1. Solutions

a)

Static Techniques

1. Cash flow calendar equals maturity overview of cash inflows/outflows
   - looks at balance sheet differences in A/L values by maturity and looks for possible shortfalls
   - Does this in a general overall sense by examining A&L difference at end of period

2. Cash Flow Matching (dedication)
   - matches A&L cash flows
   - not done for LifeCo

3. Gap Analysis
   - looks at balance sheet differences between fixed rate A's & L's and floating rate A's & L's
   - gap of approximately zero means you are doing a good job
   - they do more of a duration gap analysis (similar to immunization)

4. Segmentation
   - creates specialized P's (portfolios) designed especially to reflect and support a particular L product group
   - separate A P's for each L product line
   - LifeCo uses segmentation

Dynamic Strategies – Value Driven Strategies

Passive

1. Traditional Immunization
   - matches dollar duration of A's & L's subject to present value and, perhaps, dollar convexity constraints
   - may maximize dollar duration weighted return
   - LifeCo does this

2. Single Factor Immunization/Multiple Factor Immunization
   - not specifically performed by LifeCo

3. Key Rate Immunization
   - matches key rate durations (and convexities, if desired)
   - key rates are segmentation of the YC and measurement of price sensitivity to particular maturity segments
   - this is done and called "partial duration"

Multiscenario Analysis

- link between static techniques and dynamic strategies
- uses multiple economic/assumptions scenarios to study A/L CF's under changing conditions
- focus on problem detection
- adds multi-dimensional concept to risk analysis
- can use stochastic or deterministic scenarios
- used by LifeCo

Dynamic Strategies – Value Driven

Active

1. Contingent Immunization - not done
2. Portfolio Insurance (PI)
   - use of synthetic or actual puts to protect floor P value
   - LifeCo uses derivatives to hedge risks, if approved
3. Constant Properties – PI
   - not used

Dynamic Strategies – Return Driven

1. Required return on assets approach
   - calculate a return necessary to support L's
   - similar to total return or excess spread approach
   - not specifically used

2. Spread Approach
   - manage spread between rate earned on A's and cost of funds
   - not specifically used

Risk and Return Analysis

- choose P's on efficient frontier given client's risk/return profile
- not used

b)

Return Driven Strategies

- provides an easy and consistent way to measure relative value of lines of business
- readily available measure if spread based from accounting statements
- spread may be important as expenses may be amortized according to spread
- rating agencies are interested in spread measure
- used by actuaries to analyze distributable earnings
- encourages passive management of asset where more active management may be more appropriate
- does not take account of short-term interest rate risk
- may not identify risk acceptances i.e. add expected return to increase spread by adding more credit risk or increasing duration.

Value Driven Strategies

- protects against small changes in yield curve
- standard immunization protects against parallel shifts
- key rate duration against non parallel shifts
- requires continuous rebalancing as duration drift over time in both assets and liabilities
- insulates Co surplus from general market moves
- does not take into account credit risk of portfolio and other non interest rate risks
- unless merge convexity and place appropriate limits on allocations can end up with two asset portfolio i.e. barbell
- re-investment risk due to rebalancing

c)

Strategy: Contingent Immunization

- category involves setting a lower bound for asset value or conversely surplus value
- actively manage portfolio until reach lower bound (if reach lower bound)
- revert to immunized strategy if lower bound- reached
- provides opportunity to increase surplus through active management of portfolio while still aware of a lower bound on surplus values.
- needs to be monitored or large changes in market can breach lower bound before chance to immunize
- treat portfolio as a whole to take account of any correlations among liabilities that offset some market risk

d)

Criteria	
- return completeness	yes through risk return analysis
- risk completeness	yes through risk return analysis
- data requirements	lots of data for scenario and key rate analysis
- observability	risk/return profile required from management fairly observable
- model independence	key rate immunization is model indep and allows for arbitrary yc shifts but, most active/dynamic strategies require some kind of model

1. Provide means of seeking out all possible sources of return
   - can seek out extra return through active management
   - sector bets
   - duration bets
   - bond selection

2. Account for all risks
   - strategy takes interest rate risk into account if revert to key rate immunization then can protect against non parallel shifts

3. Not dependent on particular assumptions or model
   - strategy is independent of model and any particular assumptions
   - if used factor immunization or multi factor interest rate model then exposed to risk that factor model incorrect

2. Solutions

a)

LTD	PV	Duration	Rel Vol	Adj Dur	$ Dur	Adj $ Dur
Asset	550.90	13.50	0.815	11.00	7437.20	6059.90
Liab	532.00	8.10	0.663	5.37	4309.20	2856.80
Econ Val	18.90			169.47	3128.00	3203.10

Total Grp	PV	Duration	Rel Vol	Adj Dur	$ Dur	Adj $ Dur
Asset	660.00	12.30	0.815	10.02	8129.10	6623.70
Liab	624.00	7.00	0.668	4.67	4368.00	2915.60
Econ Val	36.90			100.49	3761.10	3708.00

Other A&H	PV	Duration	Rel Vol	Adj Dur	$ Dur	Adj & Dur
Asset	110.00	6.29	0.815	5.13	691.90	563.80
Liab	92.00	0.64	1.000	0.64	58.80	58.80
Econ Val	18.00			28.08	633.10	505.00

- PV Assets (A&H) = PV Assets (Group) – PV Assets (LTD) = 660.9 – 550.9 = 110
- PV Liab (A&H) = PV Liab (Group) – PV Liab (LTD) = 624 – 532 = 92
- EV (A&H) = PV Assets (A&H) – PV Liab (A&H) = 110 – 92 = 18
- Mod Dur Assets (A&H) = (12.3 x 660.9 – 13.5 x 550.9) / 110 = 6.29
- Adj. Dur Assets (A&H) = (11.00 / 13.5) x 6.29 = 5.13
- Mod Dur Liab (A&H) = (7.0 x 624 – 8.1 x 532) / 92 = 0.64
- Adj. Dur Liab (A&H) = 1.0 x 0.64 = 0.64
- Adj. Dur EV (A&H) = (110 x 5.13 – 92 x 0.64) / 18 = 28.08

b)

A focus on adjusted duration ignores convexity, or the curvature of the Assets or Liabilities to changes in interest rates. Adjusted duration is preferable to modified duration alone as it does account for volatility but effective duration is preferable, as effective duration will account for changes in interest sensitive cash flows. Preferable still might be key rate duration. Modified and effective duration only considers parallel shifts in the yield curve; key rates will allow for shape changes.

Adjusted duration alone will not account for the risk that your claims may have positive correlation with interest rate changes and claim inflation. In addition, interest risk is affected by many assumptions and by exposure to future and renewal business. Without an assessment that includes these factors, the measures used will not capture all the interest rate influences.

c)

LifeCo uses:

- Dollar Duration – gives a dollar value for the impact of rate changes; similar to adjusted duration but different scale of reference. Limited by only considering parallel shifts that seldom happen.

- Modified Duration – rate sensitivity in percentage terms of MV of assets and liabilities, suffers from the same problem as adjusted duration in that it assumes parallel shifts.

- Convexity – rate of change in duration; much more useful in conjunction with adjusted duration; gives the second order sensitivity to rate changes.

- Key Rate Sensitivities – address the parallel problem; much more useful than adjusted duration.

- Cash Flow Analysis – careful interpretation is necessary; no reinvestment assumption. Can help with other risks than interest rate (e.g. liquidity); useful in conjunction with adjusted duration.

- Scenario Testing – test results under various scenarios; can see what effect scenario changes have on the adjusted duration. Let us know whether adjusted duration is stable, how much it could change in the future. Some value when used together.

- Asset Quality – defaults can significantly affect adjusted duration values; high credit quality is necessary.

d)

Franchise Value is the PV of cash flows from new and renewal business in the future. It is usually discounted at risk adjusted rates. More generally, shareholder value consists of the sum of Economic Value and Franchise Value where Economic Value is PV of Assets less the PV of Liabilities of cash flows from the existing Balance Sheet.

e)

Where premiums do not vary with interest rates, the duration of franchise value is negative and the target asset duration must be increased. Where premiums do vary with interest rates, the duration of franchise value is positive and the target asset duration must be reduced.

Competitor pricing also impacts the relationship along with the elasticity of consumer demand to price differences.

3. Solutions

a)

The first step is to determine the number of put and call contracts to make the portfolio gamma and vega neutral. Must determine cheapest hedge.

Use	1 year put	and	10 year put
	or		or
	1 year call		10 year call

Gamma hedge equation:

$$x \cdot 0.00184 + y \cdot 0.00029 = 1.036$$

Vega hedge equation:

$$x \cdot 3.688 + y \cdot 5.761 = 1952$$

this solves to:

$$y = -24.04$$
$$x = 566.83$$

Use x = 1-year put
y = 10-year call
because this combination minimizes the cost

Now use the S&P future to hedge delta.

-2659	=	-.34608 x 566	+	.89472 x –24	+	100 · y
(Delta of Liabilities)		(Delta of 1-year puts)		(Delta of 10-year calls)		(Delta of futures)

$$y = -24.42$$

Use the 30-year treasury future to hedge rho:

-101,910,000	=	-3.98 · 566	+	40.51 · -24	+	-12,598 · z
(RHO of Liab.)		(RHO of 1-year put)		(RHO of 1-year call)		

$$z = 8089.1$$

b)

To hedge, one would need to short an option, which is in violation of the guidelines. The guidelines make no reference to put options, which is also needed to create the minimum cost hedge.

I recommend that the guidelines allow the purchase of put options. I don't recommend changing the guidelines to allow for shorting options. Shorting options lever downside potential. Synthetic short options can be set up without violating the guidelines.

c)

$VAR = DELTA \times \Delta S + .5 \times \Delta S^2 \cdot GAMMA$

ΔS is the drop in the S&P 500.

The 99th percentile drop in S&P is 2% VOL x –2.33 (99th percentile of N (0,1) = -0.04652

1300 x –0.04652 = -59.09

VAR = -2659 x –59.09 + $0.5x(59.09)^2 \times 1.036$

= 158982.15

Hedged
Same formula but Delta = Gamma = 0
so VAR = 0

Limitations
- results depend on model, data, parameters
- many risks ignored

d)

$RHO = \frac{\partial P}{\partial R}$ which is analogous to duration

Therefore, RHO hedging is similar to duration matching, which is good only for small, level shifts in the yield curve.

Could use other methods:
- key rate duration matching
- duration and convexity matching
- cash flow matching → probably not feasible since cash flows very volatile

Could use anything sensitive to interest rates:
bonds, swaps, swaptions, floors, etc.

e)
Operational Risks Include
1. Inadequate systems and controls
2. Human error
3. Management failure

- management is informed and involved
- policies, procedures and approved activities documented
- independent checks, balances for purchases, payments and accounting exist – e.g. two quotes required.
- back office exists – systems

Credit Risk – Risk of Counterparty Default

Manage by:

- diversification – by number and exposure
- minimum rating – BBB, should be higher
- analysis of current and potential exposure
  - current = replacement cost
  - potential = VAR or simulation
- Netting with respect to counterparty
- Monitoring of ratings
- Settlement Risk Control

Deficiencies:

- Exposure limits too high relative to surplus
- No downgrade triggers
- No requirement for netting everywhere
- No exposure limit triggers

4. Solutions

a)

Model Risks

Inapplicability of Modeling – problem does not require a model

Incorrect Model

e.g. missing a key factor or making incorrect assumptions

Correct Model but Incorrect Solution

Inappropriate use of a model -- using it in a way that its builder never intended it to be used

Badly approximated solution

Software/Hardware Bugs

Unstable data used in model

b)

Arbitrage-Free

- Model is calibrated to the current market price of zero coupon bonds
- Model respects the law-of-one-price (i.e. no arbitrage opportunities are present within the model)
- A probability measure Q (representing probabilities of the various sample spaces w_i) exists such that:

$$\frac{S_t}{B_t} = E^q\left(\frac{S_t}{B_t} / P_t\right)$$

If such Q exists, Q is said to be a risk-neutral (or martingale) probability measure

c)

$$\text{Market price} = 89 = \frac{1}{1.07}\left[q \bullet \left(\frac{100}{1.06}\right) + (1-q)\left(\frac{100}{1.08}\right)\right]$$

Solve for q to determine if q is valid probability measure

$q = 1.51$

$1 - q = -0.51$ $\therefore q$ is not a valid martingale probability measure

5. Solutions

a)

i) As average life increases, option cost increases, assuming fixed price, OAS goes down

ii) Premium securities generally have higher option costs and therefore a lower OAS. This is because they are more susceptible to prepayments than discount securities in low interest rate scenarios.

iii) Lockouts increase stability of earlier classes, lower options cost, increase OAS

iv) Minimal impact on OAS

v) Z bonds increase stability of other classes, implies lower option costs and higher OAS

b)

- Average number over space and time
  - unlikely to earn OAS
- Model dependent
  - extremely sensitive to methodology, assumptions and data
- summary #
  - no PASS price distribution provided
  - others could give indication of problem scenarios and asymmetry
- Portfolio OAS usually just a simple market value weighted average of OAS of each asset. Better to aggregate portfolio cash flows at each node and then calculate portfolio OAS
- Abuses in practice
- Constant spread added to interest rates subtly changes their distribution
- Non-interest-related events not easily modeled
- MBS's prepayment models usually assume prepays are deterministic and not path dependent

c)

- disclose sensitivity of OAS to varying volatility and prepay assumptions
- provide PASS price distribution
  - path and spread specific PASS
  - outline problem scenarios, asymmetry
- calculate true portfolio OAS by aggregating P cash flows at each node
- Use double stochastic approach to calculating prepays
- Look at other measures

6. Solutions

a)

$$P = P(O,T^*)[X \cdot N(-d2) - FxN(-d_1)]$$

P = *Put Value*

$P(O,T*)$ = price of a zero coupon bond that matures at time $T*$ *for* \$1

T^* – *time to maturity*

x = *strike price*

$$d_1 = \frac{1_n\left(\frac{F_o}{x}\right) + \frac{\sigma^2 T}{2}}{}, d_2 = d_1 - \sigma\sqrt{T}$$

$$F_o = \textit{forward price} = \frac{B_o - I}{P(O,T)}$$

B_o = *bond price* D = modified duration

I = *present value of dividends*

σ = *volatility of forward bond*

$$D = \left(\frac{-1}{B}\right)\left(\frac{\Delta B}{\Delta y}\right) \Rightarrow \frac{\Delta B}{B} = -D \cdot \Delta y$$

$$= -Dy\left(\frac{\Delta y}{y}\right)$$

$$\Rightarrow \sigma = D\, o_y\, Y_o$$

where

σ_y = *forward yield volatility*

y_o = *forward yield*

$$\Rightarrow \sigma = 10(.10)(.07) = .07$$

$$F_o = \frac{1000 - 100}{e^{-.0513x(1)}} = 947.37$$

$$d_1 = \frac{ln\left(\frac{947.37}{1000}\right) + \frac{.07^2}{2}(1)}{.07\sqrt{1}} = -.74$$

$$N(-d_1) = 1 - N(d_1) = 1 - N(-.74) = 1 - .2296 = .7704$$

$$d_2 = d_1 - \sigma\sqrt{T} = -.74 - .07 = -.81$$

$$N(-d_2) = 1 - N(d_2) = 1 - N(-.81) = 1 - .2090 = .791$$

$$p = e^{-.0513(1)}\left[1000(.791) - 947.37(.7704)\right] = 58.09$$

b)

For a put option on a stock, $\Delta = N(-d_1) - 1$

where d_1: $\frac{ln\frac{So}{x} + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$

on the same put option $\Gamma = \frac{1}{2\sigma\sqrt{T}} x\, N^1(d_1)$

However for a bond option there are many ways of calculating delta and gamma because there is not a single underlying variable (as in the case of a stock price) but multiple variables in the form of various interest rates from various points of the yield curve.

Ways of calculating delta for bond option:

1. assume a parallel shift in all interest rates
2. divide the yield curve into segments and calculate a Δ for each segment based on a parallel shift in the rate of that segment
3. Calculate a Δ for each rate by shifting only that rate
4. Use the first three factors from principal component analysis
   a. almost parallel shift
   b. twist
   c. butterfly

Ways of calculating Γ:

1. Calculate Γ for each Δ ignoring cross-partials
2. Assume parallel shift in all rates
3. Use first three factors of principal components analysis

7. Solutions

a)

Over Confidence
Over confidence in information one has, especially if in one's area of expertise.

Biased Forecasters (non-bayesian)
Too much weight is given to most recent information; leads to under- and over-reactions.

Loss Aversion
More pain is felt with $5 loss than pleasure with $5 gain.

Mental Accounting
Willingness to sell a loser if with a winner, but wouldn't sell loser by itself.

Framing
Decision made often depends on how framed, i.e. would get more upset as a customer if asked to pay surcharge for credit card use versus discount to cash paying customers.

Fads and Fashion
Fads come and go. Investors feel pressure to conform.

Regret, Responsibility and Prudence
When investors take actions to avoid regret – pain felt by decision which led to a bad outcome regardless if decision was bad.
Actions taken: hire an agent
Also describes tendency to follow prudent and responsible decision-making, i.e. buy well recognized stock even if overvalued.

b)

1. Volume of trading and active management
   Behavior explanation: everyone thinks can pick a winner

2. Contrarian Strategies
   - investing in low P/E
   - shouldn't be able to profit on publicly available information

   Behavior explanation: overconfidence, which leads to over reaction; biased forecasts and misperception of risk.

3. Asset values and Investor Sentiment
   - asset values should represent intrinsic value
   - NAV often @ discount

   Behavior explanation: non-rational trader sentiment varies over time.

4. Equity Risk Premium
   - Much larger than can be explained by risk alone

   Behavior explanation: Myopic loss aversion – riskiness of investment depends on how frequently it is valued.

5. Payment of Dividends
   - not rational

   Mental accounting: think of dividend as separate return. Avoid dipping into capital to finance spending.

8. Solutions

a)

EIA Product

- offers participation in the growth of a market index (such as S&P 500) with min interest rate guarantee
- crediting is based on a participation rate times an index increase, with possible cap on increase
- index increase may be calculated in different ways such as point-to-point, annual ratchet etc.
- may have surrender charges, free partial withdrawals
- may receive full account value on death
- may have vesting schedule on increases

b)

- it is necessary to hedge the index increase, minimum guarantee and any decrements
- use OTC or exchange options, such as calls
- type of option depends on design, e.g. point-to-point use European call
- may purchase put option to hedge free lock
- recommend to purchase call options to hedge index exposure
- minimum guarantee = use bond to hedge
  recommend to purchase corporate bonds with good spread above min guarantee

c)

- OTC options have low liquidity, high lapses can create problems if need to liquidate options
- free withdrawals when market value is lower than min guarantees
- exposures to the credit risk when OTC used
- less than full hedging creates concern if no decrements and high index increases
- excess lapses may create significant losses when interest rate up and market down

9. Solutions

a)

OAS is considered compensation for shorting options thus $OAS \downarrow$ for a MBS as *price payment risk* $\uparrow$ *or default risk* $\uparrow$

Stressed Level

Debt service coverage ratio (DSCR) = annual NOI/annual cost of debt

As DSCR $\downarrow$ => Pr*obability*(*Defaults*) $\uparrow$ => $OAS \downarrow$

Stressed Level

Loan to Value Ratio (LTV) = Loan amount/valued property

As LTV $\uparrow \rightarrow$ Pr*obability* (*Defaults*) $\uparrow$ => $OAS \downarrow$

b)

More volatile income stream =>greater prob of default.

Thus since *NOI volatility* $\uparrow$ => *prob* (*defaults*) $\uparrow$ => $OAS \downarrow$

Largest decline in OAS takes place in junior classes since they are first to absorb losses caused by default.

For Senior class, price at prem – OAS will also decrease since recoveries are used to pay this class first, but receipt of principal means lower yield since class priced at prem

B classes are most protected since they are not first to absorb losses or recoveries (OAS does not change)

c)

Since senior class is priced at discount, the OAS will benefit from call feature and thus $OAS \uparrow$

d)

Many investors do not want to buy premium bonds. Thus issuer strips away interest and the interest only class is created.

The issuer will create the stripped classed only if the proceeds from stripped class plus the par bond is greater than the proceeds from the prem bond only.

e)

- Interest is paid when principal is outstanding
- When principal is reduced, interest pald to IO declines
- Principal reduced through defaults and involuntary principal payment

- For default, loss is allocated to junior classes thus senior class insulated
- Involuntary payment arising from recoveries go first to the most senior class and thus erode the principal.
- Senior – Subordinate structure requires most senior class receive principal payments first

10. Solutions

a)

Factors Affecting Prepayment

- human behavior
- relocation
  - -economic conditions to relocate
    - -higher equity values
    - -lower mortgage rates
    - -non-economic conditions
    - -seasonality
- curtailment
  - less 1% of all mortgage payments are partial payments
  - considered as unscheduled principal payments
  - cumulative effect can be quite large

- assumptions
  - FHA/VHA mortgage can be assumed
  - can reduce prepayment if mortgaged at a low rate
  - conventional mortgages are no longer assumable
  - hence big difference in prepayment volatility between assumable and unassumable

- Refinancing
  - biggest part of the prepayment uncertainty
  - burnout could reduce this to an extent
  - willing to refinance
    - is it cost effective after penalties are taken into consideration?
  - able to refinance
    - do you have credit available
    - most people move between willing and unwilling
  - ready to refinance
  - remember lower rates usually increase refinancing
  - sometimes refinancing increases just after an increase in rates as people believe rates are continuing to increase and they were trying to catch the bottom

- Burnout
  - generally occurs when people refinance as soon as it is economically feasible
  - low or no point refinancing has reduced this to an extent
  - seasonality
    - higher prepayment in summer than winter as people prefer to move during those periods
  - seasoning

- older mortgages are more prone to refinancing
- new mortgages generally haven't had enough time to react to change in rates
- family considerations have not changed enough
- usually unable to pay origination fees

b)

• Competitiveness
- imports plus exports as a % of GDP is used as a measure
- the more competitive the less chance of political instability

• Quality of Life
- proxied by life expectancy

• Predictability
- is inflation stable
- stable inflation is a sign of political stability

• Agriculture
- if agriculture is the dominant thing in the county there is a greater chance of political instability

• Trauma
- need to create an infrastructure for future stability

• Human Capital

• GDP
- largest part
- the greater the per capita GDP the greater the stability

• Rental Income
- if there is federal aid or resource based income then political instability might be present

• Dispersion in Income
- a greater dispersion from high and low income is a sign of political instability
- proxied by infant mortality

c)

<u>Full Hedging</u>
- attempts to strip away all the currency risk to attain a risk return profile identical to local currency investor
- requires a forward in an amount equal to the foreign currency exposure

- As the foreign currency exposure fluctuates with investment returns, the investor in foreign will become over or under hedged
- investors accept a certain amount of deviation from the ideal hedge to offset rebalancing and associated costs.

Minimum Variance

- combines the effects of asset risk and currency risk to develop a hedge to minimize variance given an expected return
- goal is to minimize the overall risk of a foreign assets, while full hedging attempts to neutralize only the currency component
- requires accurate estimates of currency volatilities and correlations

Downside – option based

- minimize the probability of excessive currency losses
- can be done in many ways:
  - purchase a put on the individual currencies
  - purchase a put on a basket of currencies
  - purchase a put on the total base currency value for the asset

d)

Diversification Benefits:

- US asset =

$w = 300{,}000 \quad \tau_{DAY} = .02 \qquad P = 0.30$

$\tau_{US\ ASSET} = 6{,}000 = (300{,}000)(.02)$

- PAC

$w = 200{,}000 \quad \tau_{DAY} = .015$

$\tau_{PAC} = (200{,}000)(.015) = 3.000$

Individually: $\tau_{US\ ASSET} + \tau_{PAC} = 6000 + 3000 = 9000 = \tau_p$

Diversify

$\tau p^2 = \Sigma W^2 i \tau 2^2 + 2\Sigma W_i W_j \tau_i \tau_j \subset$

$\tau p^2 = (300{,}000)^2 (0.02)^2 + (200{,}000)^2 (0.015)^2 + 2(300{,}000)(200{,}000)x(0.02)(0.015)(0.30)$

$\tau_{p^2} = 55{,}800{,}000$

$\tau_p = 7{,}469.94$

By diversifying the τ_p is brought down from \$9000 to \$7,469.94.

$VAR_{US\ ASSET} = 1.645(6000)\sqrt{5} = 22{,}070$

$VAR_{PAC} = 1.645(3000)\sqrt{5} = 11{,}035$

VAR for the portfolio with both: $1.645\ (7470)\sqrt{5} = 27{,}477$

Before diversification: VAR = 22,070 + 11035 = 33,105

After: VAR = 27,477

Benefit of Diversification = 5,628

11. Solutions

a)

Portfolio Yield Strategy

(i) Recession Scenario

- the assets are indexed at higher yield than the yield under recession scenario, therefore, the portfolio yield will improve due to the lowering in interest rates
- the lower in yield will impact on the reinvestment income; however, it will not have instant negative impact on the portfolio yield now.
- the portfolio yield will not go down that much
- However, as the recession scenario persists, the portfolio yield will go down, the guaranteed minimum interest rate of 5% may kick in
- because crediting rates remain above current, and therefore are competitive, lower lapses should be experienced
  - the profit margin will be narrowed

(ii) Inflation Scenario

- the portfolio yield will lag the market yield, therefore, there may be some interest sensitive surrenders
- to avoid surrender, the company can either:
  1. increase the crediting rate, which erodes the profit margin or
  2. continue crediting yield lower than the market rate. If the surrenders accelerate the assets need to be sold at a loss to pay for cash outflow
     - the additional surrenders may create the need to sell assets at deflated values

Current Market Rates Strategy

(i) Recession Strategy

- the lower in interest rates will result in gains in the asset portfolio
- since the crediting rate is switched to the new money rate, the company can credit the lower rate to policyholders without sharing the benefits of gains (however, 5% minimum interest rate is guaranteed)
  - ⇨ the end result is that the profit condition is improved over the portfolio yield crediting approach, but pricing spreads are compressed due to the guarantee kicking in

(ii) Inflation Scenarios

- higher interest rates depress the company asset portfolio
- new money crediting strategy further erodes the company's profit margin
- the end result is that the company has smaller profit margins, or may even result in negative profit

b)

- it incurs transaction costs
- it may have tax consequence

- it will generate earnings and cause earnings volatility which accounting department may not like
- the sales/purchases of assets should based on the ALM, not just interest rate movement
- when an asset is purchased time and effort is spent on research, therefore, the effort will be wasted if an asset is sold
- there are other (better) methods to hedge interest rate risk

c)

(i) Recession Scenario
- to protect against declining interest rates, can do the following:

(1) buy interest rate floors on constant maturity treasury (CMT) or Constant Maturity Swap (CMS), with strike of 5%.
- payoff on interest rate floors will offset losses of buying at high prices
- good because CMT approximates rate credited on SPDA
- this will also provide a gain to offset interest rate guaranteed

(2) prepayment Options
- pays if prepayment exceeds a certain level
- works against increasing in prepayments when rates fall
- payment received would offset losses on reinvestment

(3) buy receiver swaption – option to enter into received fixed swap – protects against falling rates

(4) buy Bond Warrant – long dated calls on bonds
Go up in value as rates fall

(ii) Inflation Scenario
- strategies here are opposite of those to hedge recession scenario namely:
1. buy interest rate caps that pay difference between index rate and strike rate when rates go above the strike rate.
2. buy payor swaption – right to enter into pays fixed swap – protects against rising rates
3. buy a bull spread – long a call, short a different call with a lower strike

d)

Risks Include

1. Operational Risk
   - risks due to mismanagement or technical mistakes
   - must have good internal control
2. Credit Risk
   - risk due to counterparty default
   - must consider both current and potential exposure
3. Market Risks

- basis or correlation risk in that instrument used to hedge is not correlated with hedged instrument
- other risks include delta, gamma, vega (volatility),
- 8ho risk that market moves away from you

4. Legal Risk – contract may not be enforceable
   - ultra vires – counterparty may be incompetent to enter contract

Use netting and exposure limits to control some of these risks.

12. Solutions

a)

- Cash flow uncertainty – As loans are not repaid until death of borrower or relocation, the cash flow stream is highly uncertain. Yet, GICs have stated maturity dates, fixed returns and in general are considered "preservation of principal" products.
- GICs rates are fixed in advance. So if interest rates drop, the company may not be able to provide the required fixed return.
- GICs allow for book value withdrawals and as the liquidity of these loans is minimal, it would be difficult to sell these assets should there be significant withdrawal activity.
- Spread Risk – The company return is based on the spread between its GIC rate and the rate earned from approximately 6 months CD's + 3%. If interest rates drop, the company's margin will decrease as well.

(b)

(i) Fixed rate loans will have the advantage to better match the rate of the liabilities, namely GICs that are stable value products. However, if the company wishes to continue issuing such loans, its GIC rates may become uncompetitive as the fixed rate loans will not be able to match current market yields. In addition, fixed rate loans may not be able to match current market yields. In addition, fixed rate loans may not be a solid market. Shaky if it is determined that these homeowners prefer variable rate loans.

(ii) Swapping variable for fixed has the advantage of (b)(i) but still allow the market of the product with fixed loan rates. There is counterparty risk with swaps. However, again, if yields rise, new GIC rates may be uncompetitive given a fixed rate asset base. In addition, there is cash flow uncertainty i.e. whether the company can make floating payment given uncertainly of its cash flows.

(iii) Insurance will protect against cash flow uncertainty but it will be another cost that will reduce the spread between the loan rate and the GIC rate. In addition, this insurance will not protect the company from interest rate risk as the liability is floating but the asset is fixed.

13. Solutions

a)

Use the volatility matrix to determine the volatility at 8 months with a strike of X=35.

Interpolate:

$$\sigma(12\ mths) = 0.236$$
$$\sigma(6\ mths) = 0.230$$
$$\sigma(8\ mths) = \sigma(6) + 2/6*(\sigma(12 - \sigma(6))$$
$$= 0.230 + 2/6*(0.236 - 0.230)$$
$$= 0.232$$

$S_0 = 40$
X=35
r=0.07
T=2/3

Value of put option (European) $P = X\ e^{-rT} N(-d_2) - S_0 N(-d_1)$

$$d_1 = \left[In(S_0 / X) + \left(r + \sigma^2/2\right)T\right] / \left(\sigma\sqrt{T}\right)$$
$$= \left[In(40/35) + \left(0.07 + 0.232^2/2\right)*2/3\right] / \left(0.232\sqrt{2/3}\right) = 1.04599$$
$$d_2 = d_1 - \sigma\sqrt{T}$$
$$= 1.04599 - 0.232\sqrt{2/3} = 0.85657$$

b)

The implied volatilities decrease as strike price increases.

This matrix is consistent with the equity smile used by traders to price options. It implies a fatter left tail and thinner right tail stock price distribution versus the lognormal distribution.

c)

(i)
Compound option model
Equity is a call option on value of firm with strike price equal to debt of firm.
Stock option is an option on the option of the value of the firm.
When value of firm increases from rise in stock price, proportion of debt decreases making stock less risky.
Therefore volatility of stock decreases (consistent with implied distribution).

(ii)
Displaced Diffusion Model
Assumes assets are risky or riskless
Assumes fixed amount of default free debt

$a = \alpha(1+\beta)$ where α = % risky assets, β = debt/equity ratio
If a>1 – debt is greater than riskless assets.
Volatility falls as stock prices rise.
Model behaves like compound option model.
The difference with the compound option model is that if debt > Value of firm, equity may become negative.

(iii)
Constant elasticity of variance (σ)
Stock price volatility is $\sigma S - \alpha$ where $1 \geq \alpha \geq 0$
When stock price increases, volatility decreases
Assumes a company has fixed expenses. If firm performs poorly, earnings become more volatile because it still has to meet the fixed expenses. Stock price decreases and volatility increases.

(iv)
Any stochastic model for volatility
Once volatility negatively correlated to stock price.

14. Solutions

a)

- mortgage at floating rates, leases at fixed – will lose if rates rise
- leases may not be renewed after they expire
- market value may fall
- lessees may go belly-up
- overall economic conditions could worsen, reducing the property value, increasing insolvency among lessees, and making it difficult to replace leases
- as a single property, not diversified either by industry or geographically

b)

- enter into an interest rate swap-fixed rate for floating. Pay fixed.
- buy an interest rate cap that will pay when interest rates rise above a certain level
- diversify by buying other property types (e.g., commercial, residential) and in other geographic regions

15. Solutions

a)

Using real world probabilities, expected payoff is
(1,000) (0.85) + (700) (0.1) + (350)(0.05) = 937.50

Using risk-neutral probability

Probability	Value
0.75	1000
0.15	700
0.10	350

Expected value = 890 $PV = 8\% e^{-0.25} = 693$

$\therefore e^{5x} = \frac{937.50}{693}, x = 6.04\%$

$\therefore spread = 6.04\% - 5\% = 1.04\%$

b)

No default: Max (0,1000-1000) = 0
Default with 70% Rec: Max (0, 1000-700) = 300
Default with 35% Rec: Max(0, 1000-350) = 650
∴ Possible payoff of 0, 300, 650
If further assume that δ may default, total 9 payoff, see section C

c)

Using risk-neutral probabilities:
Assumption Z may default with recovery % applied to payoff

Let $X_1 - X$ doesn't default $Z_1 - Z$ doesn't default
$X_2 - X$ default 70% rec $Z_2 - Z$ default 70% rec
$X_3 - X$ default 35% rec Z_3 -- Z default 35% rec

Situation	**Payoff**	**Probability**
X_1, Z_1	0	0.75 x 0.75
X_1, Z_2	0	0.75 x 0.15
X_1, Z_3	0	0.75 x 0.1
X_2, Z_1	300	0.15 x 0.75
X_2, Z_2	210	0.15 x 0.15
X_2, Z_3	105	0.15 x 0.1
X_3, Z_1	650	0.1 x 0.75
X_3, Z_2	455	0.1 x 0.15
X_3, Z_3	227.5	0.1 x 0.1

∴ expected payoff = 97.9
∴ option value = 97.9 x $e^{-0.25}$
= 76.24

16. Solutions

a)

MV (Liability) Direct	MV(A) – MV (Equity) Indirect
Independent of assets and how they are calculated	dependent on assets and how they are calculated
Independent of accounting rules (statement)	relies on accounting statement
Independent of with credit quality of company	relies on credit quality of company
no adjustments need to be made to it	relies on equity (dist. earnings) which can be difficult to model
	Requires adjustments

Adjustments

(i)
must subtract out franchise value from MV equity) no adjustment in direct approach
(ii)
must subtract out corporate debt from estimate of MV(L) in indirect method
(iii)
indirect method produces higher liability value after accounting for put option in state guarantee funds

- indirect: some issues on how assets from takeover/merger should be handled
no related issue for direct method

b)

Two Approaches
First: Spread Adjustment

- ⇨ add required spread to interest rates in model to account for risk
- ⇨ problems with spreads though
  a. not necessarily additive
  b. small change in optionality leads to large change in spread
  c. no market comparables from which spread can be determined

- ⇨ too subjective – is spread correct?

- ⇨ Okay to use spread method if market comparables available
  e.g. default risk can be easily modeled using spread approach

2nd Approach

- ⇨ explicitly model the CF
- ⇨ required if options embedded in CF's
- ⇨ each CF than has a new risk-adjusted discount rate

⇨ $r^* = r_F$ + risk aversion for risks modeled
+ risk spread for those not

use of IR lattice limited to those cases when CF's not path dependent on previous results
MC simulation needed for path dependent flows e.g. MBS, rate crediting based on average rate, however MC cannot value an American Option

17. Solutions

a)

Prepayment Uncertainty
Sensitivity of security price to changes in level of prepay
speed projected by a prepayment model
can measure overall prepayment risk or that due only to refinancing or relocation

Volatility Risk (Vega)
Sensitivity of a security's price to a change in the underlying volatility of treasury rates
Hold OAS constant and recalculate price for a 1% increase and decrease in interest rate volatility.

Zero Volatility Spread (ZV0)
Spread investor expects to earn if there was no uncertainty about future path of interest rates. The constant spread over the treasury curve which equates the discounted cash flows from today's implied forward curve to the current price of the security.

Spread Duration
Sensitivity of bond price to change in its OAS

Prepayment Uncertainty
Prepayment uncertainty increases the value of an option. Since OAS is compensation for shorting options, the greater prepayment risk => ↑ option cost => ↓ OAS. Thus mortgage-backed securities (MBS, CMBS, CMO) value ↓. CMO value affected less depending on seniority at the tranche. ARM's move with the i-rate-no charge.

Volatility Risk
Debt instrument with embedded options depend on level of interest rates ↑ volatility => ↑ value of option. Investor in mortgage-backed securities (MBS, GMBS, CMO) or callable bond has granted this option at issue so price must fall since value has fallen.

- putable bond value will rise ↑ since investor has option => price ↑
- ARM will not be affected – rate resets

ZV0
related to volatility => with no volatility in interest rates time value of prepayment option drops to zero
true of callable & putable bonds as well

ZV0 is like OAS with assumptions of no uncertainty concern future term structure
Difference in ZV0 and OAS represents time value of option

- small when timing of cash flows and interest rate Δ's insensitive
- difference in ARM's and CMO's due to any cap

Spread Duration

Duration for MBS and ARM, reflect fact that a change in OAS affects the PV of (future cash flows)

But cash flows themselves unaffected by Δ's in OAS.

- Δ's in OAS for callable and putable bond does affect the cash flows an investor receives
- For MBS, measure analogous to MacCarley duration as refinancing motivated by Δ's in level of interest rates.
- For CMO, though, it could be very different.
- For corporate bonds, spread donation = effective duration
  Δ's in OAS could mean difference between refinancing or not