Solution #1

(a) **Plan A**

$$AL_{1/1/99} = \frac{\text{Service at } 1/1/99 \times 0.018 \times 1999 \text{ Salary} \times 1.045^{(64-x)} \times \ddot{a}_{65}^{(12)}}{1.07^{(65-x)}}$$

$$= \frac{15 \times 0.018 \times 80{,}000 \times 1.045^{(64-55)} \times 10}{1.07^{(65-55)}}$$

$$= 163{,}178$$

$$NC_{1/1/1999} = \frac{AL_{1/1/1999}}{\text{Service at } 1/1/1999}$$

$$= \frac{163{,}178}{15}$$

$$= 10{,}879$$

Plan B

$$NC_{1/1/1999} = \frac{PVFB_{1/1/1989}}{PVFS_{1/1/1989}} \times 1999 \text{ Salary}$$

$$= \frac{2.0\% \times \text{Svc at } 65 \times \ddot{a}_{65}^{(12)} \times 1.06^{(64-w)} \times 1.08^{-(65-w)}}{\ddot{a}_{30_j}} \times 1999 \text{ Salary where } j = \frac{1.08}{1.06} - 1$$

$$= \frac{2.0\% \times 30 \times 9 \times 1.06^{(64-35)} \times 1.08^{-(65-35)}}{23.1783} \times 75{,}000$$

$$= 9{,}408.75 \text{ (or } 12.545\% \text{ of salary)}$$

$$AL_{1/1/1999} = PVPNC_{1/1/1999} = NC_{1/1/1999} \times S_{10_j}$$

$$= 9{,}408.75 \times 11.09875$$

$$= 104{,}425$$

8P – Solution #1 - Continued

(b) Merged Plan

Normal cost under the frozen initial liability cost method.

$$NC_{1/1/2000} = \frac{\sum PVFB - FIL}{\sum PVFS} \times \sum \text{Salary}$$

$$PVFB^{A}_{1/1/2000} = \frac{1.8\% \times \text{Svc at } 65 \times \ddot{a}^{(12)}_{65} \times 1999 \text{ Salary } \times 1.045^{(64-\text{age at } 1/1/1999)}}{1.07^{(65-\text{age at } 1/1/2000)}}$$

$$= \frac{1.8\% \times (15+10) \times 10 \times 80{,}000 \times 1.045^{(64-55)}}{1.07^{(65-56)}}$$

$$= 291{,}001$$

$$PVFB^{B}_{1/1/2000} = \frac{(1.8\% \times \text{Svc after } 1999 + 2.0\% \times \text{Svc to } 2000) \times \ddot{a}^{(12)}_{65} \times 1999 \text{ Salary} \times 1.06 \times 1.045^{(64-\text{age at } 1/1/2000)}}{1.07^{(65-\text{age at } 1/1/2000)}}$$

$$= \frac{(1.8\% \times 19 + 2.0\% \times 11) \times 10 \times 75{,}000 \times 1.06 \times 1.045^{(64-46)}}{1.07^{(65-46)}}$$

$$= 272{,}838$$

$$PVFS^{A}_{1/1/2000} = 80{,}000 \times 1.045 \times \left(\ddot{a}_{\overline{9}|j} = 8.203164\right) = 685{,}785 \quad \text{where } j = \frac{1.07}{1.045} - 1$$

$$PVFS^{B}_{1/1/2000} = 75{,}000 \times 1.06 \times \left(\ddot{a}_{\overline{19}|j} = 15.487496\right) = 1{,}231{,}256$$

8P – Solution #1 - Continued

Frozen Initial Liability for Former Plan A member (under EAN):

$$NC_{1/1/2000} = \frac{PVFB_{1/1/2000} \times v^{(56-w)}}{PVFS_{1/1/1984}} \times 2000 \text{ Salary}$$

$$= \frac{291{,}001 \times 1.07^{-16}}{80{,}000 \times 1.045 \times 1.045^{-16} \times \left(\ddot{a}_{25j} = 19.09955\right)} \times 80{,}000 \times 1.045 \text{ where } j = \frac{1.07}{1.045} - 1$$

$$= 10{,}437.38 \text{ (or 12.48\% of salary)}$$

$$AL_{1/1/2000} = PVPNC_{1/1/2000} = NC_{1/1/2000} \times \ddot{S}_{16j}$$

$$= 10{,}437.38 \times 19.677489$$

$$= 205.381$$

Frozen Initial Liability for Former Plan B member (Under EAN)

$$NC_{1/1/2000} = \frac{PVFB_{1/1/2000} \times v^{(46-w)}}{PVFS_{1/1/1989}} \times 2000 \text{ Salary}$$

$$= \frac{272{,}838 \times 107^{-11}}{75{,}000 \times 1.06 \times 1.045^{-11} \times \left(\ddot{a}_{30j} = 21.741902\right)} \times 75{,}000 \times 1.06 \qquad \text{where } j = \frac{1.07}{1.045} - 1$$

$$= 9{,}675.31 \text{ (or 12.17\% of salary)}$$

$$AL_{1/1/2000} = PVPNC_{1/1/2000} = NC_{1/1/2000} \times \ddot{S}_{11j}$$

$$= 9{,}675.31 \times 12.711905$$

$$= 122{,}992$$

8P – Solution #1 - Continued

Normal Cost under the Frozen Initial Liability Cost Method

$$NC_{1/1/2000} = \frac{291{,}001 + 272{,}838 - (205{,}381 + 122{,}992)}{685{,}785 + 1{,}231{,}256} \times (80{,}000 \times 1.045 + 75{,}000 \times 1.06)$$
$$= 12.28\% \times 163{,}100$$
$$= 20{,}032$$

(c)

PUC

Expected pattern of AL
- Starts off small
- Grows rapidly as ages of plan participants increase
- AL = PV of accrued benefits at all times.

Expected pattern of NC
- Same pattern as AL
- Individual NC tend to rise more rapidly than salary
- If average age of participants remains fairly stable due to retirements of older members and entry of new, younger members, then NC will remain stable as a % of payroll.

EAN

Expected pattern of AL
- Starts off higher than PUC
- Doesn't grow as rapidly thereafter as population ages.

Expected pattern of NC
- Constant percentage of salary
- As an individual's salary increases, $NC increases by salary escalation.
- Overcomes problem of expontential increase in NC as long as projected benefit remains constant or stable for each member.

FIL

Expected pattern of AL
- Same as EAN for year 0. – Same as EAN in all years if all assumptions are realized.
- Inv. gains (losses) immediately reflected in AL.
- If consistent inv. gain (losses), $AL_{FIL} > AL_{EAN}(AL_{FIL} < AL_{EAN})$.
- If consistent salary increases greater(lower) than assumptions $AL_{FIL} < AL_{EAN}(AL_{FIL} > AL_{EAN})$.

8P – Solution #1 - Continued

Expected pattern of NC

- Same as EAN in first year – Same as EAN in all years if all assumptions are realized.
- Inv. gain (loss) reduces (increases) NC by amortizing over PVFS or PVFY
- If consistent inv. gain (losses) $NC_{FIL} < NC_{EAN}(NC_{FIL} > NC_{EAN})$
- If consistent salary increases greater (lower) than assumptions $NC_{FIL} > NC_{EAN}(NC_{FIL} < NC_{EAN})$

8P – Solution #2

(a) 1/1/2000

$AL = PVPB_x + PV \text{ Term Ben}_x$

Employee A:

$$AL \text{ Term} = 0.015 \times 40{,}000 \times 2 \times \frac{(0.1 \times 10)}{1.07^{65-30}} = 112.39$$

$$\begin{aligned} AL \text{ Retirement} &= 0.015 \times 40{,}000 \times (1.05)^{64-30} \times 10 \times 0.9 \times v^{65-30} \times 2 \\ &= 2657.04 \times 2 \\ &= 5314.08 \end{aligned}$$

$$NC \text{ Term} = \frac{112.39}{2} \times 3 - 112.39 = 56.19$$

$$NC \text{ Retirement} = \frac{AL_{net}}{2} = 2657.04$$

$$AL_A = 5426.47$$

$$NC_A = 2713.23$$

Employee B: 1/1/2000

$$AL_B = 0.015 \times 5 \times 60{,}000 \times (1.05)^{64-40} \times 10 \times v^{65-40} = 26740$$

$$NC_B = \frac{AL}{5} = 5348$$

$$AL_{Total} = 32166.47$$

$$NC_{Total} = 8061.23$$

$$UAL = 32166.47 - 30{,}000 = 2166.47$$

8P – Solution #2 -- Continued

(b) 1/1/2000

$$AL_A = 0.015 \times 3 \times 40{,}000 \times v^{65-41} \times 10 = 1803.95$$
$$NC_A = 0$$
$$AL_B = 0.015 \times 6 \times 70{,}000 \times (1.05)^{64-41} \times 10 \times v^{65-41} = 38149.05$$
$$NC_B = \frac{AL_B}{6} = 6358.18$$

Total $AL = 1804 + 38{,}149 = 39{,}953$

Total $NC = 6358$

(c) $UAL_0 = 2166.47$

Contributions $= (10{,}000 - NC)(1.07) = 2074.48$

Gain on term of Employee A $= (\text{Exp. } AL_A) - (\text{Act. } AL_A)$

$$= (5426.47 + 2713.23)(1.07) - 1803.95$$
$$= 8709.48 - 1803.95$$
$$= 6905.53$$

Loss on Mortality of Employee B $= \text{Exp. } AL_B - \text{Act. } AL_B$

$$= (26740 + 5348) \times 1.07 - 38149.05$$
$$= 3814.89$$

Actual UAL = (5047) or a surplus.

Total Gains $= UAL_0 \times 1.07 - UAL_1$

$$= 2166 \times 1.07 + 5047$$
$$= 7365$$

Exp. Fund = 42,800; Actual = 45,000

Gain on assets = 2200

Total = 2075 + 6905 – 3815 + 2200 = 7365

8P -- Solution #3

Item #1:

Present Value of Future Benefits as at 1.1.99

$$= \left\{2\% \times \$50{,}000 / 1.06 \times \left[1.05^{(64-49)} \times \left(1 - 1.05^{-3}\right) / 0.05 \times 1/3 \times 1.05\right] 40 \times 10 / \left[1.08^{(65-49)}\right]\right\}$$
$$+ \left\{2\% \times \$30{,}000 / 1.06 \times \left[1.05^{(64-29)} \times \left(1 - 1.05^{-3}\right) / 0.05 \times 1/3 \times 1.05\right] 40 \times 10 / \left[1.08^{(65-29)}\right]\right\}$$
$$= 218{,}257 + 74{,}547$$
$$= 292{,}804$$

Present Value of Future Salaries as at 1.1.99

$$= \left\{\$50{,}000 / 1.06 \times \left[1 - (1.05/1.08)^{(65-49)}\right] / [1 - 1.05/1.08]\right\}$$
$$+ \left\{\$30{,}000 / 1.06 \times \left[1 - (1.05/1.08)^{(65-29)}\right] / [1 - 1.05/1.08]\right\}$$
$$= 616{,}142 + 649{,}314$$
$$= 1{,}265{,}456$$

Initial Unfunded Liability as at 1.1.99 (Projected Unit Credit Method)
= (Present Value of Future Benefits as at 1.1.99) × (Service to 1.1.99/Service to age 65)
= 218,257 × (24/40) + 74,547 × (4/40)
= 130,954 + 7,453
= 138,409

Unit Normal Cost as at 1.1.99

$$= \frac{(\text{Present Value of Future Benefits as at 1.1.99} - \text{Initial Unfunded Liability as at 1.1.99})}{\text{Present Value of Future Salaries as at 1.1.99}}$$
$$= (292{,}804 - 138{,}409) / 1{,}265{,}456$$
$$= 12.20074\%$$

Amortization Factor for 15 years

$$= \frac{\left[1 - (1/1.08)^{15}\right]}{\left[1 - (1/1.08)\right]} = 9.244237$$

8P -- Solution #3 -- Continued

Amortization Payment of Initial Unfunded Liability as at 1.1.99

$$=\frac{\text{Initial Unfunded Liability as at 1.1.99}}{\text{Amortization Factor for 15 years}}=\frac{138{,}409}{9.244237}$$
$$=14{,}972$$

Total Employer Cost as at 1.1.99

$$= \text{Normal Cost as at 1.1.99}$$
$$+ \text{Amortization Payment of Initial Unfunded Liability as at 1.1.99}$$
$$=(\text{Unit Normal Cost as at 1.1.99} \times \text{Total Salaries})+14{,}972$$
$$=12.20074\% \times \left(\frac{50{,}000}{1.06}+\frac{30{,}000}{1.06}\right)+14{,}972$$
$$=24{,}180$$

Item #2

Present Value of Future Benefits for ABC Co. employees as at 1.1.00
= (Present Value of Future Benefits for ABC Co. employees as at 1.1.00 × 1.08) × (1.06/1.05)
= ($292,804 × 1.08) × (1.06/1.05)
= 319,240

Present Value of Future Benefits for new employee as at 1.1.00

$$=\left\{2\% \times 30{,}000 \times \left[1.05^{(64-40)} \times \left(\frac{1-1.05^{-3}}{0.05 \times \frac{1}{3} \times 1.05}\right) \times 25 \times \frac{10}{\left(1.08^{65-40}\right)}\right]\right\}$$
$$=67{,}328$$

Total Present Value of Future Benefits for Pension Plan as at 1.1.00
= Present Value of Future Benefits for ABC Co. employees and new employees
= 319,240 + 67,328 = 386,568

8P -- Solution #3 -- Continued

Present Value of Future Salaries for all employees as at 1.1.00

$$= \left\{\$50{,}000 \times \frac{\left[1-(1.05/1.08)^{(65-50)}\right]}{\left[1-(1.05/1.08)\right]}\right\}$$

$$+ \left\{\$30{,}000 \times \frac{\left[1-(1.05/1.08)^{(65-30)}\right]}{\left[1-(1.05/1.08)\right]}\right\}$$

$$+ \left\{\$30{,}000 \times \frac{\left[1-(1.05/1.08)^{(65-40)}\right]}{\left[1-(1.05/1.08)\right]}\right\}$$

$$= 620{,}343 + 677{,}081 + 545{,}975$$

$$= 1{,}843{,}399$$

Initial Unfunded Liability as at 1.1.00
= (Initial Unfunded Liability as at 1.1.99
– Amortization Payment of Initial Unfunded Liability as at 1.1.99) × 1.08
= (138,409 – 14,972) × 1.08 = 133,312

Unit Normal Cost as at 1.1.00

$$= \frac{(\text{Present Value of Future Benefits as at 1.1.00} - \text{Assets as at 1.1.00} - \text{Initial Unfunded Liability as at 1.1.00})}{\text{Present Value of Future Salaries for all employees as at 1.1.00}}$$

$$= \frac{(388{,}568 - 20{,}000 - 133{,}312)}{1{,}843{,}399}$$

$$= 12.65358\%$$

8P -- Solution #3 -- Continued

Item #3

Investment Loss in 1999
= Assets as at 1.1.00 – Expected assets as at 1.1.00
= Assets as at 1.1.00 – (Assets as at 1.1.99 + Total employer cost as at 1.1.99) × 1.08
= 20,000 – (0 + 24,180) × 1.08 = (6,114)

Loss due to salaries increasing in 1999 (for ABC Co. employees as at 1.1.99 being greater than assumed
= (Increase in the present value of future of benefits as at 1.1.99)
– (Unit normal cost as at 1.1.99 × increase in the present value of future salaries as at 1.1.00)

$$= (319{,}240 - 292{,}804 \times 1.08) - 0.1220074 \times (620343 + 677081) \times \left(1 - \frac{1.05}{1.06}\right)$$

= 1,519

Shortfall for new employee as at 1.1.00 (due to inadequate unit normal cost as at 1.1.99)
= Present value of future benefits for new employee as at 1.1.00
– (Unit normal cost as at 1.1.99 × present value of future salaries for new employee as at 1.1.00)
= 67,328 – 0.1220074 × 545,975 = 715

Cross-check results:

Increase in unit normal cost form 1.1.99 to 1.1.00

$$= (0.1265358 - 0.1220074) = 0.004528$$

$$= \frac{(6114 + 1519 + 715)}{1843399} = 0.004528$$

8P -- Solution #4

Age = 60
Service = 30
Spouse = 56

(a) Accrued benefit = 50 × 12 × 30 = 18,000/year

$$_{5|}\ddot{a}_{65}^{(12)} = \frac{N_{65}^{(12)}}{D_{60}} = \frac{55{,}300}{9{,}200} = 6.0108696$$

$$\ddot{a}_{60}^{(12)} = \frac{N_{60}^{(12)}}{D_{60}} = \frac{93{,}000}{9{,}200} = 10.1087$$

$$\ddot{a}_{56}^{(12)} = \frac{N_{56}^{(12)}}{D_{56}} = \frac{136{,}800}{12{,}800} = 10.6875$$

Actuarial equivalent equation is:

$$X\left[\ddot{a}_{60}^{(12)} + 60\%\left(\ddot{a}_{56}^{(12)} - \ddot{a}_{60:56}^{(12)}\right)\right] = 18{,}000\ {}_{5|}\ddot{a}_{15}^{(12)}$$

$$X = \frac{18{,}000(6.0208696)}{\left[10.1087 + 0.6(10.6175 - 10.6)\right]} = 10{,}648\,/\,\text{year}$$

∴ Monthly retirement benefit = 10648 / 12 = \$887

(b) PV of accrued retirement benefit $= 18{,}000\ {}_{5|}\ddot{a}_{65}^{(12)} = 108196$

Accrued liability at age 60 under EAN is

$$B(y)\ddot{a}_{y}^{(12)}\frac{Nw - Nx}{Nw - Ny} \times \frac{D_y}{D_x} = 50 \times 35 \times 12 \times \frac{N_{65}^{(12)}}{D_{65}}\left(\frac{N_{30} - N_{60}}{N_{30} - N_{65}}\right) \times \frac{D_{65}}{D_{60}}$$

$$= 50 \times 35 \times 12 \times \frac{55{,}300}{9200} \times \left(\frac{1288900 - 97200}{1288900 - 58000}\right)$$

$$= 122{,}208$$

∴ Gain due to early retirement is $122{,}208 - 108{,}196 = 14{,}012$

Solution #5

	Retirement Age	$a_x^{(12)}$	Service @ Retirement
A	57	13.6	30
B	60	13.0	29

$$NC \text{ old plan} = \frac{PVFB_e}{PVFY_e}$$

$$PVFB_{eA} = 40 \times 30 \times 13.6 \times 1.07^{-30} \times 12 = 25727$$

$$PVFY_{eA} = \ddot{a}^{62}_{\overline{30}|} = 13.2777$$

$$NC_A = 1938$$

$$NC_B = \frac{40 \times 29 \times 13.0 \times 1.07^{-29} \times 12}{\ddot{a}^{(12)}_{\overline{29}|}} = \frac{25436}{13.1371} = 1936$$

Total $NC = 3874$

<u>Under New Provisions:</u>

$$PVFB'_A = PVFB_A \times \frac{43}{40} = 27656$$

$$PVFB'_B = PVFB_B \times \frac{43}{40} = 27344$$

$$PV \text{ Contributions}_A = 520\ddot{a}_{\overline{4}|}\, v^{16} + 520 \times 16 v^{16} = 4467$$

$$PV \text{ Contributions}_B = 520\ddot{a}_{\overline{10}|}\, v^{19} + 520 \times 19 v^{19} = 3812$$

$$NC'_A = \frac{PVFB'_e - PV \text{ Contribution}_e}{PVFY_e} = \frac{27656 - 4467}{13.2777} = 1746$$

$$NC'_B = \frac{27344 - 3816}{13.1371} = 1791$$

Total $NC' = 3537$

Change in $NC = -337$