Exam QFICORE

MORNING SESSION

Date: Tuesday, October 28, 2014
Time: 8:30 a.m. – 11:45 a.m.

INSTRUCTIONS TO CANDIDATES

General Instructions

1. This examination has a total of 100 points. It consists of a morning session (worth 60 points) and an afternoon session (worth 40 points).

    a) The morning session consists of 9 questions <u>numbered 1 through 9</u>.

    b) The afternoon session consists of 6 questions <u>numbered 10 through 15</u>.

    The points for each question are indicated at the beginning of the question.

2. Failure to stop writing after time is called will result in the disqualification of your answers or further disciplinary action.

3. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions on the exam booklet.

Written-Answer Instructions

1. Write your candidate number at the top of each sheet. Your name must not appear.

2. Write on only one side of a sheet. Start each question on a fresh sheet. On each sheet, write the number of the question that you are answering. Do not answer more than one question on a single sheet.

3. The answer should be confined to the question as set.

4. When you are asked to calculate, show all your work including any applicable formulas. When you are asked to recommend, provide proper justification supporting your recommendation.

5. When you finish, insert all your written-answer sheets into the Essay Answer Envelope. Be sure to hand in all your answer sheets because they cannot be accepted later. Seal the envelope and write your candidate number in the space provided on the outside of the envelope. Check the appropriate box to indicate morning or afternoon session for Exam QFICORE.

6. Be sure your written-answer envelope is signed because if it is not, your examination will not be graded.

Tournez le cahier d'examen pour la version française.

BEGINNING OF EXAMINATION
Morning Session

1. (*7 points*) You are an investment actuary investigating the usefulness of a mean-reverting model based on the process X_t defined by:

$$dX_t = -\alpha X_t\, dt + dW_t$$

where $\alpha > 0$ and W_t is a Wiener process.

The initial value of the process is $X_0 = x_0$.

 (a)   (*1.5 points*) Verify that X_t satisfies the integral equation:

$$X_t = x_0 e^{-\alpha t} + e^{-\alpha t} \int_0^t e^{\alpha s}\, dW_s$$

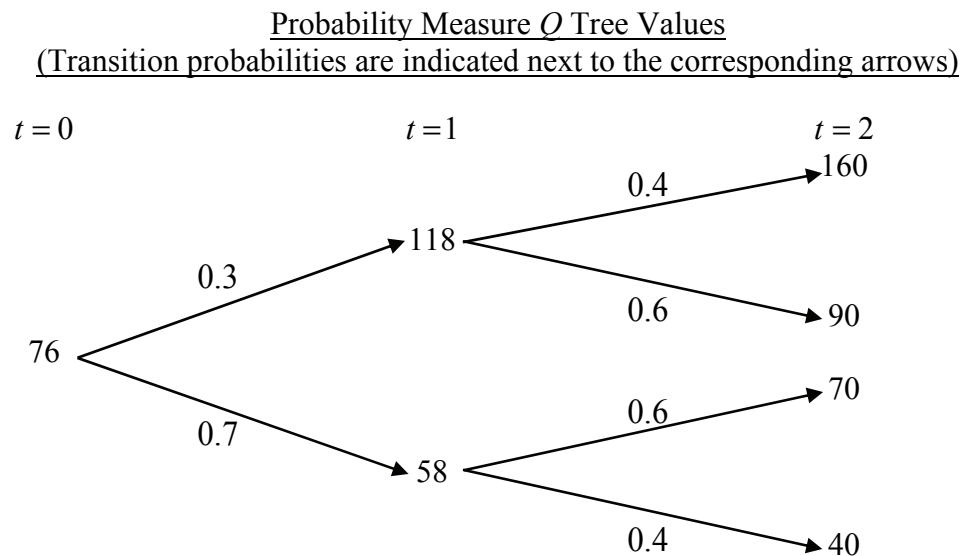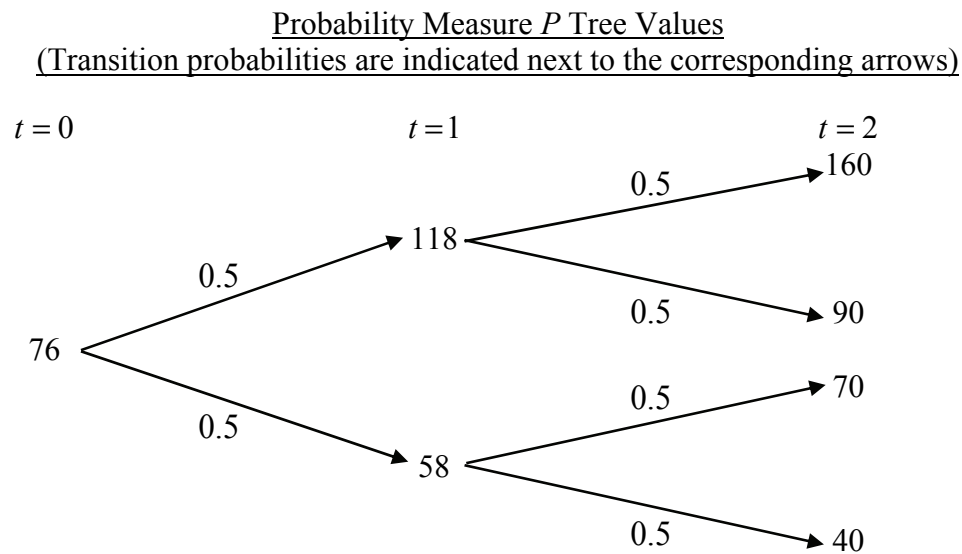based on the function $F_t = X_t e^{\alpha t}$ and using Ito's lemma.

 (b)   (*3 points*) Derive, using part (a), closed-form expressions for:

 (i)   Expected value, $E(X_t)$

 (ii)   Variance, $Var(X_t)$

 (c)   (*0.5 points*) Explain why the process $\{X_t\}$ is a zero mean reverting process and interpret the parameter α.

You read the following comment:

> *"We live in a world primarily driven by random jumps, and tools designed for random walks address the wrong problem. We should be using fractal (scalable) models."*

 (d)   (*2 points*) Compare the key characteristics of Guassian (nonscalable) and fractal (scalable) models, and their effectiveness in assessing financial strategies.

2. (*6 points*) The following 2 binomial trees show the possible values for a stochastic process at times 0, 1, and 2 for two different probability measures P and Q that apply to the same tree:

<u>Probability Measure P Tree Values</u>
(Transition probabilities are indicated next to the corresponding arrows)

<u>Probability Measure Q Tree Values</u>
(Transition probabilities are indicated next to the corresponding arrows)

(a) (*1 point*) Explain why P and Q are equivalent measures.

(b) (*2.5 points*) Determine under which measure P or Q the process is a martingale.

(c) (*1.5 points*) Calculate the one-period standard deviation from time $t = 0$ to $t = 1$ under measure P.

(d) (*1 point*) Calculate the value of the Radon-Nikodym derivative of Q with respect to P for time 1.

3. (*5 points*)

    (a) (*1 point*) Define and explain the following concepts associated with implied volatility:

        (i) Volatility smile

        (ii) Volatility surface

    (b) (*2 points*) Compare the advantages and disadvantages of using stochastic volatility vs. constant volatility in the Black-Scholes model.

You are asked to model a portfolio of equity options and currency options. The following GARCH (1, 1) model is used to set the volatility for an option pricing model that is based on a lognormal distribution.

$$\sigma_n^2 = w + \alpha \mu_{n-1}^2 + \beta \sigma_{n-1}^2, \text{ where } \alpha = 10\%, \beta = 70\%, \text{ and } w = 0.3\%.$$

You observe that $\mu_0 = \mu_1 = 15\%$.

    (c) (*2 points*) Calculate the change in σ_2^2 when σ_0^2 is decreased from 8% to 7%.

4. (*6 points*) You are given the task to model two correlated asset variables that are simulated using the same random process. They follow the following stochastic differential equations:

$$dX_t = 0.15dt + 0.25dW_t$$

$$dS_t = 0.05S_tdt + 0.15S_tdW_t$$

$$X_0 = S_0 = 1$$

W_t is a Wiener process.

(a) (*2 points*) Calculate $E\left(\ln\left(S_t\right)/t\right)$ using Ito's lemma.

(b) (*1 point*) Explain why $\ln\left(E\left(S_t\right)\right)/t$ is greater than the answer from (a).

(c) (*1 point*) Calculate the 95% confidence interval of X_5.

(d) (*2 points*) Calculate S_5, given $X_5 = 3$.

5. (*9 points*) Given the following information you are asked to estimate the price C of a European call option on a default-free zero-coupon bond using the Black-76 pricing formula:

- The maturity of the bond is U years;
- The expiration of the option is T years where T is much shorter than U;
- K = strike price of the option;
- The price at time t of a default-free zero-coupon bond maturing at time m, $P(t,m)$, satisfies the SDE below:

$$\frac{dP(t,m)}{P(t,m)} = r(t)\,dt + \sigma(t,m)\,dX_t, \text{ where}$$

  - $r(t)$ is the risk-free rate at time t;
  - $\sigma(t,m)$ is volatility of $P(t,m)$; and
  - X_t is a Wiener process.

- The Black-76 pricing formula gives the price of the option as:

$$C = P(0,T)(FN(d_1) - KN(d_2)), \text{ where}$$

  - $F = E[P(T,U)]$ is the forward price at T of the bond maturing at U;

  - $d_1 = \dfrac{\ln\left(\dfrac{F}{K}\right) + \dfrac{V}{2}}{\sqrt{V}}$, $d_2 = d_1 - \sqrt{V}$;

  - V = variance of $\ln(P(T,U))$; and
  - both the aforementioned expectation and variance are under the equivalent martingale measure.

You now need to derive formulas for F and V. First you define $G(t) = \dfrac{P(t,U)}{P(t,T)}$. You note that $F = E[G(T)]$ and V = variance of $\ln(G(T))$ since $P(T,T) = 1$.

(a) (*1.5 points*) Derive the SDE of $\ln(G(t))$.

(b) (*2.5 points*) Explain how you can use Girsanov theorem to claim that there exists a martingale measure under which the drift of $G(t)$ vanishes.

5.   **Continued**

(c)   (*3 points*)

(i)   Derive the SDE of $G(t)$ under the martingale measure in (b).

(ii)   Determine the drift-shift needed to create the martingale measure in (b).

(d)   (*2 points*)

(i)   Express F in terms of $P(0,T)$ and $P(0,U)$.

(ii)   Express V as an integral involving $\sigma(t,T)$ and $\sigma(t,U)$.

6. (*9 points*) Your CFO asked you to investigate a European asset S. After some research, you found out the asset follows a special model where the probability of an annual up-movement decreases every year for 5 years:

$$S_0 = 1$$

$$S_t = \quad 1.2 \times S_{t-1} \text{ with probability of } 0.9 - t/10 \text{ (for } t=1, 2, 3, 4, \text{ and } 5)$$

$$0.95 \times S_{t-1} \text{ with probability of } 0.1 + t/10 \text{ (for } t=1, 2, 3, 4, \text{ and } 5)$$

$$1.2 \times S_{t-1} \text{ with probability of } 0.3 \text{ (for } t \geq 6)$$

$$0.95 \times S_{t-1} \text{ with probability of } 0.7 \text{ (for } t \geq 6)$$

S_t is the value of S at time t in Euro.

An exotic option C pays $\ln(S)$ at $t = 6$ only if S moves up in the first year.

(a) Let Z represent the number of annual up-movements of asset S for $t \leq 6$:

    (i) (*1 point*) Derive the price of S at $t = 6$ as a function of Z.

    (ii) (*3 points*) Calculate the expected payout of C.

6. Continued

Assume the exchange rate X in Dollar per Euro is determined by the following stochastic differential equation:

$$dX_t = \left(i_{us,t} - i_{euro,t}\right) X_t \, dt + \sigma_t X_t \, dW$$

$$X_0 = 1.3$$

Where $i_{us,t}$ is the U.S. inflation rate from $t-1$ to t, $i_{euro,t}$ is the inflation rate in Europe from $t-1$ to t.

t	$i_{us,t}$	$i_{euro,t}$	σ_t
1	3%	4%	20%
2	3%	$i_{euro,2}$	20%
3	3%	$i_{euro,3}$	20%

W is a Wiener process.

Assume X_t and S_t are independent and S is tradable only at the end of each year. Let A_t be a simple dollar return of the investment in S over the period $t-1$ to t.

(b)

    (i)   (*2 points*) Derive a formula for $E(A_3)$ in terms of $i_{euro,2}$ and/or $i_{euro,3}$.

    (ii)   (*3 points*) Calculate $i_{euro,2}$ and $i_{euro,3}$ such that the process A_t is a martingale.

7. (*6 points*) You are responsible for a moderate-sized foundation. You are given the following investment guidelines:

- Avoid concentration risk

- Invest in a broad equity market

(a) (*1 point*) Determine, given the guidelines, which of the following indices is the most appropriate to serve as the equity benchmark to evaluate investment performance. Justify your answer.

- Dow Jones Industrial Average Index

- S&P MidCap 400 Index

- Value Line Arithmetic Composite Index

Now suppose that the foundation has given you a mandate to manage an index fund that tracks the S&P 500 index. To construct your index fund, you must choose one of the following methods:

- Full replication

- Optimization

(b) (*1 point*) Recommend the most appropriate method. Justify your answer.

Now suppose that your mandate is to invest with active managers.

(c) (*2 points*) Describe the advantages and disadvantages of the following choices:

(i) Hiring a single manager in *either* a growth *or* value style (but not both).

(ii) Hiring one growth manager and one value manager.

(iii) Hiring one manager with a market-oriented style.

7. Continued

You are analyzing the performance of an active manager whose objective is investing for growth and income, with an orientation to mid-cap stocks within the universe of U.S.-domiciled companies. You can choose from the following indices for use in a returns-based style analysis:

- The S&P/Vanguard 500 Growth and Value indices, which have a large-cap orientation.

- The Russell 2000 Growth and Value indices, which have a small-cap orientation.

- The Russell 1000 Growth and Value indices, which include large-cap and mid-cap shares.

- The Russell Top 200 Growth and Value indices, which together represent the 200 largest market-cap securities in the Russell 1000 Index.

- The Russell Midcap Growth and Value indices, which together represent the 800 smallest market-cap issues in the Russell 1000 Index.

Your assistant recommends the S&P/Vanguard 500 Growth and Value indices and the Russell 2000 Growth and Value indices for the style analysis.

(d)　(*1 point*)　Critique your assistant's selection.

(e)　(*1 point*)　Recommend a more appropriate selection of indices.

8. (*7 points*) LMN Life specializes in selling fixed deferred annuity products with no market value adjustments. The company currently has $9.6 billion of assets backing $8 billion of liabilities. You have been asked to recommend an optimal asset allocation strategy using the liability cash flow as a benchmark. You have the choice of the following four strategic asset allocations (SAAs):

Strategic Asset Allocation	Expected Return $E(R_m)$	Standard Deviation of Return (σ_m)	Asset Duration	Liability Duration
1	9%	17.3%	5.5	5
2	8%	15.0%	6	5
3	7%	10.0%	8	5
4	6%	5.0%	3	5

Portfolio Composition	SAA 1	SAA 2	SAA 3	SAA 4
Equities	30%	25%	20%	15%
Corporate Bonds (NAIC 2)	40%	45%	55%	65%
Structured Assets (NAIC 5)	30%	30%	25%	20%
Total	100%	100%	100%	100%

You are given the following investment objectives and constraints:

- Investment objectives: based on the liability's required return to maintain policy persistency, you are asked to target a 6% minimum risk-adjusted return with 1% target tracking risk. You are also given the following utility function to determine risk-adjusted expected returns:

$$E(R_m) - 0.01\sigma_m^2$$

8. Continued

- Statutory constraints: the company targets to maintain the current S&P rating of "A" by minimizing the use of Risk-Based Capital (RBC). The followings are the current RBC charges applicable to this block of business:

RBC Capital C-1 Risk Charge (Pre-tax)	
NAIC 1	0.4%
NAIC 2	1.3%
NAIC 3	4.6%
NAIC 4	10%
NAIC 5	23%
NAIC 6	30%
Other	30%
RBC Capital C-3 ALM Risk Charge	
The Statutory capital assessment concludes that the C-3 factor under the stochastic capital calculation is always smaller than the 0.6 factor prescribed by the NAIC.	

(a)   (*2 points*)

(i)   Explain what tracking risk is and the meaning of the target tracking risk specified in the investment objectives, assuming normality of excess portfolio returns.

(ii)   Suggest ways to achieve a lower tracking risk.

(b)   (*3 points*)

(i)   Recommend one of the four strategic asset allocations solely based on the investment objectives and constraints listed above.

(ii)   Critique your recommendation in part (i) from the interest rate risk perspective.

(c)   (*2 points*) Recommend two market instruments for hedging the interest rate risk inherent in the recommendation under part (b).

9. (*5 points*) The Organic Cauliflower Grower Association of Greenminded Valley (OCGAGV) represents the interest of its member producers. It advertises the producers, stabilizes the income of its members and promotes healthy living and environmental value.

They created a contingency fund to stabilize the revenue of the members. Members are expected to contribute to the fund, according to their income, during profitable years. In return, during non-profitable or less profitable years, the fund will provide support payments to the members in relation to their needs. This fund has no planned maturity.

This association also encourages among their members the use of the most ecological methods. As long as either fund is administered solely for the benefit of its members, this association is not subject to tax.

(a) (*2 points*) Develop a suitable investment policy for the contingency fund. List all elements.

You are given a universe of possible assets:

Symbol	Stocks	μ	σ
DDP	Deep dig petroleum	10%	35%
ST	Science TV	8%	30%
AIR	African Indian Rice	7%	25%

Symbol	Bonds	Rating	Coupon	Price	Yield to maturity	Term	Duration
GP	Greenpeace	A	5.00%	106.55	4.50%	20	13.40
UST	U.S. Treasury	AAA	3.00%	100.00	3.00%	20	15.41
HT	Here Telecom	BB	7.00%	105.51	6.25%	10	7.67

9. **Continued**

And the correlation matrix among them and with the cauliflower production (OCGAGV):

ρ	DDP	ST	AIR	GP	UST	HT	OCGAGV
DDP	1.00						
ST	0.50	1.00					
AIR	0.25	0.35	1.00				
GP	0.70	0.80	0.55	1.00			
UST	0.20	0.30	0.40	0.95	1.00		
HT	0.40	0.80	0.35	0.75	0.80	1.00	
OCGAGV	-0.25	0.30	-0.85	0.60	0.20	0.30	1.00

(b)   (*2 points*) Discuss the appropriateness of each asset for the contingency fund.

(c)   (*1 point*) Discuss the appropriateness of overweighting bonds relative to equities to support the liabilities:

(i)   In the presence of a deficit

(ii)   In the presence of a surplus

END OF EXAMINATION
Morning Session