Solution 1

(a) Hierarchy of models:

- If the liability exists as a traded instrument, use the market price of the instrument as the fair value
- If the liability is not an actively-traded instrument but there exists a similar instrument that is traded, use the price of this similar instrument, then adjust for differences between the two to get the fair value
- If neither the first two apply, determine the fair value of the liability as the risk-adjusted present value of future liability cash flows.

As group medical insurance is not actively traded, and there is no similar instrument that is traded, LifeCo should use the third method.

(b) Using the cost of capital approach, we first calculate the valuation interest rate (r_L), then use this to discount the cash flows:

$$r_L = r_A - \left(e \times \left(\frac{r_E}{(1-t)} - r_A \right) \right)$$

Where:
r_L = valuation interest rate
r_A = total return on risky assets
e = ratio of capital to liabilities
r_E = market's required return on capital
t = tax rate

From the case study, r_A equals 6.75% based on the book yield of the assets in LifeCo's group business.

From the case study, r_A equals 6.75% based on the book yield of the assets in LifeCo's group business.

Solution 1 **(continued)**

We assume C is the most similar competitor due to its return on assets. Hence, we will use e and r_E from competitor C.

$$r_L = 0.0675 - \left(0.20 \times \left(\frac{0.16}{(1-0.35)} - 0.0675\right)\right) = 3.18\%$$

The fair value of the group medical policy is 750 / (1.0318) = $726.91

(c) Market Value Margin (MVM) is an adjustment made to the expected cash flow that accounts for the risk of the cashflows and allows discounting at the risk-free rate.

In other words, MVM is the value that makes:

$$\frac{C + MVM}{1 + r_f} = \frac{C}{1 + r_L}$$

Where:

C = liability cashflows

r_L = valuation interest rate

r_f = risk-free rate

In this case, we have

$$\frac{750 + MVM}{1 + 0.05} = \frac{750}{1 + 0.0318}$$

MVM = 763.26 – 750

= 13.26

Solution 2

(a)

Traditional Life segment has:

1. Government bonds – long duration, low required capital, reasonable yield. Good Choice
2. Public investment grade bonds – very long duration, low required capital, high yield. Excellent choice
3. Public high yield bonds – Short duration, high required capital, high yield. Not a good option
4. Private investment grade bonds – long duration (but not as long as liability) medium required capital, high yield. Decent but not excellent
5. Private high yield bonds – Short duration, high required capital, high yield. Not a good option
6. Pass-throughs – short duration, high required capital, high yield. Not a good option.
7. CMOs – Same as pass-throughs
8. Cash & short term – very short duration, low required capital, low yield – need more to keep LifeCo away from a liquidity crisis but don't want too much
9. Commercial Mortgages – short duration, high required capital high yield not suitable for this product
10. Equities – short duration, high required capital, low yield, three strikes, it is out
11. Real estate – short duration, high required capital, high yield, Yield is great but does not meet other objectives

Given our objectives, the best investment would be the public investment grade corporate bonds. Government bonds or private investment grade bonds are also good choices. Too much is invested in equities. This money would be better invested in these bond classes. Reducing the amount invested in high yield bonds and commercial mortgages, and moving this money into public investment grade bonds would also be advised.

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Solution 2 (continued)

((b)

Trading bonds is considerably more difficult than trading stocks. Bonds are thinly traded, even the most liquid bonds are hard to move. LifeCo would have to contact a bond specialist to see if it could buy these bonds. Then the specialist would have to contact other bond dealers to effect a trade. It may take weeks to find a suitable trade partner. Also, since bonds are rarely traded, there is no set market price; the specialist likely has had to apply a sophisticated regression model to estimate the bond's price based on credit quality, coupon, time to maturity, etc. Often, though actual prices differ from model-produced prices.

Illiquidity may also have an impact on LifeCo's ability to sell some bonds. It may need to take a loss to move them.

Solution 3

(a) Trad segment

- Term and whole life insurance
- Long duration
- Guaranteed interest rates
- Maximum loan rates

Non Trad segment

- All universal life
- Minimum guaranteed rate
- Fund transfer risk
- Cashflows that vary with interest rates

(b) Market efficiency is either assumed to be high or low

- If we assume high then we need a passive PM approach. Create index to match.
- If we assume low then we need an active PM approach . Find underpriced assets.
- Real estate market data is harder to get
- I recommend an active approach for real estate

So we use an active approach

- Top-down – look at national level, then regional, then local
- Bottom-up – look at individual property

To select actual properties we could use

- Technical analysis
- or Fundamental analysis

I recommend Fundamental analysis for real estate

(c) Best Practices are

1. Secure Sr Management commitment
   - management must understand ALM
2. clear assignment of roles and responsibilities
3. leverage cashflow testing platform
4. select appropriate metrics
   - must be relevant and actionable measures
5. responsive and effective mitigation

Solution 3 (continued)

Life Co follows these practices

- ALM committee has CEO, CFO, etc involved
- ALM policy is reviewed by Board annually
- ALM committee has functional area reps (investment, pricing, etc)
- Have ALM procedure manual
- Don’t know about cf platform so this could be area for improvement
- LifeCo uses many measures (duration, key rates) and has various guidelines on them
- LifeCo has regular meetings with annuities weekly

Solution 4

(a) Life Co follows these practices

- ALM committee has CEO, CFO, etc involved
- ALM policy is reviewed by Board annually
- ALM committee has functional area reps (investment, pricing, etc)
- Have ALM procedure manual

(b) Traditional Credit Analysis

- This is similar to insurance; it collects premium for the (yield spread above treasuries) to insure against defaults
- Bondholders have been compensated for taking credit risk.
- The drawbacks of this approach
- Most investors can't diversify to get the average return.
- Want to outperform the historical market averages
- Most can't absorb short term losses.

Dynamic Credit analysis

- Equivalent to classic equity analysis; but focus on different factors
- Look at improvement in
  - Debt service coverage
  - Debt as a multiple of cash flow
  - Debt as a percentage of capital
- Analysts look at covenant protections carefully; they analyze event risk carefully
- Liquidity analysis and analyzing default likelihood as specific event is important, as in equity analysis
- Private Company Valuation
- Three main dynamic credit analysis procedures
  1) CreditMetrics: uses transition probability matrix
  2) KMV Approach: Similar to Merton model for default risk
  3) Credit Risk +: Uses the compound Poisson model for loss distribution for the entire portfolio

Solution 5

(a) Objective – Codify plan to manage/address/understand liquidity issues given liability constraints and specify action in time of crisis

Management Oversight

- Management should select those responsible for monitoring liquidity position
- Management should assign a crisis team

Liquidity Measures and Monitoring Reports

- Specify content and frequency of reporting

Constraints

- Specify assets to be retained
- Maximum realized capital loss that can be tolerated

Establish Plan

- Create an action plan to respond to liquidity needs

(b) Internal controls on assessment of credit risk before the transaction

Internal controls on continued monitoring of credit quality

Documentation of provisions to reduce credit risk and enforcement of transaction

Credit enhancement structures

(c) Gain commitment of senior management

Set up system of checks and balances throughout transaction

Make sure back office has required expertise and hardware to perform required accounting

Set up risk committee in charge of overseeing derivative transactions, setting risk limits, monitoring transaction

Independent internal audits to ensure practices adhere to policies

Thorough documentation of all policies and practices

Solution 6

(a) EIA are relatively short-term compared with VA.

The reference index in EIA is normally price index without dividend being reinvested.
VA GMMB tied to account value which get the benefits of dividend.

EIA benefit is like owning a call option
Policyholders of VA own put option.

The EIA option is usually in-the-money at maturity, while the VA GMMB option is less likely to be in-the-money at maturity.

Sellers of EIA contracts normally expect the guarantee would mature in-the-money. And they normally pass the equity risk to a third party by buying call options.

Sellers of VA may expect the contract mature out-of-the-money and the insurer may decide to run the risk without hedging.

Hedging VA guarantee is more complex than hedging EIA and may involve dynamic hedging.

(b) The ratchet premium without life-of-contract guarantee is

$$RP = P\prod_{t=1}^{n}\{1+\max(\alpha(\frac{S_t}{S_{t-1}}-1),0)\}$$

The expected value of RP is, under the iid Q-measure,
$H = E_Q\,[e^{-rn}(RP)]$

$$H = P\{e^{-r} + \alpha(e^{-d}\Phi(d_1) - e^{-r}\Phi(d_2))\}^n$$

$d_1 = (r\text{-}d+ \sigma^2/2)/\sigma$
$d_2 = d_1\text{-}\ \sigma$

$d_1 = (.06\text{-}.02+ .2^2/2)/.2 = .3$
d2 = .3 - .2 = .1

$\Phi(d_1)= \Phi(.3)=.6179$
$\Phi(d_2)= \Phi(.1)= .5398$

$$H = 100 * \{e^{-.06} + .5 * (e^{-.02} * .6179 - e^{-.06} * .5398)\}^7$$

$H = 100*(.941765+ .5*.048643)^7=93.48$

Solution 7

(a)

value of pay fixed swaption is

$LA[s_0N(d1) - s_kN(d2)]$ $\quad A = \frac{1}{m}\sum_{i=1}^{mn} P(0,T_i)$

$$d1 = \frac{\ln(\frac{s_0}{s_k}) + \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}}; d2 = d1 - \sigma\sqrt{T}$$

L = 1,000,000

s_0= [sqrt(exp(0.05))-1] * 2 = 5.06% semi-annual compounding

s_k = 5.2%

T = 5

σ= 0.2

A = ½*[exp(-0.05*5.5) + exp(-0.05*6)] = 0.75020

d1 = 0.16391

N(d1)=0.565101

d2 = -0.28329

N(d2)=0.388474

Swaption premium = $LA[s_0N(d1) - s_kN(d2)]$ = 6,309.56

(b) If the option has not been exercised
XYZ is long an option
Counterparty is short an option
XYZ holds an asset that it can lose if the counterparty becomes bankrupt

If the option has been exercised
XYZ may have an asset or liability depending on the swap value
XYZ maximum loss is the swap replacement value if it is positive
XYZ loss amount can be less if collateralization, netting provisions or downgrade triggers

Solution 7 (continued)

(c) Credit default swap is typically used to protect a bond holder against default by the issuer (reference entity) in case the issuer defaults (credit event)
Periodic payments are paid by the CDS buyer to the CDS seller during CDS term
Payments are made until the CDS term ends or the occurrence of the credit event
If credit event occurs the payment by the issuer is made in cash or by delivering a bond

This is complicated contingent payoff so simulation should be used for valuation

Solution 8

(a)

- Gain from deferred annuity line would help offset term product losses if portfolio is duration-matched in the aggregate
- Duration-matching is a good strategy for dealing with the risk of interest rate fluctuation.
- Can only lower crediting rate to guaranteed minimum rate in policy.
- marketing consideration / early surrender high lapses
- Regulation issue
- Temporary natural hedging only

(b)

- Today - Enter into a series of forward-starting interest rate swaps (pay LIBOR, receive fixed)
- At the times of each premium payment - enter into an offsetting swap; (pay fixed and receive LIBOR), and invest the premium in fixed rate
- Company earns swap rate plus spread
- Risks:
    - counterparty credit exposure
    - can't lock in today's credit spreads Rebalancing
    - Cost of derivatives
    - Accounting issue of using derivatives
    - Swap market may not offer all needed tenor

(c)

- Securitizing the premium flow
    - expected premiums are packaged and sold to the capital markets for cash
    - Benefits
        - can invest cash today at current yields
        - increased earnings
        - reduced interest rate risk
        - increased assets under management
    - Risks
        - deviation in lapse/mortality experience
        - level premium term is lapse-supported; if lapses lower than expected, must set higher reserve

Solution 8 (continued)

OR

- Sell a structured liability
  - sell into the market a liability with cash flows that match the product's cash flows
  - Benefits
    - cash flows are matched
    - lock in profit margin
    - increase assets under management
    - minimizes use of derivatives and FAS 133 implications
  - Risks: expected cash flows may not match actual cash flows

Solution 9

a)

- o The GMMB is vulnerable to poor market returns over the term, using the "actuarial method
- o Poor market returns are represented by the left tail of the stock price distribution
- o Many models when calibrated by standard technique, such as maximum likelihood, tend to be too thin tailed on the left side. This is because MLE is heavily weighted to the centre of the distribution rather then the tails.

b)

$$\frac{S_t}{S_{t-1}} \sim \log N\left(\mu . \sigma^2\right)$$

$$\Rightarrow P_r\left[\frac{S_1}{S_0} \langle 0.75\right] = 0.25$$

$$\Leftrightarrow \Phi\left(\frac{\log 0.75 - \mu}{\sigma}\right) = 0.25$$

$$\Leftrightarrow \frac{\log 0.75 - \mu}{\sigma} = -1.96 \Rightarrow \mu =$$

Similarly $\frac{S_t}{S_{t-5}} \sim \log N\left(5\mu . 5\sigma^2\right)$

$$\Rightarrow \left(\frac{\log 0.77 - 5\mu}{\sqrt{5}\sigma}\right) = -1.96$$

So we have:

1) $\mu = 1.96\sigma + \log 0.75$

and 2) $5\mu = 1.96\sqrt{5}\sigma + \log 0.77 \Rightarrow \mu = \frac{1.96}{\sqrt{5}}\sigma + \frac{\log 0.77}{5}$

Equate 10 and 2) gives

$$1.96\left(1 - \frac{1}{\sqrt{5}}\right)\sigma = \frac{\log 0.77}{5} - \log 0.75 \Rightarrow \sigma = 0.2173$$

Substitute in 1)$\Rightarrow \mu = 0.1382$

Solution 9 (continued)

c)

- o RSLN fits the whole data distribution better

  (Adv) - Calibrated logN is fitted to the left tail, may be a poor fit to the centre and right tail

- o RSLN captures dynamic process better

  (Adv) - volatility bunching

  - association of volatility and low returns

- o LgN is simpler, and is consistent with black Scholes framework.

  (Disadv) -

Solution 10

(a) Collar - range of prepayment speeds which PAC principal repayments do not vary
Narrower collar = weak protection
Prepayments faster than top collar accelerate payments and reduce future collar
Prepayments slower than bottom collar reduce payments, increase future collar and provide extension protection for longer tranches
Prepayments within collar typically widen future collar

Window – Interval over which scheduled prepayments are made
Tighter window = better guaranteed return of CF (more bullet-like), better rolldown of yield curve, experience greater avg. life variability when prepayments outside collar
Wider window likely to outperform tight windows due to wider spread
Shorter window = fewer and larger repayments

Lockout - feature of companion where PAC bonds locked out for period where principal is paid to companions (typically 12-24 months)
Companions absorb all principal in excess of scheduled payments
Lockout reduces call and extension risk

(b) Top collar = better call protection on current coupon over premium coupon
Greater avg. life variability on premium coupon vs. current coupon
Discount coupons greatest stability
Upper collar broken, discount coupon benefits, premium coupon hurt
Lower collar broken, discount coupon hurt, premium coupon benefits

(c) CFs certain as long as prepayments stay within range
MBS shorten as rates fall, lengthen as rates rise. PAC bonds behave more like corporates
PAC bonds provide more call/extension protection than MBS bonds

Solution 10 (continued)

(d) Total prepayments = relocations – assumptions + curtailments + refinancings

Relocations – default, cash paydowns, or equity refinancings
- Affected by economic considerations: home equity levels, mortgage rates, tax deductibility
- Affected by non-economic decisions: age of loan, yearly seasonal cycle, multi-year housing cycle

Assumptions – mortgages assumed by buyer rather than prepaid
- Easier to qualify, minimal transaction costs, no judgment on interest rate timing
- Incentive to assume when current market rates higher than existing mortgage and LTV ratio high

Curtailments – partial prepayments of mortgage
- Small effect at beginning of mortgage, large cumulative effect as pool seasons

Refinancing – interest rate related payment
- Path dependent, need lower lows in mortgage rates to encourage new refinancing
- Burnout – sizable short-term fluctuations in prepayment speeds around a gradual declining trend once a full refinancing has occurred
- Are home owners ready, willing, and able?
- Does pool have experienced refinancers?

(e) Need wider collars
Prefer discount bonds
Prefer presence of companions
Tighter windows
Include lockout

Solution 11

a) 1. RSA uses known quantity, the MV of assets
2. RSA = constant spread, when added to treasures, makes PV (liab + expense)=MVCA)
3. Steps
    1. Calculate market value (A)
    2. Calculate treasury forward rates on same day
    3. for interest-sensitive liabilities, develop set of treasury rate paths
    4. calculate liability + expense CF along each path

b) $RSA = \frac{\$1,000}{\$10,000 - \frac{1}{2} * \$1,000} = 105bp - 30bp$

10 years = 75bp

- need to consider p/h interest sensitive lapses
- could cost more or less

c) excess spread = asset spread – total target spread
= asset spread – (RSA + credit spread + expenses)
$= 110bp - (75bp + 25bp)$

excess spread = 10bp

- ➢ this means assets are earning 10 bps over what is spread is required – barely positive means may need to look for higher yielding assets

d) RSA measures are limited in the sense that the highest RSA may not always be the best

1. different OAS's may not be available at different sales volume – or for both assets and liabs
2. profit goal may be to maximize total excess spread

e) SPDA's are contrasts that take single premium and accumulate at give rate and then annuitize

- ➢ The book value cash out embedded option is analogus to an interest rate cap (loss to insurer if p/h lapses when rates rise after issue
- ➢ The minimum guaranteed floor in SPDA's are analogus to interest rate floors

Solution 11 **(continued)**

- by pricing an interest rate cap to mimic the BV adjustment in the SPDA, the effect of this option can be quantified instead of using RSA
- Similarly, by pricing an interest rate flow whose state = min guaranteed rate, the option under the SPDA can be priced
- The embedded options within the SPDA make it the hardest to price
- RSA simply adds a spread to the liabilities $\Rightarrow$ cap/flow actually replicate inner-workings of SPDA

f) Swap = long cap + short floor
Similar to prior reasoning, swap can be decomposed into 1) cap and 2) flow to price embedded options with SPDA

Solution 12

(a)

Company:

Long a call on assets A_t to walk away (default)
with maturity in 5 years and strike price L_5^*, the guaranteed payment

Short a call on asset A_t to share the profit
with maturity in 5 years and strike price L_5^*/α,
where α is the current ratio of liabilities to assets

Equity at time t = $C_E(A_t, L_5^*) - \delta\ \alpha\ C_E(A_t, L_5^*/\alpha)$,
where δ is the participation (bonus) level

Policyholder:

Short a put (or call) on assets A_t to default
with maturity in 5 years and strike price L_5^*, the guaranteed payment

Long a call on asset A_t to share the profit
with maturity in 5 years and strike price L_5^*/α

Liabilities at time t = $L_t^* P(t, 5) - P_E(A_t, L_5^*) + \delta\ \alpha\ C_E(A_t, L_5^*/\alpha)$

(b)

Formulas:

$E_t = A_t\left(N(d_1) - \delta\ \alpha\ N(d_3)\right) - P(t, T)\ L_T^*\left(N(d_2) - \delta\ N(d_4)\right)$ (or $= C_E(A_t, L_T^*) - \delta\ \alpha\ C_E(A_t, L_T^*/\alpha)$)

$d_1 = \left(\ln(A_t/(P(t, T)\ L_T^*)) + \sigma^2 (T - t)/2\right) / \left(\sigma\ \text{sqrt}(T-t)\right)$

$d_2 = \left(\ln(A_t/(P(t, T)\ L_T^*)) - \sigma^2 (T - t)/2\right) / \left(\sigma\ \text{sqrt}(T-t)\right)$

$d_3 = \left(\ln(\alpha\ A_t/(P(t, T)\ L_T^*)) + \sigma^2 (T - t)/2\right) / \left(\sigma\ \text{sqrt}(T-t)\right)$

$d_4 = \left(\ln(\alpha\ A_t/(P(t, T)\ L_T^*)) - \sigma^2 (T - t)/2\right) / \left(\sigma\ \text{sqrt}(T-t)\right)$

$L_T^* = L_0 \exp(r^* T)$

Inputs:

$t = 0$, $T = 5$, $r^* = 0.04$, $\delta = 0.85$, $\alpha = 0.95$, $\sigma = 0.2$, $L_0 = 95$, $A_0 = 100$, $P(0,5) = 0.8$

Calculations:

$L_T^* = 116.03$

$d_1 = 0.39005$, $N(d_1) = 0.6518$

$d_2 = -0.05716$, $N(d_2) = 0.4772$

$d_3 = 0.27536$, $N(d_3) = 0.6085$

$d_4 = -0.17186$, $N(d_4) = 0.4318$

Answer:

$E_t = 5.81$

Solution 12 (continued)

(c)

To avoid subsidy

$$\delta = (C_E(A_t, L_T^*) - (1-\alpha) A_0) / (\alpha\, C_E(A_t, L_T^*/\alpha))$$

$$C_E(A_t, L_T^*) = A_t\, N(d_1) - P(t, T)\, L_T^*\, N(d_2)$$

$$C_E(A_t, L_T^*/\alpha) = A_t\, N(d_3) - P(t, T)\, L_T^*/\alpha\, N(d_4)$$

$C_E(A_t, L_T^*) = 20.88$, $C_E(A_t, L_T^*/\alpha) = 18.66$

$\delta = 89.58\%$

Solution 13

(a)

P = 1 million
(P* - P) / P = Sum[-D(i) * d(i)]

Scenario 1. ΔLiab = - (2+1.5+2+1) * 1% * 1 mil = - $65,000
ΔAsset = - 6.5 * 1% * 1 mil = - $65,000
Change in NPV = ΔAsset – ΔLiab = - 65,000 – (-65,000) = 0

Scenario 2. ΔLiab = - (2* 0.6%+1.5*0.8%+2*1%+1*1.2%) * 1 mil = - $56,000
ΔAsset = - 6.5 * 0.8% * 1 mil = - $52,000
Change in NPV = ΔAsset – ΔLiab = - 52,000 – (-56,000) = $4,000

(b)

W(i) = D(i) / T(i)
W(0) = 1 – Sum[W(i)]

W(3) = 2/3 = $666,667 in 3-year zero
W(5) = 1.5/5 = $300,000 in 5-year zero
W(7) = 2/7 = $285,714 in 7-year zero
W(10) = 1/10 = $100,000 in 10-year zero
W(0) = 1,000,000 – 666,667 – 300,000 – 285,714 – 100,000 = - $352,381
i.e. borrowing $352,381 in cash

(c)

No
Portfolio (b) is not cash flow matching
Convexity risk remains
Need to rebalance continuously
Liability cash flow very sensitive to assumption change, e.g. lapse

Solution 14

(a)

ExposedCo receives revenues in Euros, pays expenses in C$, Yuan, Euros and US$
Reporting is in US$

So risk exposures are: strengthening of C$ or Yuan and weakening of Euro (versus US$)

(b)

Merck model applied to ExposedCo:

I Assess potential future fx movements / volatility
- Determine likely ranges of fx rates
- Consider factors affecting fx rates
- Impact of government policies
- Consider outside expert forecasts

II Examine impact of fx movements on strategic plans
- impact on financial results under various scenarios

III Determine whether to hedge
- External considerations
  - Impact on share price
  - Investor / clientele behavior
  - Impact on dividends
- Internal considerations
  - Impact on financial plans

IV Determine appropriate instrument
- Options, futures, swaps could be used
- ExposedCo wants to minimize gains AND losses from fx
- Also wants to minimize cost
- Swaps or futures most appropriate since no upfront cost and lock in rates
- (no gains or losses if fully hedged)

V Implement hedging program
- Determine term and amount of instruments
- Develop simulation model

monitor effectiveness of hedge and rebalance as needed

Solution 15

(a)

Long-run real GDP growth = labor growth + productivity growth
= 2.50% + 0.90% = 3.40%

Equilibrium real interest rate = long-run real GDP growth = 3.40%

Expected inflation = Nominal bond yield – inflation-indexed bond yield – inflation risk premium = 4.50% - 1.80% - 0.30% = 2.40%

Equilibrium nominal GDP growth = 3.40% + 2.40% = 5.80%

Equilibrium Fed Funds rate = equilibrium nominal GDP growth = 5.80%

Equilibrium 10-year treasury yield = Fed Funds rate + inflation risk premium
= 5.80% + 0.30% = 6.10%

Equilibrium yield on aggregate portfolio = equilibrium 10-year treasury yield plus 0.35% = 6.45%

(b)

Unbiased expectations theory (UEH) – forward rates are unbiased predictors of future spot rates

(c)

Empirical evidence mostly rejects the UEH – when yield curve is rising, short rates tend to rise but yields on long bonds tend to fall over their remaining lives. Forward rates usually overestimate actual future rates. The term premium, which is composed of the risk premium and the convexity premium, is not zero: the two premiums do not net to zero.

(d)

The trustees should lower the assumption for the long-term expected return on fixed income from 7.50% to 6.45%. The UEH does not hold empirically

Solution 16

The values of the option on the boundary are given by
f(S, 3)=max(1300 - S,0) when S > 1200
f(1200,t) = 0 when 0<=t<=3
Use one unit of Option L to match the first boundary.

Then match at t=2.
Choose a regular 3-year European put option with a strike price of 1200 (Option I).
It is worth 75.91 at the 2-year point when S=1200.
The position in option L is worth 126.00 at this point.
The position we require in option I is therefore -126/75.91=-1.66.
Next to match the second boundary condition at t=1.
Use Option H.
It is worth 75.91.
Our position in Option I and L is worth -16.32 at this point.
We require a position in option H 16.32/75.91=0.22.

At t=0, use option G.
It is worth 75.91. Our position in Option H, I and L is worth -5.19 at this point.
We require a position in option G -5.19/75.91=0.07.

Solution 17

a) The stock pays no dividends. Stocks do pay dividends. Adjustments to the formula can now account for dividends

An investor's trades do not affect the taxes paid. Different investors pay taxes at different rates.

Investor pays no transaction costs. Trading costs make it hard for an investor to create an option-like payoff by trading in the underlying stock. It becomes harder to take a gain from the arbitrage.

Volatility is known and doesn't change. In reality the volatility is only an estimate and can change over the life of the option (called volatility of volatility).

Interest rates remain constant. Interest rates may change over time and it is unknown how they will change.

Stock price changes smoothly. Stocks actually can jump up or down quickly when major news is released.

Unrestricted borrowing and lending at a single rate. Borrowing rates will be higher than lending rates for an individual investor.

No penalties or costs associated with short selling a stock. In reality you can only short sell after an up tick in the stock. Investor must go thru the expense of borrowing the stock before he can sell it, may require collateral.

Exercise occurs only at maturity. (Option is European.) Most of options are American options.

No takeovers or other event to end the life of the option early.

Solution 17 **(continued)**

b) Present Value of dividends is $\sum_t dividend_t e^{-r_f t}$

$= 0.25e^{-0.25^{*5\%}} + 0.25e^{0.5^{*}5\%} + 0.25e^{-0.75^{*}5\%} + 0.25e^{-15\%^{*}} = 0.9693$

Reduce the current stock price by the present value of dividends.
S=50-0.9693=49.0307

K=50,r=5%, $\sigma = 20\%, T = 1$

$$d_1 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} = \frac{\ln\left(\frac{49.0307}{50}\right) + \left(0.05 + \frac{0.2^2}{2}\right)*1}{0.2\sqrt{1}} = .25$$

$d_2 = d_1 - \sigma\sqrt{T} = .25 - 0.2\sqrt{1} = .05$

N(-0.25)=0.4013, N(-0.05)=.4801

Put Price = $\mathrm{K}e^{-r^{*}T} \; N(-d_2) - S * N(-d_1)$
Put Price = $50 \; *.4801 - 49.0307 * 0.4013 = 3.1582$

c) Implied volatility

In actual markets, option prices are determined by supply and demand. Implied volatility is the volatility that is implied by option prices observed in the market by iteratively solving for the volatility that equates the BS value to the market value of the options.

Volatility Smile of Equity Options

Volatility smile is a description of implied volatilities at different strike prices.

Implied volatility usually decreases as strike price increases.

Implied distribution has a heavier left tail and a thinner right tail than a lognormal distribution. Possible explanations include equity leverage of a company or “Crashaphobia”

Volatility surface

Volatility surface as two dimensions: Strike price and time to maturity.

Solution 18

(a) Options in Projects:
Option to abandon :
Can end the project at any time.
Similar to an American put option on the value of the project with a strike price equal to the project value less liquidation costs.

Option to expand:
Can make further investment in project if conditions are good.
Similar to an American call option on value of additional capacity with
a strike price equal to cost of expansion.

Option to contract:
Can reduce the scale of the project's operation.
Similar to an American put value on the lost capacity with a strike price equal to the costs saved by contracting the project

Option to extend:
At end of the project's life you have the option to extend the life of the
project by paying a fixed amount.
Similar to a European call option on the project's future value.

Option to defer:
The option to defer the project to a later date.
Similar to an American call option on the value of the project.

(b) First calculate the value of the option to expand the project at the end of year 1.

Node E = Additional Gold Revenue – Expense on Additional Gold Extracted – Expense on Original Gold extracted
Node E = (650-60)*10,000-10*50,000 = 5,400,000
Node F = (570-60)*10,000-10*50,000 = 4,600,000
Node G = (520-60)*10,000-10*50,000 = 4,100,000
Node H = (430-60)*10,000-10*50,000 = 3,200,000
Node I = (360-60)*10,000-10*50,000 = 2,500,000

Option Vale at Node B:
=Exp(-.05)*[(.167)*(5,400,000) + (.333)* 4,600,000 + (.5)* 4,100,000] – 4,000,000
=264,932
Therefore the option would be exercised at node B

Solution 18 **(continued)**

Option Value at Node C:
=Exp(-.05)*(1/3)*[4,600,000 + 4,100,000 +3,200,000] - 4,000,000
=-226,790
Negative so the option will not be exercised at node C

Option Value at Node D:
=Exp(-.05)*[(.5)* 4,100,000 + (.333)* 3,200,000 + (.167)* 2,500,000] - 4,000,000
=-639,211
Negative so the option will not be exercised at node D

After determining the point of exercise now calculate the total project:

Value at Node B = PV of Nodes E, F, G + Value at node B + value of the option
= Exp(-.05)*[(.167*(650-50) + .333*(570-50) +.5*(520-50))*50,000-5,000,000] + (550-50)*50,000 – 5,000,000 + 264,932
=39,687,134

Value at Node C
=Exp(-.05)*[(1/3)*[(570-50) + (520-50) + (430-50)]*50,000-5,000,000] + (510-50)*50,000 - 5,000,000
= 34,963,591

Value at Node D
=Exp(-.05)*[(.5*(520-50)+.333*(430-50)+.167*(360-50))*50,000-5,000,000] + (420-50)*50,000 – 5,000,000
=28,401,485

Value at Node A
=Exp(-.05)*(1/3)*[39,687,134 + 34,963,591 + 28,401,485]
=32,675,431
This is greater than the initial investment of 30,000,000 so recommend opening the mine.

(c) May not open the mine after all due to loss aversion as people view losses looming larger than gains.
Regret avoidance
Avoidance of feeling remorse about a decision that leads to a bad outcome
Fashions and Fada
Non-Bayesian Forecasting
Instead of Bayes rule, people make their own probability predictions

Solution 19

(a)

- sample bivariate random values for S1 and S2
- adjust the random values for correlation
- project stock price at end of period using selected process and random values
- use risk-free rate instead of expected rate to project stock prices
- calculate the payoff of the call option
- repeat previous steps for a large number of times to obtain the payoff distribution
- take the average of these payoffs to get the expected payoff at maturity
- discount the expected payoff at the risk-free rate to obtain the call price

(b)

e1=Random Normal Number 1
e2=Rho*Random Normal # 1 + Random Normal # 2*[(1-(Rho^2))^0.5]
e1=0.4
e2=0.8*0.4 + 0.7*[(1-(0.8^2))^0.5]
e2=0.74

S1 = S0*exp[(mu - 0.5*(sigma^2))*dt + sigma*Normal Random #*(dt^0.5)]
Stock 1: S1 = 100*exp[(0.04-0.5*(0.1^2))*1 + 0.10*0.4*(1^0.5)] = 107.79
Stock 2: S1 = 50*exp[(0.04-0.5*(0.2^2))*1 + 0.20*0.74*(1^0.5)] = 59.14

Payoff of the option is MAX[0, Portfolio Value at time 1 - Portfolio Value at time 0]
Portfolio Value at time 1 = 107.79+59.14 = 166.93
Portfolio Value at time 0 = 100.00+50.00 = 150.00
Payoff = MAX[0,166.93-150.00] = 16.93

(c)

The antithetic variate technique pairs each standard normal deviate (Y) with (-Y), so an estimator f1 using Y would be paired with an estimator f2 using (-Y)

Estimator fr would be the average of f1 and f2 (fr = 0.5*(f1 + f2))

This works well because when one value is above the true value, the other tends to be below.

Taking the mean of the fr's rather than the f's reduces variance as each high estimator is paired with a low estimator

Solution 20

(a) Define the following Greeks:

i) Delta

Delta is the change of the option price with respect to change of the underlying stock price, dC/dS

ii) Gamma

Gamma is the second derivative of the option price with respect to the stock price, d^2C/dS^2

iii) Rho

Rho is the change of the option price with respect to change of the interest rate, dC/dr

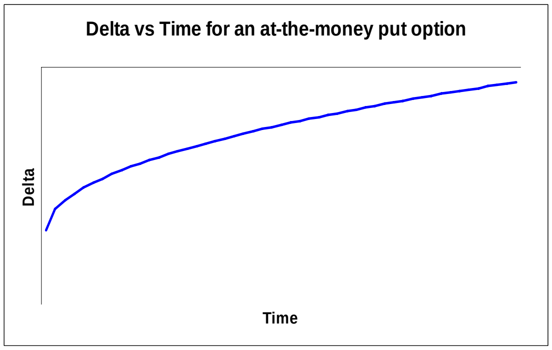
iv) Vega

Vega is the change of the option price with respect to change of the volatility of the underlying stock, $dC/d\sigma$

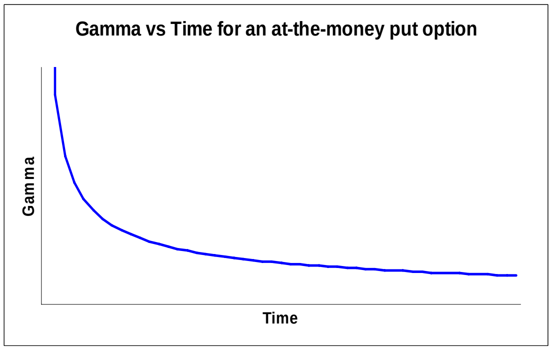
(b) Sketch the curve of each of the above Greeks as a function of time to maturity for an at-the-money put option on a non-dividend paying stock.

Solution 20 (continued)

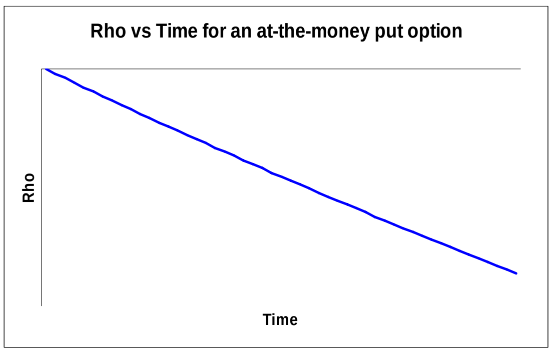
i) Delta



ii) Gamma

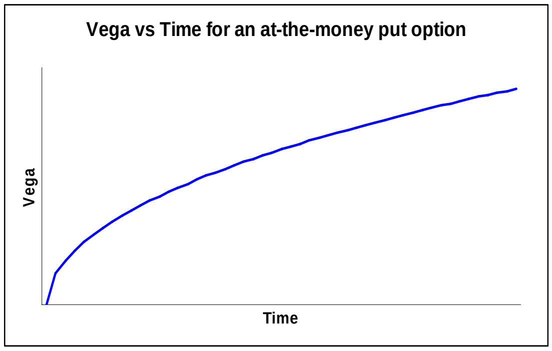


iii) Rho



Solution 20 **(continued)**

iv) Vega



(c) Explain the reasons for the shapes of the curves in b).

i) Delta

Delta for a put option is negative.
It gets less negative as the time to maturity grows longer, because delta = $N(d_1)-1$, and $N(d_1)$ increases as T increases.

ii) Gamma

Gamma is the slope of the delta graph.
Slope of the delta goes to zero, therefore, gamma tends to go to zero when the time to maturity increases.
Short life at-the-money options are highly sensitive to jumps in the stock price, they show very high gammas

iii) Rho

Rho decreases as T increases.
Rho is negative for a put.
At-the-money put options with longer maturities have a long duration (value drops significantly when interest rate increases)

iv) Vega

As volatility increases, price of puts increases.
Vega is positive.
As maturity increases, price of puts increases.
Vega is more positive as T increases.