Solution 1

(a) $EAN\ AL = PVFB_x - PVFNC_x$

$\left(\text{or } PVFB_x \times \left(N_w - N_x\right) / \left(N_w - N_y\right) \text{ or equivalent}\right)$

where $EAN\ NC = PVFB_w / PVFS_w \times S_x$

Member A

$PVFB_w = 2\% \times \$50,000 \times (1.04)^{17} \times 32 \times \ddot{a}_{60}^{(12)} \times v^{32}$
$= \$115,907$

$PVFS_w = \$50,000 / (1.04)^{14} \times \ddot{a}_{\overline{32}|j}$ where $j = 1.06/1.04 - 1$
$= \$28,874 \times 24.1892 = \$698,433$

$EAN\ NC = \$115,907 / \$698,439 \times \$50,000 = \$8,298$

$PVFB_x = PVFB_w \times 1.06^{14}$
$= \$262,055$

$PVFNC_x = \$8,298 \times \ddot{a}_{\overline{18}|j}$
$= \$127,653$

$EAN\ AL = \$262,055 - \$127,653 = \$134,402$

Member B

$PVFB_w = 2\% \times \$80,000 \times 25 \times \ddot{a}_{60}^{(12)} \times v^{25}$
$= \$111,839$

$PVFS_w = \$80,000 / (1.04)^{24} \times \ddot{a}_{\overline{25}|j}$ where $j = 1.06/1.04 - 1$
$= \$31,210 \times 20.0798 = \$626,685$

$EAN\ NC = \$111,839 / \$626,685 \times \$80,000 = \$14,277$

$PVFB_x = PVFB_w \times 1.06^{24}$
$= \$452,830$

$PVFNC_x = \$14,277 \times \ddot{a}_{\overline{1}|j}$
$= \$14,277$

$EAN\ AL = \$452,830 - \$14,277 = \$438,553$

$UAL = AL - F$
$= \$134,402 + \$438,553 - \$525,000 = \$47,955$

$NC = \$8,298 + \$14,277 = \$22,575$

Solution 1 (continued)

(b) Member A

$PVFB_w$ $= 2\% \times \$70,000 \times (1.04)^{16} \times 32 \times \ddot{a}_{60}^{(12)} \times v^{32}$
$= \$156,029$

$PVFS_w$ $= \$70,000 / (1.04)^{15} \times \ddot{a}_{\overline{32}|j}$ where $j = 1.06/1.04 - 1$
$= \$38,869 \times 24.1892 = \$940,198$

$EAN\ NC$ $= \$156,029 / \$940,198 \times \$70,000 = \$11,617$

$PVFB_x$ $= PVFB_w \times 1.06^{15} = \$373,932$

$PVFNC_x$ $= \$11,617 \times \ddot{a}_{\overline{17}|j} = \$170,310$

$EAN\ AL$ $= \$373,932 - \$170,310 = \$203,622$

Alternatively:

$EAN\ NC$ $= \$8,298 \times \$70,000 / \$50,000 = \$11,617$

$EAN\ AL =$
$(\$134,402 + \$8,298) \times 1.06 \times \$70,000 / \$50,000 / 1.04 = \$203,622$

Member B

$EAN\ AL = \$40,000 \times \ddot{a}_{60}^{(12)}$ or roll-forward: $(\$438,553 + \$14,277)$*1.06
$= \$480,000$

Fund $= (\$525,000 + \$25,000) \times 1.04$
$= \$572,000$

UAL $= \$203,622 + \$480,000 - \$572,000$
$= \$111,622$

NC $= \$11,617$

(c) Total losses = Actual UAL – Expected UAL
$= \$111,622 - \$47,955 \times 1.06$
$= \$60,789$

Asset gain/(loss) = Expected Fund – Actual Fund

Expected Fund $= (\$525,000 + \$25,000) \times 1.06 = \$583,000$

Asset loss $= \$583,000 - \$572,000 = \$11,000$

Solution 1 (continued)

Retirement gain/(loss) = Expected AL – Actual AL

$= PVFB_{60} - \text{Actual AL} = \$111{,}839 \times 1.06^{25} - \$480{,}000$

$= \$0$

Salary gain/(loss) = Actual AL – Expected AL

$= \$203{,}622 - \$203{,}622 \times \$50{,}000 \times 1.04 / \$70{,}000$

$= \$52{,}360$

Contribution gain/(loss) = Actual contribution – Expected NC

$= \$25{,}000 \times 1.06 - \$22{,}575 \times 1.06$

$= \$2{,}571$

Total losses $= \$11{,}000 + \$52{,}360 - \$2{,}571 = \$60{,}789$

Solution 2

(a) $AL_x = B_x \ddot{a}_r^{(12)} \, v^{r-x} \; {}_{r-x}p_x$

$NC_x = b_x \ddot{a}_r^{(12)} \, v^{r-x} \; {}_{r-x}p_x$

Before negotiations.

$$\begin{aligned} AL_{54} &= B_{54}\, \ddot{a}_{65}^{(12)} \, v^{11} \\ &= 50 \times 12 \times 25 \times 9.6 v^{11} \\ &= 72,031 \end{aligned}$$

$$\begin{aligned} NC_{54} &= AL_{54} / (x-e) \\ &= 72,031/25 \\ &= 2,881 \end{aligned}$$

After negotiations.

$$\begin{aligned} AL_{54} &= 25\% @ 55 + 75\% @ 62 \\ &= 0.25 \times \left[B_{54}\ \mathrm{ER}_{55}\ \ddot{a}_{55}^{(12)} v + \mathrm{Bridge}_{55} \mathrm{ER}_{55} \ddot{a}_{55:\overline{10}|} v \right] \\ &\quad + 0.75 \times \left[B_{62} \mathrm{ER}_{62} \ddot{a}_{62}^{(12)} v^8 + \mathrm{Bridge}_{62} \mathrm{ER}_{62} \ddot{a}_{62:\overline{3}|} v^8 \right] \\ &= 0.25 \times \left[57 \times 12 \times 25 \times (1 - 0.04 \times 7) \times 11.8 v + 10 \times 12 \times 25 \times (1 - 0.04 \times 7) \times 7.2 v \right] \\ &\quad + 0.75 \times \left[60 \times 12 \times 25 \times 1 \times 10.3 v^8 + 10 \times 12 \times 25 \times 1 \times 2.7 v^8 \right] \\ &= 37754 + 87689 \\ &= 125,443 \end{aligned}$$

$$\begin{aligned} \mathrm{NC} &= 0.25 \times \left[b_{54} \mathrm{ER}_{55} \ddot{a}_{55}^{(12)} v + \mathrm{Bridge}_{55} \mathrm{ER}_{55} \ddot{a}_{55:\overline{10}|} v \right] \\ &\quad + 0.75 \times \left[b_{62}\ \ddot{a}_{62}^{(12)} v^8 + \mathrm{Bridge}_{62}\ \ddot{a}_{62:\overline{3}|} v^8 \right] \\ &= 0.25 \times 6041 + 0.75 \times 4677 \\ &= 5,018 \end{aligned}$$

$\Rightarrow$ Increase in AL $= 125,443 - 72,031 = 53,412$

$\Rightarrow$ Increase in NC $= 5,018 - 2,881 = 2,137$

Solution 2 (continued)

(b)

$$B_{55} = 57 \times 12 \times 26 \times \left(1 - 0.04 \times 7\right) = 12,804$$

$$\text{Bridge}_{55} = 10 \times 12 \times 26 \times \left(1 - 0.04 \times 7\right) = 2,246$$

$$\begin{aligned} \text{AL}_{55} &= B_{55}\ddot{a}_{55}^{(12)} + \text{Bridge}_{55}\ddot{a}_{55:\overline{10|}}^{(12)} \\ &= 12,804 \times 11.8 + 2,246 \times 7.2 \\ &= 167,258 \end{aligned}$$

$$\begin{aligned} \text{Expected AL}_{55} &= \left(AL_{54} + NC_{54}\right)\left(1 + i\right) \\ &= (125,443 + 5,018) \times 1.065 \\ &= 138,941 \end{aligned}$$

$\Rightarrow$ loss $= 28,317$

Solution 3

(a) svc_{65} = Years of service at age 65

$= 30(\text{A})$

$= 40(\text{B})$

NRB = Projected Normal Retirement Benefit = $1.75\% \times 2006\text{ Salary} \times$
$1.04^{(65\text{-Age-1})} \times svc_{65} =$

Participant A's NRB $= 1.75\% \times \$120,000 \times 1.04^{9} \times 30$
= $89,668.64

Participant B's NRB $= 1.75\% \times \$30,000 \times 1.04^{34} \times 40$
= $79,680.64

PVFB = NRB $\times a_{65} \times (1.07)^{(\text{Age}_{\overline{65|}})}$

Participant A's PVFB $= 89,668.64 \times 10 \times 1.07^{-10} = \$455,829.90$

Participant B's PVFB = $79,680.64 \times 10 \times 1.07^{-35} = \$74,631.23$

Total = $455,829.90+$74,631.23 = $530,461.13

$j = 1.07/1.04 - 1 = 2.88\%$

$a_{65-age:j} = (65 - \text{Age})$ year certain annuity using j% interest =
= 8.8282 (A)
= 22.4842 (B)

PVFS = 2005 Salary $x\ a_{65-age:j} =$

Participant A's PVFS = $120,000 \times 8.8282$
= $1,059,384

Participant B's PVFS $= 30,000 \times 22.4842$
= $674,526
total = $1,733,910

Aggregate Normal Cost = (Total PVFB – Assets) / Total PVFS ×Total 2006 Salary
$= (530,461.13 - 300,000)/1,733,910 \times 150,000$

= $19,937.12

(b) Allocate Assets = Individual PVFB / Total PVFB × Assets

Participant A's Allocated Assets = 455,829.90/530,461.13×300,000
$= \$257,792.63$

Participant B's Allocated Assets $= 74,631.23/530,461.13 \times 300,000$
= $42,207.37

Solution 3 (continued)

Individual Aggregate Normal Cost = (Individual PVFB – Allocated Assets)/ Individual PVFS × 2006 Salary

Participant A's NC $= \left(455{,}829.90 - 257{,}792.63\right) / 1{,}059{,}384 \times 120{,}000$

= \$22,432.35

Participant B's NC $= \left(74{,}631.23 - 42{,}207.37\right) / 674{,}526 \times 30{,}000$

= \$1,442.07

Total = \$23,874.42

Solution 4

(a) **Pension at Retirement under normal form =**

B_x = 1.5% of final salary × service × early retirement reduction
Final salary = 50,000
Service = 20
Early retirement reduction = (65 - 57) $\times$ 0.04 =32%

Pension = 1.5% $\times$ $50{,}000 \times 20 \times (1 - 0.32)$
= 10,200

Pension at retirement under optional form

Pension under normal form $\times \ddot{a}_{57}^{(12)} / \ddot{a}_{57:55(75\%)}^{(12)}$

$\ddot{a}_{57:55(75\%)}^{(12)} = \ddot{a}_{57}^{(12)} + 0.75 \times \left(\ddot{a}_{55(\text{spouse})}^{(12)} - \ddot{a}_{57:55}^{(12)} \right)$

$= 12.1 + 0.75 \times (13.5 - 11.2)$

$= 13.825$

Annual pension at January 1, 2006 = 10,200 $\times$ 12.1/13.825
= $8,927

(b) **<u>Assuming retirement</u>**:

AL = $\$8,927 \times 13.825 = \$123,420$
Normal Cost = 0

Funded Ratio = 117,500/123,420 = 95.2%
Thus no extra contribution in 2007
Company contribution in 2006 (scenario A) = $0

<u>Assuming no retirement</u>:

AL $= B_{60} \times \ddot{a}_{60}^{(12)} \times v^{(60-57)}$
NC = AL/service

$\text{B}_{60} = 1.5\% \times \text{salary} \times (1 + ss)^{(59-57)} \times \text{service} \times (1 - \text{ERR})$

$= 1.5\% \times 50{,}000 \times (1.04)^2 \times 20 \times (1 - 0.04 \times (65 - 60))$

= $12,979

AL $= \$12{,}979 \times 11.4 \times 1.06^{-3}$
= $124,230

Solution 4 (continued)

Normal Cost = $124,230/20
= $6,211
Funded Ratio = 117,500/124,230 = 94.6%
Extra contribution in 2006 = 124,230 – 117,500= $6,730
Total Company contribution = $6,211 + $6,730
= $12,941

Difference = $12,941

Solution 5

(a) $NC_{FIL} = \dfrac{\text{PVFB-UL-Assets}}{\ddot{a}}$

Initial liability for FIL method is based on
UAL = accrued liability under EAN

$PVFB_{J,e} = (50 \times 12) \times 40 \ddot{a}_{60}^{(12)} v^{40} = 28000$

$NC_J = \dfrac{28000}{\ddot{a}_{\overline{40|}}} = 1756$

$AL_J = 1756 \ddot{s}_{\overline{10|}} = 24534$

$PVFB_{Ke} = (50 \times 12) \times 40 \ddot{a}_{60}^{(12)} v^{40} = 28000$

$NC_K = \dfrac{28000}{\ddot{a}_{\overline{40|}}} = 1756$

$AL_K = 1756 \ddot{s}_{\overline{20|}} = 68471$

$AL = 24534 + 68471 = 93005$

$PVFB_{J,30} = (50 \times 12 \times 40) \ddot{a}_{60}^{(12)} v^{30} = 50144$

$PVFB_{K,40} = (50 \times 12 \times 40) \ddot{a}_{60}^{(12)} v^{20} = 89800$

PVFB = 139944

$NC_{FIL} = \dfrac{\text{139944-93005-0}}{\underbrace{\tfrac{1}{2}\left(\ddot{a}_{\overline{30|}} + \ddot{a}_{\overline{20|}}\right)}_{13.374419}} = 3510$

$\text{Contribution} = 3510 + \dfrac{93005}{\ddot{a}_{\overline{15|}}} = 12544$

Solution 5 **(continued)**

(b) $\text{Assets}_{1/1/07} = 12544 \times 1.15 = 14426$

must find UL at 1/1/07

$$\text{UL} = 93005 \frac{\ddot{a}_{\overline{14}|}}{\ddot{a}_{\overline{15}|}} = 89009$$

At 1/1/2007:

$$\text{PVFB}_{\text{J},31} = \left(50 \times 12 \times 40\right) \ddot{a}_{60}^{(12)} v^{29} = 53153$$

$$\text{PVFB}_{\text{K},41} = \left(50 \times 12 \times 40\right) \ddot{a}_{60}^{(12)} v^{19} = 95188$$

PVFB = 148341

$$\text{NC} = \frac{\text{148341-89009-14426}}{\underbrace{\tfrac{1}{2}\left(\ddot{a}_{\overline{29}|} + \ddot{a}_{\overline{14}|}\right)}_{13.116884}} = 3424$$

$$\text{Contrib} = 3424 + \frac{93005}{\ddot{a}_{\overline{15}|}} = 12458$$