Question 1

a)

Optimal hedge rates (% of currency hedges) in general depends on:

- % of asset portfolio that's invested in foreign assets
- Potential hedging costs which include:
  - contract trading cost
    - bid ask spreads on forwards
    - transaction costs
  - custody costs
  - custody fees

  In total these costs range in 20-50 bp range. These costs reduce expected return directly on hedges portfolio.
- Risk reduction benefits must be higher than hedging costs
- General level of risk aversion for LifeCo.

  This is the utility function = $S(R) - \frac{\sigma^2}{RT}$

  Where RT is risk tolerance level.
- The correlations and volatilities of different currency exposure, for LifeCo. For example, if source currency exposures have negative correlations, they don't need to be hedged, because they offset each other.

- Based on net foreign CR and volatility/correlation projection
- Depends on type of investors
- Depends on consumption mix of foreign products
- Foreign exchange exposure can be inflation hedge for imported goods
- Consider impact on strategic plan

Depends on forecast of foreign exchange rates

b)

The objective of LifeCo is to eliminate currency risk. This is consistent with LifeCo objective of delivering stable earnings. Currently they use currency swaps and forwards to hedge currency risk. This is consistent with Full Hedging Approach, where certain exposure is hedged using forwards, swaps. The problems with this approach are that:

- Because under/over hedged when foreign assets appreciate/depreciate, need frequent hedge adjustments.
- Only hedges currency movement. Ignores foreign asset impairment in context of total portfolio

Alternatives are:

- <u>Minimum Variance Hedging</u>
  where asset class volatilities and correlations are integrated with currency volatilities and correlations to derive a minimum risk portfolio per a given level of expected return. This would be consistent with minimizing total earnings volatility.

- Downside-Option Based Hedging
  useful to truncate downside exposure but still retain upside potential , 3 alternatives:
  - Buy puts on individual currencies
  - Buy puts on basket of currencies
  - Buy puts on base $ currency value of foreign exposure.

  Second approach is probably cheaper because it accounts for offsetting correlations of currencies.
- Can also create synthetic put positions, via Delta hedging, where forward contracts are traded.
- No hedging
- Set up foreign subsidiaries to improve foreign currency revenue/expense mismatch
- Downside/Option based approach retains upside potential but at cost = option premium

Could use semi-variance to reflect downside risk aversion

c)

Historical Simulation

- Simulation based on historical currency movements applied to current portfolio.
  - How long is historical period (60 days, 360 days) or more?
  - What did the market do during historical period, where there any crisis events?
- Can use bootstrapping to alleviate this to an extent.

Estimation error for historical greater than for Delta normal

d)

- Transfer pricing involves the determination of a price or cost for funds transferred internally from one business unit to another
- Can be used to allocate performance into meaningful components e.g. credit risk, interest rate mismatch, etc.
- Need (n-1) benchmarks for n components

(e)

- Actual Foreign Income = (Actual Foreign Income – Benchmark) + Benchmark
  Benchmark = Income from foreign operations of a fully hedged portfolio, reflecting all hedging costs

Question 2

(a)
Fair value based:
- asset & liability market value-based
- total return included realized and unrealized gain/loss
- good for long term view
- more realistically captures firm value

Book value based:
- asset & liability book value
- asset may partially mark-to-market under FAS 115
- ignore unrealized gain/loss
- false feeling of security

(b)
(i)
- long term portfolio return should be better
- company would not invest a lot in high yield bonds to pursue higher income, because fair value based would capture the risk
- fair value would mark asset to market
- future impact would be captured
- equity offers high long term return but with long current income

(ii & iii)
- fair value based measurement would mark both A & L to market
- it captures both current and future income
- by maximizing total return, future accounting income likely to be better
- more likely to invest in projects with future potential, thus future economic profit will be better

(c)
Investment division: compare asset return to benchmark return
Product division: compare benchmark return to liability

Asset book income = 700 * ((1 + 0.0728)^0.25-1) +g/L = 12.4
Similarly, liability book income = 12.0 and benchmark book income = 12.3

Investment performance = 12.4 – 12.3 = 0.1
Product performance = 12.3 – 12.0 = 0.2

Book value @ end of quarter = beg book + book income – CF
Market value = book value * market/ book ratio
Market return = change of market value + CF

Question 3

(a)

Option Pricing Stochastic Models

1) Monte Carlo Simulation

- Simulate paths for interest rate movements that are arbitrage free
- Project cash flows along each path and discount at short rates to derive expected cost
- Can handle path dependent cash flows, complete payoffs
- Good for derivatives dependent on several variables
- Cannot handle American option
- Not a problem for GMAB since most guarantees are at specified times

2) Binomial Trees

- Each period stock can move up by a proportional amount 'u' or decrease by 'd'
- Expected payoff is discounted at risk neutral rate to derive price
- Can handle American option but not path dependency
- If GMAB has more than one guarantee at different times, can use backwardization to see if GMAB sill be exercised early

3) Finite Differences

- Solve Black-Scholes Merton differential equation by difference formula
- Not very good for GMAB

4) Black's Model

- GMAB is essentially a put on the accumulated account value (bond) as if interest rate rise, annuitant would exercise (sell the bond) to the insurer
- P= $p(O,T^*)\left[XN(-d_2)-F_ON(-d)\right]$
- Requires bond price to lognormal at time T*, payoff of option
- Fo is expected forward bond price =

$$= \frac{B_O - I}{P(O,T)}$$

I = present value of coupon payments

(b)

(i)Running the risk naked

1) Take the view that accumulated guarantee payoffs will be less than accumulated guarantee fees

2) Problems are:

- The view may be wrong
- Potentially volatile earnings
- Potentially severe capital requirement
- Have marketing risk of having to increase premium
- Exposure is high downside risk with limit to upside (receiving premium)
- Put exposure
- Insurance risk remains
- Limitation of model misspecification

(ii) Static hedging

1) Hedge guarantee risk by purchasing customized options from a third party over the counter market
2) Considerations are:

- Exposure to counterparty credit risk
- Restrictions on volume
- Unwillingness of investment bank to transact in certain market and strike prices
- High expense and profit margins built on the options
- Bid ask spread
- Insurers must generate enough premium income
- Provide only partial protection against guarantee risk

(iii) Dynamic Hedging

1) Hedge the guarantee risk by creating synthetic option using traded underlying securities, interest rate futures and short dated options
2) Requires dynamic rebalancing
3) Increase holdings in underly securities when security price increase – buy high and sell low
4) Require expertise and supporting system
5) Internal management cost may be higher than that in price in option purchased
6) Volatility is uncertain and may change – hedging cost may be higher than expected
7) Liquidity risk and transaction costs extreme events may cause trading difficulties
8) Revenue risk – option cost and risk charge move opposite direction
9) There is basis risk and model risk

(c)

Key Considerations

1) Management and stakeholders risk attitude toward specific risk classes
2) Willingness to manage or sell risks and specific risk classes
3) Presence and absence of expertise and supporting system to manage specific risks
4) Market price dynamics (price taker or setter) and marketing strategy (penetration, skimming, price leader or differential)
5) Risk size and its correlation with other company risk

(d)

LifeCo must be able to identify the many risks that exist in the GMAB. Should take an active approach to the modeling of the guarantee. Should change its ACM process into 5 steps:

- Identify risk exposure
- Determine how much exposure is acceptable
- Determine appropriate hedging instruments
- Create the hedging portfolio
- Evaluate effectiveness

(e)

Reserving	Dynamic Hedging
Process: Brownian motion. The drift is adjusted to reflect fund mana. Fees and guarantee fees	Same
Process parameters should reflect real world expectations, risk aversion and market imperfections.	Process parameters: capital market. Risk free rate of return reflecting the no arbitrage assumption
For an elective reset, option election process should be included	Same
Modification to the scenario generator needed to reflect American nature of the reset	Same
Mortality and policyholder behavior	Same
	Bid ask spread and transaction cost upon rebalancing

Question 4

Integrated Risk Management

No integration with market risk, i.e. Default rates are not a function of market level or behavior

Risk Reduction

Derivative type restrictions reduce volatility
Policies set by senior management
Master agreement netting across counterparties

Exposure Limits

Limit by class only, not counterparty
No limits for some classes (futures)
Current exposure (vs. potential) only monitored
Limits not function of credit rating or defaults

Monitoring

Frequency unspecified
Accountability not specified
No downgrade procedure
No marking to market

b)
Risk
Variance
VAR
Expected shortfall
Regret
Put value
Expected counterparty credit exposure
Expected counterparty credit loss
Expected cross-counterparty credit loss

Reward
Expected profit and loss
Expected return
Expected upside
Call value

c)

i) Counterparty exposure models
Economic loss on immediate default of all transactions for a given counterparty
Credit migration can be included
No future changes in exposure accounted for
Doesn't highlight "wrong-way" exposures
Almost exactly LifeCo's model

ii) Portfolio Credit Risk Models
Measure portfolio effects, specifically obligor correlations
Include default and migration correlation
Deterministic interest rates
Bigger impact on derivatives
LifeCo has none of these elements

d) Risk factors and Scenarios
Over analysis period project scenarios of systemic risk factors
Include joint evolution of both credit and market factors

Obligor exposures, recoveries and losses
Compute in each scenario
Based on market level
Account for netting, mitigation and collateral

Joint default/migration model
Develop scenario dependent default/migration probabilities
Relationship to scenario done through "Creditworthiness index"
Correlations are driven by joint variation of conditional probabilities

Conditional portfolio loss distribution in a scenario
Computers using Monte Carlo or statistical tools

Unconditional loss aggregation across scenarios
Average conditional losses across all scenarios
Must assign probabilities to each scenario

Question 5

a)

Warehousing assets sometimes is beneficial if the assets could be acquired at favorable price since GIC is usually sold at discrete time interval and opportunities for GIC sale may not coincide with opportunity to acquire assets. However, there is a risk that asset value may go down between the time acquired and time of a GIC sale due to for example an interest rate increase. Normally futures are sold against the acquired assets to minimize risk, and then unwound as GICs are issued.

b)

If assets have not been warehoused when GIC rate is committed, there is a risk that interest may go down and suitable assets may not be available to fund the GIC since there is usually a lag between commitment and the deposits are made.

To hedge can enter a pay floating/receive fixed swap as soon as GIC is committed and then unwind swap when the assets are finally acquired. There is still a risk that spread between the asset acquired and the futures will change (basis risk) resulting in loss. Basis risk can be hedged with spread locks or CMT/CMS swaps.

Need to estimate the price volatility
- need to know the acceptable level of volatility
- need to choose hedge instrument
- need to calculate the hedge ratio
- need to look at the price (cost) of available instruments

c)

i) Government Bond

Advantage

No default risk
Good protection against interest rate
Liquid

Disadvantage

On-balance sheet transaction
"On-the-run" or "on special" issue more expensive to borrow
Basis risk not hedged

ii) Bond Future

Advantage

Off-balance sheet transaction
Liquid

No "on special" / "on the run" issue as Treasury

Favorite hedge accounting treatment by FASB
No upfront cash deposit required

Disadvantage

Cheapest to delivery (CTD) $\Rightarrow$ Negative convexity
CTD difficult to evaluate/monitor interest ratio risk
Does not hedge basis (spread) risk

Interest Rate Swap
Advantage

Off-balance sheet transaction
No "on special" issue as training
Flexible maturity
Better interest rate risk protection. Swap rate closely related to corporate spread
Priced off Treasuries, so give same interest rate protection

Disadvantage

OTC mostly
Credit risk involved
Bid/Ask spread (less liquid)

Question 6

A

i) Costs

OTC Call Option	**Delta Hedging**
Usually more expensive	In theory cheaper, as implied volatility of synthetic option less than what investment bankers use
	But significant risks that could increase costs
	Significant systems/expertise required, which if you don't have already is expensive

ii) Risks

Liquidity Risk	Model Risk – is S&P going to move like the model predicts?
Credit Risk - counterparty default - long term option riskier	Basis Risk – minimal, with futures
Operational Risk - will OTC transaction be executed properly?	Market Risk – major index moves might not get hedged
	Volatility Risk – impacts rebalancing frequency, also transaction costs
	Consider hedging Gamma & Rho as well
	Huge operational risk - many complicated transactions take place with great frequency

iii) Effectiveness in Matching Liability

Perfect match if no decrement	
But, how to handle withdrawals?	- won't know until end of term
hedge less than 100%	
hedge fully, but sell off excess as necessary? (liquidity issues, and selling when out of the money)	May fail to replicate the option
Hold excess for speculative purposes? (legal?)	

b)

strike price = \$10M × 90% × $(1.03)^{10}$

x = \$12.0952 M

$S_o = \$10M$

T = 10

$\sigma = .15$

r = .05

Dividend is assumed to be zero.

Delta of option $\Delta = N(d_1)$

$$d_1 = \frac{In\left(\frac{S_o}{x}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$d_1 = \frac{In\left(\frac{10}{12.0952}\right) + \left(.05 + \frac{15^2}{2}\right)10}{.15\sqrt{10}} = .8902$$

$N(d_1) = .8133$ = amount of index required

$$\frac{.8133 \times e^{-rt} \times \$10M}{100{,}000}$$

$$= \frac{.8133 * e^{-.05(1)} * \$10M}{100{,}000} = 77.4 \text{ contracts}$$

Question 7

Modern Finance Theory indicates that investors are risk averse, utility maximizes and Bayesian forecaster

Empirical Studies of corporate management

1. **Dividend Policy**

According to modern finance theory (PMT) that dividends don't matter since dividends are taxed at income ratio

Companies do manage a smoothing dividend policies due to:

Investors don't like to dip in capital gains

Dividends can be additional income in a rising market or a silver lining when markets are down

Client issue and some investors like dividends

2. **Earnings Management**

According to PMT, the PV of earnings matters more than quarter-to-quarter to quote earnings but corporate spends a lot effort to manipulate earnings. Reasons.

a. Management reveals internal information through earnings

b. There is a cost associated with earnings volatility

Higher funding costs since external funding is higher than internal funding $\Rightarrow$ earnings volatility may influence companies ability to execute business plan or loss of investment opportunity

Cost of insolvency costs - extreme earnings may cause the company to go insolvent

c. Target for takeover if earnings are bad

d. Manager's ability is judged by producing stable earnings

3. Expansion; reorganization

PMT suggests the goal is to create value for shareholders; however, the companies tend to expand the company more than PMT suggest. Reasons:

a. Managers benefit from managing large companies and more assets
b. Managers believe they can do a better job than the company being acquired
c. Increase opportunities for current employees
d. Personal gain = some managers will overpay upon acquisition or merger for personal benefits

** BEGINNING OF EXAMINATION **
AFTERNOON SESSION

Question 8

a)
For Dividends

Some shareholders prefer continuous stream of payments as income

Higher dividend means company will be viewed as doing well

Companies should pay dividend based on its investment needs and financing opportunities

Pay dividends if there are not any attractive investment opportunities

Against Dividends

It is considered that the company failed to find appropriate investment opportunities to invest all the available cash

If the shareholders needed income they should diversity to fixed income a percentage of their portfolio or sell their shares

Historically there is no relation between company's dividend policy and stock performance

Share price decline equal to the amount of the dividend paid, never to be recovered

Dividend income is taxed at a higher rate from capital gain

Remaining capital gains are riskier

b)
Market Myopia

The company has to perform to provide good accounting results on a short-term basis (quarterly)

The myth is not true because:

companies with long term pr aspects and profitability command high P/E ratios

Insurance companies and pension funds invest more in R&D intensive companies than blue chip

Historically it has been proven that the markets:

Realistically factor in the effects of long term management actions on the stock price

Does not care if the accounts expense or capitalize value building out-lay

Can distinguish between value neutral and value adding opportunities

If the myth were true:

All the companies will sell for same P/E ratio

Share value should depend on CF expected to be generated over firm's life time

Simple strategy to buy depressed shares and short sell overpriced stocks should always out perform

Supply and Demand

Myth that the supply and demand of the stock effects the stock price

∴ company should promote their stock to increase demand

It is based on the simple assumption that the supply of shares is fixed and the demand affects the price. Supply can be created by traders with derivatives and short selling. Institution investors buys stocks to perform in a certain way, they can use any proxy stock.

Trading is not an indicator of demand, it is the change in outlook that changes stock price

Stock price determined by intrinsic value

Stock price determined by lead steers

Volume will increase but price will not change

Supply is not fixed due to short selling and synthetic securities

Question 9

Answer to question 9 – 2001 – 8V

a) Martingale is a process of the form $d\theta = \sigma dz$ where dz is a Weiner process

Let f and g be the prices of two derivative securities

that depend on a single source of uncertainty

Then $\theta = \frac{f}{g}$ is a martingale
for all securities f

if the market price of risk is the volatility of numeraire g

Hence $\frac{f_0}{g_0} = E_g(\frac{f_T}{g_T})$

b) Set g equal to the money market account, where $g_0 = 1$ and

g grows at instantaneous interest rate r, at any given time.

And follows the process $dg = rgdt$ and

$$g_T = e^{\int_0^T rdt}$$

The volatility of g is 0,

and therefore this is risk-free world

And $f_0 = g_0 \hat{E}\ (\frac{f_T}{g_T})$ under risk-neutral expectation

Hence $f_0 = g_0 \hat{E}\ (\frac{f_T}{g_T}) = 1 \cdot \hat{E}\left[\frac{f_T}{e^{\int_0^T rdt}}\right] = \hat{E}(e^{-\bar{r}T} f_T)$

Question 10

a)

Seasoning – prepayment rates increase as mortgage ages and then level off or decrease slightly with age

Interest Rates – low rates produce higher prepays due to refinancings and relocations

Seasonality – prepays are higher in summer and lower in winter

Burnout

- as rates reach a certain low a second or third time, prepayments decrease
- path dependence of prepay rates versus interest rates
- after lots of prepays, these left in pool are less likely to prepay

b)

Seasoning (aging) –

- yes, base assumption has seasoning
- five years from issue, house rate is flat
- takes time to reach fully seasoned state

Interest rates – yes

- competitor rates are included in excess lapse function

Seasonality – not present

Burnout – not present

- high credited rates continue to affect lapses

c)

Interest Rates – if ignored, lapse cannot reflect interest sensitive lapses at a lower interest rate level environment

Seasoning – if this term is ignored, the lapse for early years will be overstated, which does not reflect the underlying situation of newly issued policy

Seasonality – if ignored, the total lapses will be the same but the timing is different

Burnout – if ignored, the lapses for a prolonged period of lower interest rates will be overstated while the lapse rate at that moment will be quite stable.

d)
generate a set of arbitrage-free interest rate paths
project cash flows along each path and calculate present values
add a spread to all the paths and repeat the process and fin the average of $PV(CFs)$

e)
Interest rates – if ignored will understate the lapse, as in a steep YC enfironment the future credited rate for competitors is high. This would lead to earlier lapse and the acquisition expense unamortized. The RSA is underestimated.

Seasoning – if ignores should not have effect on RSA

Burnout – if ignored will lead to overstated lapses. The RSA is bigger than actual.

Question 11

a)

i)

Straddle payoff max $(S_T - X, X - S_T)$

ii)

Pays off $(X - S)_t$ *if* $S < H$ at some point before T

iii)

At end of term pays $(X - S)_t$ where S_{ave} is average strike price over period

iv)

Payoff if $(S_1 - S_2)_2$ where S_1 is price at time T_1 (strike price) and S_2 stock price at time T_2

v)

Pays $(S_{\max} - X)_t$ where $S_{\max}$ is the maximum value of S over term of option

b)

i)

Will lose money if stocks go down so we want put options

Straddle also pays if stock goes up so unnecessary option

Management fees are earned continuously so in retrospect are a function of the average level of stocks of the period. Average price put would be the best fit.

Forward start at-the-money average price put would be great for guaranteeing fees for next period

Forward start put not a good fit to hedge management fess earned on average stock price during the period

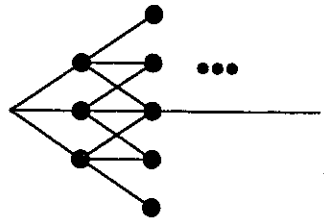
Look back put provides downside protection but would be too expensive

ii)

Above arguments remain the same. We need a forward start at-the-money put option.

c)

Trinomial valuation method: we have a trinomial tree with branches for each node that can go up, down or remain the same. See graph.



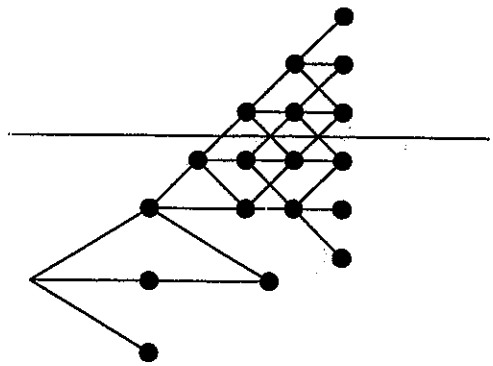
Moving along an up branch multiplies current stock price by $u = e^{\sigma\sqrt{3\Delta t}}$, the middle branch remain at same level, and the down branch multiplies by 1/u. The probabilities are

$$p_m = \frac{2}{3}$$

$$p_d = \sqrt{\frac{\Delta t}{12\sigma^2}\left(r - \frac{\sigma^2}{2}\right)} + \frac{1}{6}, p_u = \sqrt{\frac{\Delta t}{12\sigma^2}\left(r - \frac{\sigma^2}{2}\right)} + \frac{1}{6}$$

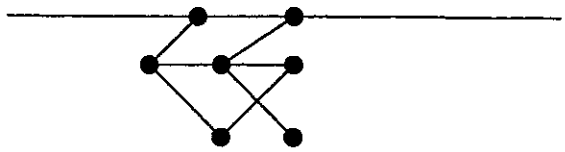
For barrier options, there are 3 ways one can value if the barrier does not lie on nodes of the tree.

1. Adaptive Mesh method decrease step size closer to the barrier (see example)



2. Calculate value of option assuming barrier is on inner and outer nodes around barrier and average.

3. Adjust tree or so that barrier is on nodes (see example)



For American options and trinomial method: use backwardisation like on binomial tree. The value at each node is f_{ij} = max (intrinsic value, $e^{-r\Delta t}\left[p_u f_{i+1,j+1} + p_m f_{i+1,j} + p_d f_{i+1,j-1}\right]$)

Question 12

a)
normally distributed return, $\mu = 0.15,\ \sigma = 0.20,\ T = 0.25$

$$\text{mean} = \left(\mu - \frac{\sigma^2}{2}\right)(T) = \left(0.15 - \frac{(0.20)^2}{2}\right)(0.25)$$

$$= 0.0325$$

std. deviation = $\sigma\sqrt{T} = 0.20\sqrt{0.25} = 0.10$

expected end of period assets = 700,000 - $e^{0.0325}$ = 723,124

b)

$$\text{average assets} = \frac{700{,}000 + 723{,}124}{2} = 711{,}562$$

i)
pre tax income – mgmt. fees + earnings on fixed assets – credited i on fixed
- fixed expenses

ii)
$$(711{,}562)(0.015)(0.25) + (100{,}000)(0.06)(0.25) - (100{,}000)(0.05)(0.25) - (3{,}000)(0.25)$$

$$= 2168$$

c)
95% < CI is ± 1.96 std. Deviations

$$0.0325 \pm 1.96 \pm 1.96(0.10) = [-16.35\%, 22.85\%]$$

d)
for both scenarios, only change is in management fees, constant pretax baseline of –500

high estimate: $$-500 + (0.015)(0.25)(700{,}000)\left(\frac{1 + e^{0.2285}}{2}\right) = 2{,}462$$

low estimate: $$-500 + (0.015)(0.25)(700{,}000)\left(\frac{1 + e^{-0.1635}}{2}\right) = 1{,}927$$

95% CI = $[1{,}927, 2462]$

e)
high: $2{,}462 - 2{,}168 = 294$
low: $1{,}927 - 2{,}168 = (241)$

high: $\frac{294}{2{,}168} = 13.6\%$

low: $\frac{(241)}{2{,}168} = (11.1\%)$

f)
$$\frac{pre-tax\ income}{range} = \frac{2{,}168}{294-(241)} = 4.05$$

g)
$P = Xe^{-rT}N(-d_2) - SN(-d_1)$

Assume $S_o = 100$

$$P = Xe^{-(0.045)(0.25)}(0.4751) - (100)(0.4355)$$

for x = 100, P = 3.431
x = 95, P = 1.567
x = 90, P = 0.567

h)
i)

$$\frac{2{,}168}{100}(1567) = 33.97$$

ii)

Index = 84.92
Option Payoff = $(10.08)(21.68) = 218.53$

Lower income limit: $1{,}927 - 33.97 + 218.53 = 2{,}134$

Upper income limit: $2{,}462 - 33.97 = 2{,}428$

iii)

$$\frac{2{,}168}{2{,}428-2{,}134}=7.37$$

Question 13

Primary Source: The Four Faces of an Interest Rate Model, Chap. 11, Investment Management for Insurers, Babbel and Fabozzi

1(a): The realistic, arbitrage-free term structure model proposed by the vendor is NOT appropriate for ALM and reserve adequacy work.

Such models are affected by confounding, where it is impossible to discriminate between model misspecification error and the term premia. As a result, such models are generally not of practical use

1(b): Arbitrage-free models take certain market prices as given or input, and adjust model parameters in order to fit the prices exactly.

Equilibrium term structure models are truly models of the term structure process. Rather than interpolating among prices at one particular point in time, they attempt to capture the behaviors of the term structure over time

Risk Neutral: The principle of risk neutral valuation asserts that, regardless of how risk averse investors are, we can identify a set of spot rates that values discount bonds correctly relative to the rest of the market by changing the probability distribution of the short term rates so that the expected rate of return on any security over the next instant is the same.

The important aspect of a risk neutral model is that the expected return on all securities is the risk-free rate, i.e., there is no extra expected return to compensate investors for the extra price risk in bonds of longer maturity.

Realistic: Realistic simulation generates scenarios that bear resemblance to observed changes in interest rates and risk premia in the real world.

Describe the four classes of interest rate modeling approaches:

1) Risk Neutral and Arbitrage-Free: This type of model is risk adjusted to use for pricing derivatives.

2) Risk Neutral and Equilibrium: Equilibrium models capture the global behavior of the term structure over time, so security-specific effects are treated in the appropriate way, as noise.

3) Realistic and Arbitrage-Free: Such a model starts by exactly matching the term structure of interest rates implied by a set of market prices on an initial date, then evolves that curve into the future according to the realistic probability measure

4) Realistic and Equilibrium: In contrast, this type of model does not take observed market prices at a particular point in time as given, rather it uses a statistical approach to capture the behavior of the term structure over time.

Uses and limitations of the four classes of interest rate models:

1) Risk Neutral and Arbitrage-Free: It is appropriately used for current pricing when the set of market prices is complete and reliable. Useless for horizon pricing whereby future prices are unknown

2) Risk Neutral and Equilibrium: For current pricing, such models can be estimated from historical data when current market prices are sparse Can also be used for horizon pricing, since the horizon prices obtained under the different values of the state variables are available in an equilibrium model

3) Realistic and Arbitrage-Free: However, such models are affected by confounding, where It is impossible to discriminate between model misspecification error and the term premia. As a result, such models are not of practical use.

4) Realistic and Equilibrium: Since the arbitrage-free form of a realistic model is not available, the equilibrium form must be used for stress testing, VAR calculations, reserve and asset adequacy testing, and other uses of realistic scenarios.

Question 14

a)
Determine the position in S needed to hedge a long position in G.

$dS = .1S\,dt + .2S\,dz$

$G = S^2 + S + 1$

By Ito's lemma

$$dG = \left(\frac{dG}{dS}\mu S + \frac{dG}{dt} + \frac{1}{2}\frac{d^2G}{dS^2}r^2S^2\right)dt + \frac{dG}{dS}Sdz$$

$\mu = .1 \quad \tau = .2$ $\qquad \frac{dG}{dS} = 2S + 1 \qquad \frac{dG}{dt} = 0 \qquad \frac{d^2G}{dS^2} = 2$

$$dG = \left((2S+1)(.1)S + \frac{1}{2}2(.21^2S^2)dt + (2S+1).2Sdz\right)$$

Need a position in S, X, so the coefficient of dz is zero

$(2S+1).2Sdz + x(.2)Sdz = 0$

$x = -(2S+1)$

S=5

X=-11

Short 11 shares of S

a) Use of Ito's Lemma

Determine the process that the hedge position would follow under each student's formula.

$H(S) = 1 - S^2$

$$dH_A = \left(\frac{dH}{dS} US + \frac{dH}{dt} + \frac{1}{2} \frac{d^2H}{dS^2} r^2 S^d \right) dt + rSdz$$

$$u = .1 \quad \frac{dt}{ds} = -2S \quad \frac{d^2H}{dS^2} = -2 \quad r = .2$$

$$dH_A = \left(-2S^2 + \frac{1}{2}(-2)(.21^2)S^2 \right) dt + (-2S)(.2)Sdz$$

$$= (-.24S^2)dt - .4S^2 dz$$

b)

$$H(S) = S^2 - 10S + 1$$

$$\frac{dH}{dS} = 2S - 10 \qquad \frac{d^2H}{dS^2} = 2 \qquad \frac{dH}{dt} = 0$$

$$dH_B = ((2S - 10)(.1)S + \frac{1}{2}(2)(.2)^2 \frac{S^2}{dt} + (2S - 10)(.2)Sdz$$

$$= (.24\,S^2 - S)dt + [.4S^2 - 2S]dz$$

ii)
Use Student As.

At S=5, the coefficient of dz is zero for student B and cannot offset the risk in G.

Question 15

Performance Measurement

A. Asset:

$$R_a = R_f + OAS - DOAS\ \Delta OAS - \Delta D(i) + \frac{r}{c} + pa + E_a$$

R_f = risk free rate
OAS = option-adjusted spread, at the beginning of the period
ΔOAS = change in the OAS during the period
$DOAS$ = price sensitivity of the security (or asset) to the change in the OAS
$D(i)$ = ith key-rate duration
r(i) = shift of the ith key-rate
$\frac{r}{c}$ = rich/cheap rate one period change the rich/cheap value
$\frac{p}{a}$ = portfolio adjustment = change in total return due to trading
E_a = investment expenses

B. Liability:

$$R_l = R_f + ROAS - \Delta D1(i)\Delta r(i) + E_l$$

R_f = risk free rate
ROAS = option-adjusted spread required by the liability's pricing
$D(i)$ = ith key-rate duration MVL (or LV) price sensitivity, to the ith key-rate
r(i) = shift of the ith key-rate
E_l = insurance (or admin) expenses

C. Attribution:

Net profits = $R_a - R_l$

C1 Risk = credit risk = OAS – DOAS ΔOAS = skill in sector rotation

C2 risk = pricing risk = -ROAS

C3 risk = duration management = interest rate risk = ALM risk
= $\sum(Da(i) - DI(i))\Delta r(i)$ = skill in interest-rate anticipation

Intraweek changes = $\frac{r}{c} + pa$ = skill in bond/security selection

Expense management = expense underrun = $-(E_a - E_1)$

Question 16

a)
i)

expected portion of no-default value lost through defaults

$$h(0,T)=\frac{e^{-y^*(T)T}-e^{-Y(T)T}}{E^{-y^*(T)T}}$$

$$h(0,5)=\frac{e^{-y^*(5)5}-e^{-y(5)5}}{e^{-y^*(5)5}}$$

$y\times(5)=0.055 \qquad 100\,e^{-y(5)5}=70.47\Rightarrow e^{-y(5)5}=0.7047$

$$h(0,5)=\frac{e^{-0.055x5}-0.7047}{e^{-0.055x5}}=\frac{0.7586-0.7047}{0.7596}$$

= 0.07226 or 7.226%

ii)

$$h(0,10)=\frac{e^{-y^2(10)10}-e^{-y(10)10}}{e^{-y^*(10)10}}$$

$y\times(10)=0.055 \qquad 100e^{y(10)10}=48.675\Rightarrow e^{-y(10)10}=0.48675$

$$h(0,10)=\frac{e^{-0.055x10}-0.48675}{e^{-0.055x10}}$$

$$=\frac{0.57695-0.48675}{0.57695}$$

=0.1563 or 15.634%

iii)

$$h(T_1,T_2)=h(O,T_2)-h(O,T_1)$$

$$=h(0,10)-h(0,5)$$

=15.634% - 7.226%

= 8.408%

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STOP

b

i)

default probability during first five years

$$= \frac{h(0,5)}{1 - \text{recovery rate}}$$

$$= \frac{0.07226}{1-0.4} = 0.12043 \text{ or } 12.043\%$$

= 4 times historical default rate

Historical default rate for 1st 5 years = 3%

ii)

default probability 1st 10 years = $\frac{0.1563}{1-0.4}$

= 0.14013 or 14.013%

= 1.4 times historical default rate

historical default rate –1.5 & 10 = 13% - 3% = 10%

iii)

default prob. Between 5 & 10 years = $\frac{0.08408}{1-0.4}$

0.14013 or 14.013%

= 1.4 times historical default rate
historical default rate –1 5 & 10 = 13% - 3% = 10%

c)

Reasons for discrepancies

- traders may be pricing in possibility of recession or depression
- part of higher return may be compensation for lower liquidity

Specific Uses

- risk-neutral (based on bond prices)
- used to value credit derivatives
- estimate impact of default risk on the pricing of derivatives
- real-world (historical)
- used when carry out scenario analysis to calculate future losses from defaults

d)

i)

5 year

Total Excess Return = (0.12043-0.03)x (1-0.4)

$$= 0.05426$$

$$\text{Annual Excess Return} = \frac{5.426\%}{5} = 1.085\%$$

ii)

10 year

$$\text{Total Excess Return} = (26.057\% - 13\%)x(1-0.4)$$

$$= 7.8342\%$$

$$\text{Annual Excess Return} = \frac{7.8342\%}{10}$$

$$= 0.7834\%$$

Question 17

Fair Value Accounting

OPM : direct method

$$MVL = \sum (L_t + E_t) / (1 + r + s)^{-t}$$

Discount liability and expense cash flows at the risk free rate plus a spread

AAM : indirect method

$$DDE = \sum DE_t / (1 + k)^{-t}$$

Discount distributable earnings at the cost of capital

$MVL = MVA^* - DDE$ where MVA* means all assets, not just product assets

Market value of liabilities is the market value of assets less DDE

$$DEt = I_t - \Delta RS_{t-1}$$

Distributable earnings equal statutory earnings less the change in required surplus

Equivalence of AAM and OPM: when a consistent discount rate is used

When $\theta_t^L = \theta_t^A - (RP_t / MVL_{t-1})$ is the required profit margin and θ_t^L is the real spread above the risk free rate for discounting liability cash flows

Necessary assumptions include:

Statutory accounting
Taxes
Risk based capital
Investment strategy

Question 18

a)
i)

point-to point with a 2-yearAsian end

$$AV_5 = 100{,}000x\left\{1+\left[\frac{\frac{(150+140)}{2}}{100}-1\right]x\,80\%\right\}$$

= $136,000

ii)

Annual discrete lookback

$$AV_5 = 100{,}000\,x\left\{1+\left[\frac{150}{100}-1\right]x\,50\%\right\}$$

= 125,000

iii)

annual discrete lookforward

$$AV_5 = 100{,}000x\left\{1+\left[\frac{140}{100}-1\right]x\,60\%\right\}$$

=124,000

iv)

annual simple ratchet (assume minimum = 0)

$$AV_5 = 100{,}000x\left\{1+\left[\max\left(\frac{130}{100}-1\right),.12\right]\right\}+\left[\min\left(\frac{110}{130}-1\right),0\right]$$

$$+\left[\max\left(\frac{120}{110}-1\right),.12\right]+\left[\max\left(\frac{150}{120}-1\right),.12\right]$$

$$+\left[\min\left(\frac{140}{150}-1\right),0\right]$$

$$=100{,}000[1+.12+0+.0909+.12+0]=133{,}090$$

a)

Point-to-Point Ladder Design

Since 125 level was not reached at year 3,

$$AV_5=100{,}000\,x\left\{1+\left[\frac{140}{100}-1\right]x\ 90\%\right\}$$

=136,000

b)

Advantages of deterministic scenario testing
Ease of interpretation of results
Can incorporate subjective opinion
Extreme case can be included
Similar to the method used in cashflow testing

Disadvantages of deterministic scenario testing:
Difficult to create a large number of scenarios
Difficult to allocate probability to each scenario
Difficult to reflect the full range of variability
Difficult to create scenarios where the economic variables are consistent