Solution #1

(a) Aggregate $NC = \frac{PVFB - F}{PVFS} \times \text{Salary}$

$$NC = \frac{67{,}500 - 13{,}500}{PVFS} \times 80{,}000$$

$$PVFS = SAL_{2000} \times \ddot{a}_{\overline{65-x}|} \qquad i = \frac{1.06}{1.04} - 1$$

$$PVFS_{\text{Pat}} = 50{,}000 \times \ddot{a}_{\overline{5}|1.92\%} = 240{,}742$$

$$PVFS_{\text{Chris}} = 30{,}000 \times \ddot{a}_{\overline{25}|1.92\%} = 602{,}393$$

$$PVFS = 240{,}742 + 602{,}393 = 843{,}135$$

$$NC = \frac{67{,}500 - 13{,}500}{843{,}135} \times 80{,}000$$

$$= 5{,}124$$

(b) $\sum NC_{IIA} = \sum \frac{PVFB_X - F_X}{PVFS_X} \times SAL_X$

Allocate assets:

- Kelly gets 7,500

- Sue gets $\frac{50{,}000}{60{,}000} \times (13{,}500 - 7{,}500) = 5{,}000$

- Chris gets $\frac{10{,}000}{60{,}000} \times (13{,}500 - 7{,}500) = 1{,}000$

- Check total = 7,500 + 5,000 + 1,000 = 13,500

$$NC_{\text{Pat}} = \frac{50{,}000 - 5{,}000}{240{,}742} \times 50{,}000 = 9{,}346$$

$$NC_{\text{Chris}} = \frac{10{,}000 - 1{,}000}{602{,}393} \times 30{,}000 = 448$$

$$NC_{\text{Total}} = 9{,}346 + 448 = 9{,}794$$

(c) If plan will be terminated in five years, the aggregate method will not provide sufficient *NC* for Sue. This is due to the "pooling" of their salaries in the calculation of *Agg NC*. The individual aggregate method may be more appropriate since it handles each individual separately. Given the small size of the plan and the short time horizon, this method may be better.

Solution 2

Lifetime pension at retirement age x:

$$(LP_x) = 0.01 \times \text{Sal}_{2001} \times (1+SS)^{x-47} \times [1-0.05(65-x)] \times (x-w)$$

Bridge pension at retirement age 62:

$$(BP_{62}) = 0.005 \times \text{Sal}_{2001} \times (1+SS)^{62-47} \times [1-0.05(65-62)] \times (x-w) = LP_{62} \div 2$$

$$LP_{62} = 0.01 \times 60{,}000 \times 1.03^{62-47} \times [1-0.05(3)] \times (62-32) = \$23{,}836.90$$

$$BP_{62} = 0.005 \times 60{,}000 \times 1.03^{62-47} \times [1-0.05(3)] \times (62-32) = \$11{,}918.45$$

$$LP_{65} = 0.01 \times 60{,}000 \times 1.03^{65-47} \times 1 \times (65-32) = \$33{,}708.17$$

$$NC_{1.1.2002} = \frac{PVFB_w}{PVFS_w} \times \text{Sal}_{2001} \times 1.03$$

$$PVFB_w = \left[LP_{62} \times \ddot{a}^{(12)}_{\overline{62:\overline{5}|}} + BP_{62} \times \ddot{a}^{(12)}_{62:\overline{3}|} \right] \times q_{62} \times \left[\frac{D_{62}}{D_{32}} = \frac{{}^{S}D_{62}}{{}^{S}D_{32}} \div 1.03^{62-32} \right]$$
$$+ LP_{65} \times \ddot{a}^{(12)}_{\overline{65:\overline{5}|}} \times \left[\frac{D_{65}}{D_{32}} = \frac{{}^{S}D_{65}}{{}^{S}D_{32}} \div 1.03^{65-32} \right]$$

Where:

$$\begin{aligned}\ddot{a}^{(12)}_{\overline{62:\overline{5}|}} &= \ddot{a}^{(12)}_{\overline{5}|} + {}_5p_{62} v^5 \ddot{a}^{(12)}_{67} \\ &= 4.348 + \frac{0.93 \times 9.4}{1.06^5} \\ &= 10.8805\end{aligned}$$

$$\begin{aligned}\ddot{a}^{(12)}_{\overline{65:\overline{5}|}} &= \ddot{a}^{(12)}_{\overline{5}|.06} + {}_5p_{65} v^5 \ddot{a}^{(12)}_{70} \\ &= 4.348 + \frac{0.9 \times 8.5}{1.06^5} \\ &= 10.0645\end{aligned}$$

$$\ddot{a}^{(12)}_{62:\overline{3}|} = \ddot{a}^{(12)}_{62} - {}_3p_{62}v^3\ddot{a}^{(12)}_{65}$$
$$= 10.7 - \frac{0.96 \times 9.9}{1.06^3}$$
$$= 2.7203$$

$$PVFB_w = [23836.90 \times 10.8805 + 11{,}918.45 \times 2.7203] \times 0.5 \times \left[\frac{119}{396} \div 1.03^{30}\right]$$
$$+ 33{,}708.17 \times 10.0645 \times \left[\frac{54}{396} \div 1.03^{33}\right]$$
$$= 18{,}061.73 + 17{,}442.05$$
$$= 35{,}503.78$$

$$PVFS_w = \frac{\text{Sal}_{2001}}{1.03^{15-1}} \times \frac{{}^SN_w - {}^SN_{65}}{{}^SD_w}$$
$$= \frac{60{,}000}{1.03^{14}} \times \left[\frac{8084 - 54}{396}\right]$$
$$= 804{,}360$$

$$NC_{1.1.2002} = \frac{35{,}503.78}{804{,}360} \times 60{,}000 \times 1.03 = 2{,}727.80$$

$$AL_{1.1.2002} = NC_{1.1.2002} \times \left(\frac{{}^SN_{32} - {}^SN_{47}}{{}^SD_{47}}\right)$$
$$= 2{,}727.80\left(\frac{8084 - 3234}{250}\right)$$
$$= 52{,}919.32$$

Solution 3

- *AL* and *NC* for Current Provisions:

$$AL_{CP} = B(x)v_{0.065}^{65-x}\ \ddot{a}_y^{(12)}$$
$$= \frac{20 \times 10 \times 12 \times 9.6}{1.065^{65-47}}$$
$$= 7{,}416.34$$

$$NC_{CP} = AL_{CP} \div \text{Service} = \Delta B(x)\ v_{0.065}^{65-x}\ \ddot{a}_{65}^{(12)}$$
$$= 7416.34 \div 10$$
$$= 741.63$$

- *AL* and *NC* for Proposed Provisions:

$$AL_{PP} = 0.01 \times Sal_{2001} \times (1.045)^{60-47} \times \text{Service} \times v_{0.065}^{60-47}$$
$$\times \left[0.15 \times \ddot{a}_{60}^{(12)} + 0.85 \times \left[\ddot{a}_{60}^{(12)} + 0.75\left(\ddot{a}_{55}^{(12)} - \ddot{a}_{60:55}^{(12)}\right)\right]\right]$$
$$= 0.1 \times 36{,}000 \times 1.045^{13} \times 10 \times \frac{1}{1.065^{13}} \times \left[0.15 \times 10.8 + 0.85\left[10.8 + 0.75(11.8 - 9.7)\right]\right]$$
$$= 34{,}154.14$$

$$\Delta AL = AL_{PP} - AL_{CP}$$
$$= 34{,}154.14 - 7{,}416.34$$
$$= 26{,}737.80$$

$$NC_{PP} = AL_{PP} \div \text{Service}$$
$$= 34{,}154.14 \div 10$$
$$= 3{,}415.41$$

$$\Delta NC = NC_{PP} - NC_{CP}$$
$$= 3{,}415.41 - 741.63$$
$$= 2{,}673.78$$

Solution 4

(a) Company contribution at January 1, 2002:

(i) $$IPL\ NC = \sum \frac{(PVFB - AL)}{PVFS} \times S$$

$AL = 0$

Member A

$$PVFB = 1\% \times 30{,}000 \times 1.04^{(60-35)} \times 30 \times \ddot{a}_{60}^{(12)} \times v^{(60-35)} = 56{,}655$$

$$PVFS = \ddot{a}_{\overline{25}|(1.065/1.04-1)} \times 30{,}000 \times 1.04$$
$$= 19.0764 \times 30{,}000 \times 1.04 = 595{,}184$$

$$ILP\ NC = \frac{(56{,}655 - 0)}{595{,}184} \times (30{,}000 \times 1.04) = 2{,}970$$

Member B

$$PVFB = 1\% \times 50{,}000 \times 1.04^{(60-55)} \times 30 \times \ddot{a}_{60}^{(12)} \times v^{(60-55)} = 151{,}850$$

$$PVFS = \ddot{a}_{\overline{5}|(1.065/1.04-1)} \times 50{,}000 \times 1.04$$
$$= 4.7707 \times 50{,}000 \times 1.04 = 248{,}076$$

$$ILP\ NC = \frac{(151{,}850 - 0)}{248{,}076} \times (50{,}000 \times 1.04) = 31{,}830$$

Total Normal Cost = 2,970 + 31,830 = 34,800

Contribution = 34,800 (since no UAL)

(ii) Modified Aggregate $NC = ILP\ NC$ (First year)

Modified Aggregate $NC = 34{,}800$

Contribution = 34,800 (since no UAL)

(b) Accrued liability and company contribution at January 1, 2003:

(i) $$ILP\ NC = \sum \frac{(PVFB - AL)}{PVFS} \times S$$

Member A

$$AL = 2{,}970 \times 1.065 = 3{,}163$$

$$PVFB = 56{,}655 \times 1.065 \times \left(\frac{1.1}{1.04}\right) = 63{,}819$$

$$PVFS = \ddot{a}_{\overline{24}|(1.065/1.04-1)} \times 30{,}000 \times 1.10 \times 1.04$$
$$= 18.5110 \times 34{,}320 = 635{,}298$$

$$ILP\ NC = \frac{(63{,}819 - 3{,}163)}{635{,}298} \times 34{,}320 = 3{,}277$$

Member B

$$AL = 31{,}830 \times 1.065 = 33{,}899$$

$$PVFB = 151{,}850 \times 1.065 \times \left(\frac{1.1}{1.04}\right) = 171{,}050$$

$$PVFS = \ddot{a}_{\overline{4}|(1.065/1.04-1)} \times 50{,}000 \times 1.10 \times 1.04$$
$$= 3.8613 \times 57{,}200 = 220{,}866$$

$$ILP\ NC = \frac{(171{,}050 - 33{,}899)}{220{,}866} \times 57{,}200 = 35{,}519$$

Total Normal Cost = 3,277 + 35,519 = 38,796

$F = 34{,}800$ (0% return on fund)

$UAL = AL - F = 37{,}062 - 34{,}800 = 2{,}262$

$$\text{Amortization} = \frac{UAL}{\ddot{a}_{\overline{15}|}} = \frac{2{,}262}{10.0138} = 226$$

Contribution = 38,796 + 226 = 39,022

(ii) Modified Aggregate $NC = \dfrac{(PVFB - F)}{PVFNC} \times NC$

$AL \quad = F = 34{,}800$ (0% return)

$NC \quad = 38{,}796$ (same as *ILP*)

$PVFB \ = 63{,}819 + 171{,}050 = 234{,}869$

$$\begin{aligned} PVFNC &= 3{,}277 \times \ddot{a}_{\overline{24}|(1.065/1.04-1)} + 35{,}519 \times \ddot{a}_{\overline{4}|(1.065/1.04-1)} \\ &= 3{,}277 \times 18.5110 + 35{,}519 \times 3.8613 \\ &= 197{,}810 \end{aligned}$$

Modified Aggregate $NC = \dfrac{(234{,}869 - 34{,}800)}{197{,}810} \times 38{,}796 = 39{,}239$

Contribution $= 39{,}239$ (since no *UAL*)

Solution #5

(a) Accrued liability and normal cost at January 1, 2002:

$$AL = \sum B(x)\ddot{a}_y^{(12)} v^{(y-x)} \quad \text{(Same as Unit Credit in first year)}$$

$$AAN\ NC = \frac{\left(\sum PVFB - \sum AL\right)}{\sum PVFY} \times n$$

Member X

$$AL = 50 \times 12 \times 20 \times \ddot{a}_{62}^{(12)} \times (1 - 0.04 \times 3) \times v^{(62-54)} = 70{,}187$$

$$PVFB = 70{,}187 \times \frac{28}{20} = 98{,}262$$

$$PVFY = \ddot{a}_{\overline{8}|} = 6.4845$$

Member Y

$$AL = 50 \times 12 \times 4 \times \ddot{a}_{62}^{(12)} \times 0.88 \times v^{(62-30)} = 3{,}097$$

$$PVFB = 3{,}097 \times \frac{36}{4} = 27{,}873$$

$$PVFY = \ddot{a}_{\overline{32}|} = 14.2006$$

$$AAN\ NC = \frac{(98{,}262 + 27{,}873 - 70{,}187 - 3{,}097)}{(6.4845 + 14.2006)} \times 2 = \frac{52{,}851}{20.6851} \times 2 = 5{,}110$$

(b) Accrued liability and normal cost at December 31, 2002:

$$UAL_1 = (UAL_0 + NC_0) \times 1.065 - C - I_0 \text{ (Interest @ 6.5\%)}$$

$$C + I_C = 15{,}975$$

$$UAL = (70{,}187 + 3{,}097 + 5{,}110) \times 1.065 - 15{,}975 = 67{,}515$$

$$AL = UAL + F = 67{,}515 + 17{,}500 = 85{,}015$$

$$\begin{aligned} FVFB_x &= 50 \times 12 \times 21 \times \ddot{a}_{55}^{(12)} \times (1 - 0.04 \times 10) \\ &= 12{,}600 \times 12 \times 0.60 = 90{,}720 \end{aligned}$$

$$PVFB_y = 27{,}873 \times 1.065 = 29{,}685$$

$$FVFB_{\text{Total}} = 90{,}720 + 29{,}685 = 120{,}405$$

$$FVFY = \ddot{a}_{\overline{31}|} = 14.05868$$

$$AAN\ NC = \frac{(120{,}405 - 85{,}015)}{14.05868} \times 1 = 2{,}517$$

(c) Reconcile the change in Normal Cost:

NC at $1/1/02\,(U_{31/12})$	=	\$2,555 per active member
NC at $31/12/02\,(U_{1/1})$	=	\$2,517 per active member
Change in NC	=	(\$38) per active member

$$\begin{aligned} U_{31/12} = U_{1/1} - \left(\frac{1}{PVFY_{12/31}}\right) \times \Big\{ &(I - i \times F_{1/1} - I_C - I_P) \\ &- \sum \left(q_x \times \left(PVFB_{\exp} - U_{1/1} \times PVFY_{\exp}\right)\right) \\ &+ \sum \left(\Delta PVFB - U_{1/1} \times \Delta PVFY\right) \Big\} \end{aligned}$$

<u>Investment:</u>

Exp'd F	=	\$15,000 × 1,065 = \$15,975
Act'l F	=	\$17,500

$$\text{Decrease in } NC \text{ due to experience} = \frac{(17{,}500 - 15{,}975)}{14.05868} = \$109$$

Retirement:

$$\Delta PVFB = \$98{,}262 \times 1.065 - \$90{,}720$$
$$= \$13{,}929$$

$$\Delta PVFY = \ddot{a}_{\overline{7}|} = 5.841$$

$$\text{Decrease in } NC \text{ due to experience} = \frac{(13{,}929 - 5.841 \times 2{,}555)}{14.05868} = (\$71)$$