer is often different from the real world asset allocation.
- (b) Calculate the optimal asset allocation using the mean-variance/trackingerror (MVTE) utility function.
- (c) Predict the results if the tracking-error tolerance goes to:
  - (i) infinity
  - (ii) zero

- **16.** (*7 points*) You are given the following information for a GIC-type product issued by a life insurance company.
  - The corporate tax rate is 35%.
  - The current estimate of the company's credit risk premium is 1.5%.

Liability information:

Initial deposit	\$1000
Maturity date	3 years
Credited rate	4.2%
Interest payments	paid annually
Valuation interest rate	equal to credited rate

Asset information:

Allocated capital	8.0% of statutory reserves
Risk-free rate	3.0%
Expected asset portfolio yield, before expected defaults	6.3%

Assume no expenses and that the tax adjustment is calculated without risk premiums.

- (a) (2 points) Describe the steps used in determining the company's credit risk premium to be used with the direct method of performing a fair valuation of liabilities. For each step, indicate the basis for determining appropriate assumptions.
- (b) (5 points) Determine the fair value of the liability at time 0. Use the direct method.

**17.** (6 points) Gedda Life's SPDA product has a surrender charge that starts at 6% and declines 1% per year for 6 years. The minimum guaranteed credited rate is 4%. The SPDA block is currently supported by the following portfolio of assets, with maturities ranging from 2 to 10 years:

10% government bonds50% corporate bonds and private placements15% mortgage-backed securities25% commercial mortgages

Explain how each of the following derivatives can be used as part of the ALM strategy for this line and describe the risks being hedged.

- (a) Interest Rate Cap
- (b) Spread Lock
- (c) Yield Curve Swap
- (d) Interest Rate Floor
- (e) Prepayment Cap

- **18.** (*4 points*) Your company's CFO is considering pricing interest-sensitive insurance liabilities using the required spread on assets (RSA) generated by simulating interest rate paths from an arbitrage-free stochastic interest rate model.
  - (a) Compare the key characteristics of MBS securities and insurance company SPDA contracts that support the use of similar pricing and valuation methodologies.
  - (b) Explain under what circumstances the option adjusted duration (OAD) must equal the mean term of liabilities (MTL) for an SPDA product.
  - (c) Predict the impact of each of the following changes taken separately on the RSA, OAD and MTL of a typical SPDA product.
    - (i) Increasing the credited rate at issue and reset by 50 basis points.
    - (ii) Increasing the base level of surrenders that are not related to the level of interest rates.
    - (iii) Increasing the level of surrenders that are related to the level of interest rates.
    - (iv) Increasing the rate credited by all competing insurance companies by 50 basis points.

## **\*\*END OF EXAMINATION\*\***