QFI ADV Model Solutions
Fall 2014

1. Learning Objectives:

2. The candidate will understand and be able to apply a variety of credit risk theories and models.

Learning Outcomes:

(2b) Demonstrate an understanding of the basic concepts of credit risk modeling such as probability of default, loss given default, exposure at default, and expected loss.

(2k) Demonstrate an understanding of measuring and marking-to-market counterparty credit risk in credit derivatives.

(2l) Understand and apply various approaches for managing credit risk in a portfolio setting.

Sources:

QFIA-104-13: Asset/Liability Management of Financial Institutions, Tilman, Leo M., 2003, Ch. 9

Commentary on Question:

This question tests the candidates knowledge on various aspects of counterparty risk and the measurement and modeling of it via a largely retrieval question.

Solution:

(a) Define:

(i) Counterparty risk
(ii) Counterparty exposure
(iii) Potential future exposure (PFE)

Commentary on Question:

Candidates did well on this part of the question. Candidates generally knew the appropriate definitions. The two items most frequently omitted were:

1. *In counterparty exposure, not referencing that it is the market value that defines the exposure.*
2. *In the PFE, not relating it to some sort of statistical measure.*

1. Continued

(i) Counterparty risk is the risk that a party to a derivatives contract may fail to perform on its contractual obligations, causing losses to the other party. Losses are usually quantified in terms of the replacement cost of the defaulted derivatives.

(ii) Counterparty exposure is the larger of zero and the market value of the portfolio of derivative positions with a counterparty that would be lost if the counterparty were to default and there were zero recovery.

(iii) Potential future exposure (PFE) is the maximum amount of exposure expected to occur on a future date with a high degree of statistical confidence. For example, the 95% PFE is the level of potential exposure that is exceeded with only 5% probability.

(b) Describe two mitigants that reduce counterparty risk, and explain how each reduces this risk.

Commentary on Question:
Candidates did well on this part of the question. Most candidates provided both the correct mitigants and a description. Credit was given only for the discussion of the first two mitigant because the question asked for only two. Only a description of a sentence or two on each of the two mitigants was required for full credit.

1. Netting agreements or rights allow trades to be offset when determining the net payable amount upon the default of the counterparty. Without netting, the position of the non-defaulting party would be a loss of the full value of the out-of-the-money trades against a claim on the total value of the in-the-money trades. With netting, positives and negatives are added first to determine the net payment due.

2. Collateral agreements require counterparties to periodically mark to market their positions and to provide collateral (that is, to transfer the ownership of assets) to each other as exposures exceed pre-established thresholds. Collateral agreements do not eliminate all counterparty risks: market movements can increase the exposure between the time of the last collateral exchange and the time when default is determined and the trades are closed out.

3. Early settlement provisions like liquidity puts and credit triggers reduce credit exposures by shortening the effective maturities of trades.
   a. Liquidity puts give the parties the right to settle and terminate trades on pre-specified future dates.

1. Continued

b. Credit triggers specify that trades must be settled if the credit rating of a party falls below pre-specified levels

(c) Describe four main specifications of a PFE measurement model.

Commentary on Question:
Candidates did relatively poorly with this portion of the question. Several candidates referenced risks instead of specifications. Some candidates provided significant detail on the specifications of the simulation engine but not the other. Credit was also provided for mentioning and describing databases and reporting tools.

Simulation engine: Different market instruments require the specification of different stochastic processes to characterize their evolution through time

Trade pricing calculators: Once a future market scenario is generated, in order to calculate the exposure in that scenario, all trades with the counterparty must be priced

Exposure calculators: After all trades with a counterparty have been re-priced at a scenario/date, exposures can be computed. There are two fundamental concepts for the calculation of exposures: netting and margin nodes

Model validation and control: All the computer code underlying a PFE model is extensively tested during the implementation phase, and re-tested on an ongoing basis via regression tests

(d) Describe two main uses of a PFE model.

Commentary on Question:
Candidates did relatively well on this question. Most candidates provided the two uses, but often the description of one or the other was not sufficient to get full credit. Only two of the uses were considered in giving credit if more than two were provided.

1. Trade approvals against credit line limits: Credit officers set limits on PFE profiles. The limits tend to be wider for short terms and lighter for long terms. In the process of approving new trades, the PFE profile to a counterparty is re-computed including the new trades. The PFE profile is then compared with the limit schedule
2. PFE models also generate the inputs for credit risk valuation. When exposures are uncorrelated with the credit quality of the counterparty, the unconditional expected exposure profile is used for valuation

1. Continued

3. Another application of PFE models is the calculation of economic capital to support the risk of a portfolio of counterparties. The variability of exposures and the possible concentrations on certain market risk factors increases the risk of the portfolio

2. Learning Objectives:

1. The candidate will understand the standard yield curve models, including:
   - One and two-factor short rate models
   - LIBOR market models

   The candidate will understand approaches to volatility modeling.

7. The candidate will understand various investment related considerations with regard to liability manufacturing and management.

Learning Outcomes:

(1k) Define and explain the concept of volatility smile and some arguments for its existence.

(7c) Demonstrate understanding risk management and dynamic hedging for existing GMXB and it embedded options – including:
   (i) Hedgeable components including equity, interest rate, volatility and cross Greeks
   (ii) Partially Hedgeable or Unhedgeable components include policyholder behavior, mortality and lapse, basis risk, counterparty exposure, foreign bonds and equities, correlation and opration failures
   (iii) Static vs. dynamic hedging

Sources:

Volatility Correlation – The Perfect Hedger and the Fox, Rebonato, R., 2nd Edition, Ch 6 (6.2), p. 168-169.

Volatility Correlation – The Perfect Hedger and the Fox, Rebonato, R., 2nd Edition, Ch 7 (7.3), p. 206.

QFIA-116-13: The Impact of Stochastic Volatility on Pricing, Hedging and Hedge Efficiency of Withdrawal Benefit Guarantees in Variable Annuities, p. 535-543.

Commentary on Question:

The objective of the question was to test the understanding of implied volatility derived from the Black Scholes model, how it can vary and create a smile pattern when plotted against the strike price and the magnitude of this smile based on the type and moneyness of the option studied. It also tested the application of this behavior of implied volatility in a hedging situation.

Solution:

(a) Explain the concept of volatility smile.

2. Continued

Commentary on Question:
The candidates performed relatively well on this section. Most candidates were able to partially explain the concept of volatility smile but only a few were able to obtain full credit. Many candidates explained the smile as the relationship between the implied volatility and the strike price while a more accurate explanation is that the smile is the pattern created by plotting implied volatility against strike price. Also, many candidates failed to mention that the implied volatility is derived from the Black Scholes model.

In the Black-Scholes model, stock price volatility is modeled using a constant (sigma).
When trying to fit the model by equating theoretical to market option prices, all other parameters (risk-free rate, maturity, current stock price, strike price) are known or can be observed.
However, if we use market prices and try to solve for the volatility, we get different values for different values for different options.
These implied volatilities depend on the strike and the maturity of the option.
For a given maturity, volatilities depend on the strike price. For strikes close to the stock price, the implied volatilities are typically lower than when the strikes are far from the stock price, which can look somewhat like a smile.

(b)

(i) Identify the empirical fact about equity volatility smiles that is well illustrated in the graph below.

(ii) Identify the empirical fact about equity volatility smiles that is well illustrated in the graph below.

Commentary on Question:
The candidates performed well on this section. The answers for part ii) were generally excellent. In part i), a few candidates described the opposite relationship to what should have been observed between the at-the-money towards out-of-the-money put and call options while others failed to identify that relationship at all.

(i) The volatility smile is more pronounced going from at-the-money puts towards out-of-the-money than in the other direction. For call options, the smile is much less pronounced when going from at-the-money towards out-of-the-money.

(ii) This graph shows that the smile is more pronounced for short maturities (10 days) than for long maturities (1 year).

2. Continued

(c) Describe the hedging strategy of each portfolio, highlighting potential problems related to volatility hedging, if any.

Commentary on Question:
The candidates performed relatively well on this section. Most candidates were able to describe the correct strategy for both portfolios. For portfolio I, some candidates highlighted the lack of volatility hedging while very few stated that the expected profit of the insurer should not change. For portfolio II, only a few candidates explained that the vega was not calculated correctly and even fewer pointed out that this suggested strategy overestimates the change in value caused by a change in volatility.

Hedging portfolio I is a Black-Scholes delta hedge. The hedge accounts for changes in the price of the underlying, but does not account for varying stock volatility. Under this strategy, the expected profit of the insurer should not be affected, but he is exposed to high risks since changes in volatility are not hedged.

Hedging portfolio II is a Black-Scholes delta-vega hedge. It accounts for changes in the price of the underlying and for varying volatility. However, the vega is not calculated correctly and this creates additional risk. The suggested vega overestimates the change in the value of the variable annuity contract caused by a change in volatility, because it assumes constant volatility. In reality, a change in volatility would be short-term and have a smaller effect on the option value. This exposes the insurer to important risks.

(d) Recommend one of the hedging portfolios.

Commentary on Question:
The candidates performed relatively poorly on this section. Most candidates did not recommend the best strategy in this section since most candidates had not identified the main problem of portfolio II strategy. Those candidates who did recommend portfolio II still earned partial credit when they did so in order have a volatility hedge. Very few candidates recommended the best strategy with the right explanation.

Since the delta-vega portfolio (Portfolio II) is incorrectly built, the insurer would be exposed to increased risk by using this hedging strategy. Thus, if these are the only choices, the insurer should use the delta hedging portfolio (Portfolio I), even if it does not protect him from changing volatility.

(e) Suggest improvements to Portfolio II.

2. Continued

Commentary on Question:
The candidates performed poorly on this section. Again, most candidates did not identify the main problem with portfolio II. However, some candidates that had not identified the problem in c) still suggested to change vega and obtained partial credit. A common answer was to suggest that the volatility be modeled stochastically (often with Heston model) which earned partial credit as it implied that vega should be changed. Very few candidates correctly suggested the appropriate changes to the vega hedging ratio and obtained full credit.

In Portfolio II, the vega hedging ratio needs to be modified. It should reflect the fact that not all the maturities react the same way to a change in today's volatility. The derivative of the contract value Ψ_t with respect to σ should be replaced by a sum of similar derivatives taken at different times τ between time *t* and maturity. Each derivative should be weighted by a function of $(\tau - t)$.

3. **Learning Objectives:**

5. The candidate will understand the behavior characteristics of individuals and firms and be able to identify and apply concepts of behavioral finance.

Learning Outcomes:

(5a) Explain how behavioral characteristics of individuals or firms affect the investment or capital management process.

(5c) Identify and apply the concepts of behavioral finance with respect to individual investors, institutional investors, portfolio managers, fiduciaries and corporate managers.

Sources:

Barberis and Thaler, A Survey of Behavioral Finance, Section 3.2 p.15-20. (part (c); 3 points)

Wood. Behavioral Finance and Investment Committee Decision Making, p.6-7 (parts (a) and (b); 3 points)

Commentary on Question:

This question tests the candidate's ability to apply prospect theory and committee bias to a real-world situation.

Solution:

(a) Identify and explain how committee bias applies to the PDC and causes the PDC to repeatedly design poor products.

Commentary on Question:

The candidates performed relatively well on this section. Most candidates identified general committee bias but they did not relate specifically to the PDC situation and identify what caused PDC to repeatedly design poor products.

- Individuals compromise committees, and the behavioral biases of individuals are present. Applies to PDC since applies to all committees
- Committees do not learn from experience. Applies to PDC since new products have repeatedly flopped
- Because feedback was not provided, the committee could not use this learning mechanism.
- Because these outcomes were not provided, the committee could not keep track of its decision.

3. Continued

(b)

(i) Identify and explain characteristics of crowds that differ from the PDC.

(ii) Recommend changes to the PDC to take advantage of these crowd characteristics.

Commentary on Question:
The candidates performed well on this section. In part b(i), most candidates identified the differences between crowds and the PDC. Some candidates only commented on crowds and were not able to contrast both. In part b(ii), most candidates pointed out the benefits of introducing new people to the committee but they missed the recommendation to fix the PDC committee overconfidence issue.

(i)

- Crowds are diverse; PDC consists of ten actuaries who have been with ABC for their entire career.
- Crowds gather information from a wide swath of external sources; PDC has more limited information available.
- Crowds think more individually (*implication from pg 33*); Committee members are overconfident: When asked to give an expected range for new sales, PDC has repeatedly given too narrow a range.

(ii)

- Committees are homogenous: Introduce new people to the committee that have different characteristics.
- Committee needs to gather info from a wide variety of sources: Ensure committee has access to much more information and feedback.
- Committee members are overconfident: Remove some of the overconfident members of the committee.

(c) Explain each of the above observations in the context of Prospect Theory.

Commentary on Question:
The candidates performed relatively poorly on this section. For observation 1, most candidates understood that utility function is concave. For observation 2, many candidates had trouble grasping that one of the main features of prospect theory that differs from rational thinking is that smaller probabilities are given more weight. For observation 3, many candidates confused gain aversion and loss aversion at times. Only few candidates successfully explained all 3 observations.

3. Continued

Observation 1:

- Small bonuses represent many gains while a single large bonus represents a single gain, and this is preferred because
  - Utility is defined over gains and losses rather than final wealth position

Observation 2:

- People overweight unlikely events because:
  - The nonlinear probability transformation
  - Small probabilities are overweighted

Observation 3:

- Bank accounts are less risky than equity markets, and people take less risky actions with gains because
  - People are risk averse over gains
  - The utility function is concave in the domain of gains

4. Learning Objectives:

6. The candidate will understand and be able to describe the variety and assess the role of alternative assets in investment portfolios. The candidate will demonstrate an understanding of the distinguishing investment characteristics and potential contributions to investment portfolios of the following major alternative asset groups:
   - Real Estate
   - Private Equity
   - Commodities
   - Hedge Funds
   - Managed Futures
   - Distressed Securities
   - Farmland and Timber

Learning Outcomes:

(6a) Demonstrate an understanding of the types of investments available in each market, and their most important differences for an investor.

(6c) Demonstrate an understanding of the investment strategies and portfolio roles that are characteristic of each alternative investment.

Sources:

QFIA-111-13: Maginn & Tuttle, Managing Investment Portfolios, 3rd Ed. 2007, Ch. 8, p. 477-478, 498-516

Commentary on Question:

This question tests the candidates' understanding of alternative assets, and their ability to apply their knowledge by recommending an appropriate investment strategy.

Solution:

(a) Describe briefly the four major features of alternative investments.

Commentary on Question:

The candidates performed excellently on this section. Most candidates earned full credit as they were able to list and define the correct features. Candidates who did not receive full credit either provided a list of features with no description or simply did not provide enough features.

Liquidity/Illiquidity

- Relatively illiquid and usually higher premium to compensate

Diversification

- Potential benefits due to low correlation with traditional asset classes (e.g., stocks bonds)

4. Continued

Due Diligence
- Costs tend to be high due to complexity and lack of transparency

Performance Review/Appraisal
- Difficult due to few if any benchmarks that are relevant and well-established

(b) Evaluate whether an investment in private equities meets each of the four goals.

Commentary on Question:
The candidates performed excellently on this section. Most candidates demonstrated sufficient understanding of the investment goals and the private equity asset class. A few candidates failed to achieve full credit by not making a definitive conclusion about whether the asset class met the goal. For the "moderate increase to risk" goal, candidates were given credit whether they said it met the goal, or cautioned that it might be too much risk, i.e., more than moderate.

1. Higher Overall Returns
   a. YES, goal is met
   b. Higher returns are common, vs. other classes to compensate for higher risk, e.g., illiquidity

2. Willingness to transfer a larger proportion of the portfolio to long-term investments
   a. YES, goal is met
   b. Typical structures are long-term (7-10 years) commitments

3. Moderate increase to risk
   a. YES or POSSIBLY, goal is met
   b. Much higher risk due to volatility, illiquidity, many are startups

4. Good background and supportive information about the asset class
   a. NO, goal is not met
   b. Lack of information and expertise and experience will be limited

(c) Propose a different alternative asset class that aligns with the CFO goals.

Commentary on Question:
The candidates performed relatively well on this section. The majority recognized real estate, directly or indirectly invested, as the asset class meeting all the goals and explained how. Some candidates received partial credit for suggesting an alternate class if they provided support of meeting the goals.

4. Continued

Real estate is the best alternate

- Generally, returns are higher, especially in the long-term
- Higher risk than bonds while more stable than equities, so moderate increase to risk
- Strong history and good background with established concepts, so ease of communication

5. Learning Objectives:

1. The candidate will understand the standard yield curve models, including:
   - One and two-factor short rate models
   - LIBOR market models

   The candidate will understand approaches to volatility modeling.

Learning Outcomes:

(1g) Understand and explain the features of the G2++ model, including: The motivation for more than one factor, calibration approaches, the pricing of bonds and options, and the model's relationship to the two-factor Hull-White model.

(1h) Explain the set up and motivation of the Lognormal Forward LIBOR Model (LFM).

(1i) Describe the calibration of the LFM to Cap and Floor prices.

Sources:

Brigo, D and Mecurio F, Interest Rate Models – Theory and Practice, 2nd Edition, Sections 6.1-6.3, p.195-220.

Commentary on Question:

This question test knowledge of the basic set up and motivation of LFM. Also this question asks the candidates to describe how to calibrate the LFM model to caplet prices.

Solution:

(a) Compare and contrast the two models on:

Commentary on Question:

The candidates did relatively poorly on this section. For part i) many candidates listed the formula but did not provide a comparison of how each model handles interest rate dynamics. Most candidates were able to identify that LFM models interest on a forward rate basis. Very few candidates were able to specify the aspects relative to volatility, correlation, simulation and valuation. For part ii) some candidates commented on Caplets or Swaptions but only a few for both. Few candidates covered the last component of appropriateness of model relative to market instruments. Most candidates missed the most important parts of the question: compare and contrast. Elements were listed but not compared and contrasted.

(i) How they model interest rate dynamics

We compare dynamics under different characteristic as to interest rate determination, volatility, correlation, Monte Carlo simulation and tree valuation.

5. Continued

For the interest rate:

- LFM model interest on a forward rates basis. Forward rates are more natural and representative coordinates of the yield curve. (p204).
- G2++ model is a short rate model and based on modeling the instantaneous spot interest rate (p 196-197).

For volatility:

- LFM is very specific and provide a volatility level for each forward rate.
- G2++ provides a constant volatility for each factor.

Correlation:
The correlation structure is important especially with large number of swaptions. A realistic correlation pattern with a two-factor short-rate model is not always possible. (p 205) With LFM if assign a different Brownian motion to each forward rate and assume the motion to be instantaneously correlated this is a solution.

Monte-Carlo Simulation:
Both models can price derivative through a Monte Carlo simulation approach (p 112).

Tree Valuation:

- G2++ permit the use of a binomial tree when pricing exotic interest rate derivative(p 206)
- With LFM, very difficult and almost impossible due to high number of rates and no recombining nature.

(ii) How they can be calibrated to market instruments

Caplet are priced under LFM in agreement with Black's formula.The market has been pricing caplets with Black's formula for years (p 198). This provides immediate and intuitive values since simply input volatility given by the market (p220).

With G2++ a short rate, in general no such model can lead to Black's formula for caplet (p 203). Price are complicated nonlinear function of parameters in the 2 factors Gaussian model.(p 203,197).

5. Continued

With LFM, determination of swaption prices using instantaneous-correlation structure is a delicate task and an approximated formula is proposed for simplification but is still accurate for practical purposes.(p 205-206). With G2++ the two-factor model requires numerical integration (p 283).

Valuation of swaption and other payoffs are in general with LMF using Monte-Carlo method (p 206). Under G2 ++ the binomial tree method is a fundamental tool for pricing exotic interest rate derivatives (p163). For LMF the joint dynamics usually does not lead to recombining lattice for short rate so not clear to evaluate some product with a Tree in the LFM.

(b) Recommend one of the two models and justify the choice.

Commentary on Question:
The candidates performed relatively well on this section. Many candidates correctly suggested using LFM. Few candidates justified their choice with the correlation and volatility parameterization in line with the market.

I recommend LFM.
This model has the capacity to calibrate swaption efficiently.
Also, LFM prices caps with Black's cap formula is compatible with the standard formula employed in the cap market.
Parameters in the correlations structure can be obtain directly from the market, so make it possible to model realistic market correlation.
LFM volatility surface parameterization allows for better fitting of market instruments.

(c) Calculate η_3 based on the above tables.

Commentary on Question:
The candidates performed relatively well on this question. The candidates generally got this part either completely correct or completely wrong.

$v_i^2 = \frac{1}{Ti-1}\sum_{j=1}^{i} \tau_{j-2,j-1}\eta_{i-j+1}^2$ *(6.20 p. 223) right, they typically had a perfect score on that question.*

5. Continued

$\eta_1 = 12\%$ from $v_1^2 = \eta_1^2 \ or \ \sigma_1 = \eta_1$
$\eta_1^2 + \eta_2^2 = .13^2 * 2$
$\eta_2 = 13.93\%$

$\eta_1^2 + \eta_2^2 + \eta_3^2 = .16^2 * 3$
$\eta_3 = 20.74\%$

Or

$F_1(t) = (1/1) \text{ x } \eta_1^2 = .12^2$
$F_2(t) = (1/2)\text{x}(\eta_1^2 + \eta_2^2) = .13^2$
$F_3(t) = (1/3) \text{ x } (\eta_1^2 + \eta_2^2 + \eta_3^2) = .16^2$

(d) Describe the features of this parameterization.

Commentary on Question:
The candidates performed poorly on this section. The key item here was to discuss the proprieties of volatility but very few did so.

- There will be a volatility hump as the forward rate approaches its fixing date
- The structure of the volatility hump is shared between all expirations
- Each forward rate has an unique volatility level determined from the Φ

(e) Describe how you would calibrate this model to caplet prices.

Commentary on Question:
The candidates performed poorly on this section. The most common error was suggesting parameters "a" to "d" calibrate to caplets when they can't.

$$\Phi^2_i = \frac{(v^{MKT}_i)^2}{I^2(T_{i-1}; a, b, c, d)}.$$

The Φ^2_i are calibrated based on the parameters a to d.
Parameters a to d can't be calibrated through caplets.
Parameters a to d can be calibrated using swaption calibration.

6. Learning Objectives:

2. The candidate will understand and be able to apply a variety of credit risk theories and models.

Learning Outcomes:

(2h) Demonstrate an understanding of credit default swaps (CDS) and the bond-CDS basis, including the use of CDS in portfolio and trading contexts.

(2i) Demonstrate an understanding an understanding of CDS valuations

Sources:

V-C183-10, J.P. Morgan, Bond CDS Basis Handbook, February 2009

Commentary on Question:

The question tested the understanding and knowledge of basis trades and underlying calculations.

Solution:

(a) List and explain two reasons why investors enter negative basis trade.

Commentary on Question:

The candidates performed well on this section. Full credit was awarded for candidates properly listing and describing any two of the bellows.

Lock-In "Risk-free" Spread - If bond and CDS share the same credit risk but they are pricing it differently, it might be possible to construct something akin to an "arbitrage-free" trade to profit from it.

Trade the Basis - A negative basis trade (buy bond and buy CDS protection) can be used to bet that an already negative basis will disappear, or to bet that the basis will become positive. For example, CDS spreads might react faster to negative news regarding corporate events. In those cases, the basis can become positive until bond spreads catch up. A negative basis trade established prior to the negative news should profit from it.

Profit from Default - If the bond and CDS legs of a basis trade are done in the same notional, the investor can, after a default, deliver the bond to the CDS counterparty and both legs of the trade will terminate with no further payment. In that case, the investor's gain will be the net cash flows the trade generated up to that point. If the investor expects the default to happen soon, a short maturity CDS can be more economical if the CDS spread curve is steep enough.

6. Continued

(b) Calculate the CDS Spread (assuming a simplified one-step time period approach) and Bond-CDS basis for each bond.

Commentary on Question:
The candidates performed excellently on this section. Very few candidates made mistaskes on this section.

CDS spread = (Probability of default) (1 – Expected recovery rate)
Bond-CDS Basis = CDS spread – Bond spread

Maturity (years)	**1**	**2**	**3**	**4**
Bond spread (bps)	125	130	135	140
Probability of default	3%	4%	4%	5%
Expected recovery rate	70%	70%	60%	60%
CDS spread (bps)	(3%)(1-70%) = 90	(4%)(1-70%) = 120	(4%)(1-60%) = 160	(5%)(1-60%) = 200
Bond-CDS basis (bps)	90-125 = -35	120 – 130 = -10	160 – 135 = 25	200 – 140 = 60

(c) Identify the bond that offers the best *negative* basis trade arbitrage opportunity and describe the strategy.

Commentary on Question:
The candidates performed well on this section. Some candidates described a negative basis trade as a positive basis trade. Also some candidates did not state that the maturity of the CDS has to match the maturity of the bond.

Negative basis trade - If the basis is negative, the CDS spread is lower (tighter) than the bond spread. To capture the pricing discrepancy when a negative basis arises, an investor could buy the bond (long risk) and buy CDS protection (short risk) with the same maturity as the bond.

The 1-year maturity bond offers the greatest negative basis trade opportunity. The underlying positions for the strategy include buying both the 1-year maturity bond the 1-year CDS.

6. Continued

(d) Identify the bond that offers the best *positive* basis trade arbitrage opportunity and describe the strategy.

Commentary on Question: *The candidates performed well on this section. Some candidates described a positive basis trade as a negative basis trade. Also some candidates did not state that the maturity of the CDS has to match the maturity of the bond.*

Positive basis trade - If the basis is positive, the CDS spread is higher (wider) than the bond spread. An investor could borrow and short the bond (if possible) and sell CDS protection (long risk) with the same maturity (or as near as possible) as the bond. Thus the investor is not exposed to default risk but still receive a spread equal to the Bond-CDS basis.

The 4-year maturity bond offers the greatest positive basis trade opportunity. The underlying positions for the strategy include selling both the 4-year maturity bond the 4-year CDS.

(e) Calculate the market price of the 3-year Pai Solar bond.

Commentary on Question:
The candidates performed well on this section. Two common mistakes were to incorrectly calculate the 4% coupons and to use the wrong denominators in the formulae.

$$\text{Bond Price} = \frac{CF_1}{(1+rf_1+z)^1} + \frac{CF_2}{(1+rf_2+z)^2} + \cdots + \frac{CF_t}{(1+rf_t+z)^t}$$

$$\text{Bond Price} = \frac{40}{(1+0.01+z)^1} + \frac{40}{(1+0.02+z)^2} + \frac{1040}{(1+0.02+z)^3} = 978.37$$ = or 0.97837 per $ of face value

(f) Calculate the par asset swap spread of the 3-year Pai Solar bond.

Commentary on Question: *The candidates performed relatively well on this section. Most candidates were able to appropriately state the asset swap spread formulae, however only a few accurately performed the calculations. A common mistake was improperly calculating the risk free annuity as the as a 1% coupon rather than as 1 basis point coupon.*

6. Continued

Asset Swap Spread - is a way of trading a bond in which its fixed coupons are exchanged for floating payments that fluctuate in line with Libor (or some other agreed rate). Essentially, this transforms a fixed coupon bond into something analogous to a floating rate note. In doing this, the investor is able to hedge out the interest rate risk inherent in owning a bond. The spread over Libor received on the floating side is called the asset swap spread, and can be considered to give some measure of the bond's credit risk.

$$\text{Asset swap spread} = \frac{\text{PV(Coupon+Principal)} - \text{Bond Price}}{\text{Risk Free Annuity}}$$

where the annuity used here is the risk-free annuity (present value of a 1bp annuity stream) and PV represents the present value of the bond's future cash flows using the risk-free discount curve.

$\text{PV(Coupon + Principal)} = \frac{40}{1.01} + \frac{40}{1.02^2} + \frac{40}{1.02^3} + \frac{1000}{1.02^3} = 1058.07$ or 1.05806 per $ of face value

$$\text{Risk Annuity} = \frac{0.0001}{1.01} + \frac{0.0001}{1.02^2} + \frac{0.0001}{1.02^3} = 0.000289359$$

$$\text{Asset swap spread} = \frac{(\frac{1058.07}{1000}) - (\frac{978.37}{1000})}{(0.289359/1000)} = \frac{1058.07 - 978.37}{0.289359} = 275\ bps$$

7. Learning Objectives:

3. Candidate will understand the nature, measurement and management of liquidity risk in financial institutions.

Learning Outcomes:

(3a) Understand the concept of liquidity risk and the threat it represents to financial intermediaries and markets.

(3b) Measure and monitor liquidity risk, using various liquidity measurement tools and ratios.

Sources:

Ben Dor Ch 5 – Quantifying the Liquidity of Corporate Bonds

Commentary on Question:

This question requires the candidate to understand Liquidity Cost Score (LCS) and how it may be applied in portfolio construction.

Solution:

(a) Calculate the Liquidity Cost Score™ (LCS) for each bond.

Commentary on Question:

The candidates performed well on this section. Some candidates failed to apply adjustment factor to Bond C and/or did not specify the correct units (i.e. divide by 10000 or specify in basis points).

Basic LCS formula = (Bid – Ask Spread) * Option Adjusted Spread Duration
Bond A LCS = 50/10000*5*1.6 = 4%
Bond B LCS = 37/10000*5 = 1.85%
Bond C LCS = 20/10000*10*1.6*1.08 = 3.456%

(b) Assess the potential impact of these market conditions on the bonds' LCS.

Commentary on Question:

The candidates performed relatively well on this section. Most candidates were able explain the impact on the LCS of increasing market illiquidity and credit crunch.

1) *Reduced Trading Volume*: With slow-down in trading activities, trading volume will be reduced which leads to higher LCS values for bonds. Traders are more likely to issue wider bid-market indications in recognition of the lower trading volumes. This negatively affects the bond's LCS value by increasing it.

7. Continued

2) *Higher DTS/OAS*: With the pending marketing illiquidity and sovereign crisis, bonds with greater excess return volatility will likely have higher LCS values because of the risk the market maker must bear until the trade can be covered.

(c) Assess the appropriateness of each strategy and recommend which one to use.

Commentary on Question:
The candidates performed relatively poorly on this section. Most candidates were able to correctly identify strategy (ii), but often without an adequate explanation of how a low LCS portfolio will outperform in an illiquid market condition. Some candidates incorrectly identified strategy (iii) by not identifying the uselessness of the strategy due to high costs of maintaining exact LCS match in an illiquid environment. A few candidates merely restated the question instead of providing additional commentary and/or did not choose a strategy.

The manager should choose strategy ii.

Strategy (i)

- Large recent issues are typically the one's that suffer the most during financial crisis.
- LCS is an overall better measure of actual liquidity as compared to traditional portfolio liquidity management approaches based on issue size or volume

Strategy (ii)

- The mandate is to minimize the tracking error volatility with respect to the portfolio.
- Portfolio B will be rebalanced more often as compared to Portfolio A.
- During times of liquidity crisis, severe mark to market impacts of holding illiquid bonds produce large negative excess returns. Constant rebalancing keeps LCS low, and as liquidity deteriorates, a lower LCS portfolio outperforms in terms of cumulative excess returns (or returns).
- More frequent rebalancing under strategy ii to manage liquidity also helps to hedge against the curve (term structure) risk keeping the TEV low.

Strategy (iii)

- Mandate is to lower tracking error volatility compared to IG index while protecting against increasing liquidity concerns. In order to do this, would need to construct the portfolio such that it has better liquidity than the IG index. So this strategy doesn't accomplish anything

Costs

- Costs associated are turnover costs to constantly manage the LCS to a level lower than the index. High LCS bonds need to be sold and low LCS bonds need to be bought.

7. Continued

- High turnover costs however does not negate all the benefits achieved from a low LCS as bonds in a low LCS portfolio are cheaper to trade than the other portfolio.
- There is still an overall benefit to a low LCS portfolio in times of illiquid markets.

8. Learning Objectives:

4. The candidate will understand important quantitative techniques for analyzing financial time series Performance Measurement and Performance Attribution

Learning Outcomes:

(4a) Demonstrate an understanding of the mathematical considerations for analyzing financial time series.

(4b) Apply various techniques for analyzing factor models including Principal Component Analsis (PCA) and Statistical Factor Analysis.

Sources:

Principal Component Analysis on Term Structure of Interest Rates, Antii Malava.

Commentary on Question:

This question guides the candidate to go through a numerical exercise similar to the analysis done in the Malava paper.

Solution:

(a) Determine which of the principal components above are necessary in order to explain at least 90% of the variability of the term structure of interest rates.

Commentary on Question:

The candidates performed excellently on this section. A few that did this incorrectly used all the principle components instead of just 1 and 4.

Rank the eigenvalues:

I	4	1	5	2	3
Eigenvalues (λi)	3.63	1.11	0.18	0.06	0.03
Eigenvectors (βi)	0.39	0.60	0.40	-0.55	0.15
	0.44	0.48	-0.08	0.69	-0.31
	0.50	-0.12	-0.57	-0.02	0.63
	0.48	-0.36	-0.23	-0.39	-0.66
	0.41	-0.52	0.67	0.27	0.19

Calculate the explanatory degrees using the formula

$$\frac{\lambda_i}{\sum_{i=1}^{5}\lambda_i}$$

i	4	1	5	2	3
Eigenvalues (λi)	3.63	1.11	0.18	0.06	0.03
Explanatory Degree	72.46%	22.16%	3.59%	1.20%	0.60%

The first and the second PCs add up to 95% therefore two principal components are required to explain at least 90% of the variability.

8. Continued

(b) Compute the factor structure for each of the principal components you determined in (a).

Commentary on Question:
The candidates performed relatively poorly on this section. Many candidates did not provide the factor structure and instead provided a weighted average of numbers.

The factor structure is calculate using the formula
$\sqrt{\lambda_i}\beta_i$

I	4	1
Factor structure	0.74	0.63
	0.84	0.51
	0.95	-0.13
	0.91	-0.38
	0.78	-0.55

(c) Identify which component from your principal component analysis corresponds to your portfolio's adverse scenario.

Commentary on Question:
The candidates performed well on this section. Many candidates identified the correct component but often did not provide a correct justification. As a result, they did not get the full credit.

The first PC explains a shift in the term structure as the loadings are relatively flat and of the same sign.

(d) Compute the two-standard-deviation term structure movement related to your portfolio's adverse scenario.

Commentary on Question:
The candidates performed poorly on this section. This question requires students to go beyond the material in the text to provide an answer and most did not. There was no common pattern to the errors. A small handful of candidates provided perfect answers.

2 Standard Deviation Shock = μ_i+ stddev($PC_i = y_i$) × (-2) × Factor Loading x stddev(x_i)

= 0 + $\sqrt{\lambda_i}$ x (-2) x Factor Loading x stdev (xi)

8. **Continued**

Maturity	6 Months	1 Year	5 Year	10 Year	20 Year
Shock	-0.64%	-0.69%	-0.48%	-0.31%	-0.19%

"Worst case" term structure": Shocked Rate = Rate + 2 Standard Deviation Shock

Maturity	6 Months	1 Year	5 Year	10 Year	20 Year
Rate	0.29%	0.87%	2.03%	2.69%	3.35%

9. Learning Objectives:

1. The candidate will understand the standard yield curve models, including:
   - One and two-factor short rate models
   - LIBOR market models

   The candidate will understand approaches to volatility modeling.

Learning Outcomes:

(1g) Understand and explain the features of the G2++ model, including: The motivation for more than one factor, calibration approaches, the pricing of bonds and options, and the model's relationship to the two-factor Hull-White model.

Sources:

Brigo, D and Mecurio F, Interest Rate Models – Theory and Practice, 2nd Edition, Sections 4.1-4.2, p.137-138.

Brigo, D and Mecurio F, Interest Rate Models – Theory and Practice, 2nd Edition, Sections 4.1-4.2, p.142, 158

Brigo, D and Mecurio F, Interest Rate Models – Theory and Practice, 2nd Edition, Sections 4.1-4.2, p. 166-169

Brigo, D and Mecurio F, Interest Rate Models – Theory and Practice, 2nd Edition, Sections 4.1-4.2 p.147

Brigo, D and Mecurio F, Interest Rate Models – Theory and Practice, 2nd Edition, Sections 3.3.1 p.74

Commentary on Question:

This question tests the candidate's understanding of various interest rate models. Specifically, it tests the candidate's knowledge of the G2++ model, including pros and cons and main features of the model.

Solution:

(a) Describe the primary shortfall associated with using a one-factor model.

Commentary on Question:

Candidates performed well on this section. Most candidates were able to identify the main shortfall of the model. However, to get full marks, we were also looking for them to explain why it is a disadvantage, which was not consistently addressed.

The shortfall of one factor interest rate models is that all changes for all maturities depend on a single stochastic factor. Multiple factors are required to properly describe movements in the interest rate curve.

9. Continued

For example, they assume the thirty-year interest rate at a given instant is perfectly correlated with the three month rate at the same instant. This percent correlation assumption is not realistic. Interest rates are known to exhibit less than 100% correlation.

(b) Describe the components of the G2++ model and explain why they lead to efficient procedures for pricing zero coupon bonds and caplets.

Commentary on Question:
Candidates performed well on this section. However, a small proportion of candidates simply showed the components of the model, without detailed descriptions of the various components and how they are useful.

The instantaneous short rate process is given by the sum of two correlated Gaussian mean reverting factors plus a deterministic function that is properly chosen to exactly fit the current term structure of the discount factors.

The Gaussian distribution allows for a closed form solution for fitting the deterministic function to fit the current structure of zero coupon bonds. Caplet pricing under the g2++ model leads to a closed formulation of the price that is based off of the Black Scholes model.

(c) Identify three potential approaches that can be used to calibrate a two factor G2++ model to real-market volatility data. For each, outline key considerations.

Commentary on Question:
Candidates performed relatively poorly on this section. Many candidates listed other calibration approaches that were not specifically discussed in the text. To get full marks, candidates needed to list the calibration approaches discussed in the text including key considerations for each.

Calibrating to cap volatilities:

- the G2++ model can reproduce market cap volatility data very accurately
- it often happens that the ρ value is quite close to minus one, which implies that the G2++ model tends to degenerate into a one-factor (non-Markov) short rate process
- even with the model degenerating into a one-factor short rate process, it is still usually non-Markovian (because a does not equal b) so it still outperforms the one-factor version

Calibrating to swaption volatilities:

- it may be a good idea to calibrate the G2++ model only to the most significant swaptions data, leaving out illiquid entries

9. Continued

- when in need to price a particular product that is influenced only by a certain set of swap rates, it may be reasonable to calibrate the model only to the relevant swaptions
- these calibrations can require a reasonable amount of time to run, which can be reduced by either running Globally or only calibrating to a subset of the whole swaption table

Joint calibration to caps and swaptions data:

- results are not usually completely satisfactory
- markets may be misaligned and the model may not have enough parameters
- the LIBOR market model may be more appropriate

(d) Calculate the risk-neutral probability of a negative short rate at time 10.

Commentary on Question:
Candidates performed excellently on this section. A few candidates failed to identify the correct formula or made computational errors in performing the calculation. To get full marks, they needed to identify the correct formula to use and properly compute the resulting probability.

The risk neutral probability of negative rates at time t is

$Q\,(r(t)<0) = \phi(-\mu r(t)/\theta r(t))$

In this case, we have
$Q\,(r(t)<0) = \phi(-2\%/1\%)$
$Q\,(r(t)<0) = 2.3\%$

(e) Outline a brief response to your manager on the possibility of negative rates.

Commentary on Question:
Candidates performed relatively well on this section. Most candidates pointed out that the probability of negative rates was low, however many candidates failed to formulate a response to their boss that included a recommendation or further explanations as to why the model is still appropriate. To get full marks, candidates were expected to formulate a well-stated response to their boss based on the result of the calculation in part (d).

It is important to consider the risk / reward trade-off. The probability of negative rates is small (approximately 2.3%) but this model has many advantages (discussed in parts (a) and (b)) and thus there is a strong argument for using this model.

9. Continued

There have been also cases of real world negative interest rates that have happened, which makes negative rates less of a concern.

10. Learning Objectives:

2. The candidate will understand and be able to apply a variety of credit risk theories and models.

Learning Outcomes:

(2a) Demonstrate an understanding of events and causes of the recent global credit crisis.

Sources:

QFIA-102-13 Credit Risk Measurement In and Out of the Financial Crisis, Saunders, A., Allen,L., 3rd Edition, Ch1-2

Commentary on Question:

This question tested whether the candidate was familiar with the details of the 2008 global credit crisis specifically as discussed in the Saunders and Allen's paper (part (a)) and how well they understood the paper (part (b).)

A piece of advice to the candidates is for them to clearly separate their key points and sections of their solutions. For this particular question there were a few papers that answered part (a) in a single paragraph and did not clearly distinguish between the three phases of the financial crisis. In part (b) some candidates did not clearly identify which points from the futurist's report they were addressing in their solution. The solutions outlined below have been laid out in a clear and fully formed solution.

Solution:

(a) Describe and explain how the events within each of the above phases contributed to the 2008 Global Credit Crisis.

Commentary on Question:

Candidates performed well on this section. Generally the answers to phase 3 were the best.

This question focused specifically on the views expressed by Saunders and Allen and their definition for three phases of the Credit Financial Crisis of 2008. A few candidates described events outside of these three phases however these points did not benefit or penalize the candidate in anyway.

There are a number of points that describe each of the three phases. The solution below does not cover all possible answers but does provide enough detail to get full credit.

Phase I

- Result of Phase 1 was widening credit spreads and decreased liquidity
- Geographic diversification assumed investors were protected from defaults (historically this was true) but this was not the case during Phase I.

10. Continued

Phase II
- In Phase 2, Market participants hoarded capital and liquidity and global financial markets shut down
- Flight to quality assets reduced liquidity of credit assets at all credit levels

Phase III
- Large financial institutions thought to be To Big To Fail (TBTF) because of their importance but the government showed otherwise – this haphazard application contributed to the third phase (e.g. Lehman's)
- It became apparent there was something unsound in way that underwriters implemented the securitizations during the bubble years

(b) Compare and contrast the futurist's scenario with the 2008 Credit Crisis.

Commentary on Question:
Candidates performed well on this section.

Candidates that did poorly failed to provide supporting material for key points. For example, some candidates would restate the points in the question but not provide any commentary about the point. (e.g. The candidate stated 'Stock markets crashed' without any further commentary about whether this occurred in both the 2008 Financial Crisis or just in Futurist's scenario. They could also comment on whether it would crash more, less, or unknown if it crashed in both.)

The solution below is just a few of the possible answers. If the candidate provided an answer with reasonable supporting commentary then full credit was also granted.

Terrorist attacks result in significantly reduced supply of crude oil to the United States;
- Similar to the 911 terrorist attacks that led up to the 2008 crises
- Differs in that this deals with a traded commodity that will impact the financial markets more directly

Stock markets collapsed;
- Similar to the 2008 Credit crises since it crashed then too
- Could crash more or less than what was observed in the 2008 Crisis

Auto fuel rates skyrocketed;
- No real similarities to the credit crises
- People would not be able to afford gasoline to get to work

10. Continued

Consumer demand for hybrid and electric cars skyrocketed, driving the price of these vehicles upward from their already high rates;

- People would not be able to afford a hybrid/electric car and look to credit to purchase a vehicle

Automakers' leasing rates dropped to all-time lows and terms were extended from traditional 5 years to terms to greater than 10 years;

- Similar to mortgage rates dropping during 2008 Credit crisis
- Different in that mortgages were generally long term assets whereas here they are short term assets

Automakers' finance departments were collateralizing these loans as ABS and selling them in the open market;

- Similar to 2008 Credit crisis and the mortgage lending market collateralizing mortgages to create MBS.
- Differs because automobiles are expected to depreciate in value over time whereas houses were expected to appreciate

To manage inflation and liquidity, the Federal Reserve dropped the Federal Funding Rate to record lows;

- Similar to the 2008 Credit crises since Fed's did the same later in the crises
- Differs in that the Fed raised rates early in the crisis

11. Learning Objectives:

4. The candidate will understand important quantitative techniques for analyzing financial time series Performance Measurement and Performance Attribution.

Learning Outcomes:

(4f) Calculate and interpret performance attribution techniques.

Sources:

QFIA-107-13: *Handbook of Fixed Income Securities*, Fabozzi, F.J., 8th Edition, 2012, Ch. 69 and 71.

Commentary on Question:

This question asks the candidates to calculate and interpret performance attribution metrics for a given asset and portfolio.

Solution:

(a) Calculate the asset allocation return and security selection return in Quarter 1 for the portfolio before any scaling coefficient is applied.

Commentary on Question:

The candidates performed well on this section. Most candidates used the alternative solution below. Those that did not receive full credit often just plugged in the wrong numbers to the correct formula.

Asset Allocation $\sum \left(w_s^P - w_s^B\right) \cdot R_s^B$ (69–4)

Fabozzi, Ch 69 Pg 1640

= (40%-40%) x 4% + (30%-20%) x 5% +(30%-40%) x 8%
= -0.3%

Alternatively,

Asset Allocation $\sum \left(w_s^P - w_s^B\right) \cdot \left(R_s^B - R^B\right)$ (69–6)

= (40%-40%) x (4%- 5.8%) + (30%-20%) x (5% -5.8%)
+ (30%-40%) x (8% -5.8%)
= -0.3%

Note that (69-6) is equivalent to (69-4) in our case as $\sum_s w_s^P = \sum_s w_s^B$ =1

Security Selection $\sum w_s^P \cdot \left(R_s^P - R_s^B\right)$ (69–5)

Fabozzi, Ch 69 Pg 1640

= 40% x (4%-4%) + 30% x (6%-5%) + 30% x (10%-8%)
= 0.9%

11. Continued

(b) Calculate β for Quarter 1.

Commentary on Question:
The candidates performed relatively poorly on this section. Many candidates used T=1 instead of T=4. Others used quarterly values instead of annual.

$$A = \frac{(R^P - R^B)/T}{(1+R^P)^{1/T} - (1+R^B)^{1/T}}$$

Fabozzi, Ch 71 Pg 1737

A = (0.048-0.038)/4/((1.048^(1/4)-(1.038)^(1/4))
= 1.0320745

$$\sum_{t=1}^{T}\left(R_t^P - R_t^B\right)$$

= (6.4% -5.8%) + (-2.9% - (-2.6%)) + (-2.9% - (-2.6%)) + (4.2% -3.2%)
= 1.0%

$$\sum_{t=1}^{T}\left(R_t^P - R_t^B\right)^2$$

= $(6.4\% -5.8\%)^2 + (-2.9\% - (-2.6\%))^2 + (-2.9\% - (-2.6\%))^2 + (4.2\% -3.2\%)^2$
= 0.0154%

$$C = \frac{R^P - R^B - A\sum_{t=1}^{T}\left(R_t^P - R_t^B\right)}{\sum_{t=1}^{T}\left(R_t^P - R_t^B\right)^2}$$

C = (0.048-0.038 - 1.03020745 x 0.01)/0.000154 = -2.08276

β for first quarter=A+ C x $(R_t^P - R_t^B)$
= 1.0320745-2.08276 x (0.064-0.058)=1.019578

(d) Calculate the scaled asset allocation and security selection in Quarter 1 for the portfolio.

11. Continued

Commentary on Question:
The candidates performed poorly on this section. Most candidates did not calculate β properly. Others did not plug in correct values into the formula which was provided on the formula sheet.

$$R^P - R^B = \sum_i \beta_i \left(R_i^P - R_i^B\right)$$

Fabozzi, Chapter 71 Page 1737
Asset allocation after scaling**= (**β for first quarter) x asset allocation before scaling
= (1.0320745-0.0125) x (-0.3%) = -0.30587%
Security selection after scaling**= (**β for first quarter) x security selection before scaling
= (1.0320745-0.0125) x (-0.9%) = 0.91762%

(e) Assess whether Model M meets each criterion specified by your company.

Commentary on Question:
The candidates performed relatively well on this section. Most candidates identified that criteria i) was met. Many candidates did not identify that criteria ii) was not met. For criteria iii) the scaling β needed to be calculated and often it was incorrectly calculated.

Model M does not satisfy the guidelines set up by the Company.

Reasons:
1. The sum of the return splits remains equal to the total return.
2. It is clear that even when the periods are arranged in reverse order, Menchero method produces results identical to the original results. The order of periods does not have a bearing on the cumulative attribution results (i.e. It is order independence).
3. The scaling β for first quarter =1.019578>1.01

12. Learning Objectives:

1. The candidate will understand the standard yield curve models, including:
   - One and two-factor short rate models
   - LIBOR market models

   The candidate will understand approaches to volatility modeling.

2. The candidate will understand and be able to apply a variety of credit risk theories and models.

Learning Outcomes:

(1a) Identify and differentiate the features of the classic short rate models including the Vasicek and the Cox-Ingersoll-Ross (CIR) models.

(2f) Demonstrate an understanding of modeling approaches for correlated defaults.

Sources:

Brigo Interest Rate Models Ch. 3.2 (7 points)

Bluhm Ch. 2, 6 (1 point)

Commentary on Question:

This question tests candidates' understanding of default probability term structure and features of the Cox, Ingersoll, and Ross (CIR) model.

Solution:

(a) Express the probability of default before time *t* conditional on survival to time $s < t$.

Commentary on Question:

Candidates performed poorly on this part. A few candidates received partial credit for providing the correct formula for the probability of default.

Probability of default = 1 – Es[exp ($-\int_s^t \lambda u \, du$)], where Es[] is the expectation given all information available at time s.

λu is the intensity.

Default risk term structure is in close analogy to forward interest rate with zero coupon prices corresponding to survival probabilities.

(b) Calculate A, B, C and D in the above table.

12. Continued

Commentary on Question:
Candidates did well on this part. For the most part, candidates either got this completely correct or completely wrong as this was basically a number substitution problem.

From the formula sheet:
Var (λt) = λ0 σ^2/κ (exp(-κt) – exp(-2κt)) + θ σ^2/2κ (1 – exp(-κt))^2

Using the parameter values from Set 1 results in:
A = (2%*(10%^2)/.7)*(exp(-1*0.7*1) – exp(-2*0.7*1)) + (2% *(10%^2)/1.4)* (1- exp(-0.7))^2 = 0.011%.

Using the parameter values from Set 3 results in:
B = (2%*(10%^2/1.5)* (exp(-1*1.5*9) – exp(-2*1.5*9)) + (2% *(10%^2)/1.5)* (1- exp(-1.5*9))^2 = 0.0067%

As *t*-> *infinity*, E[λt] -> Θ, therefore from the values given, C = 2%.

As *t*-> *infinity*, VAR(λt) -> θ σ^2/2κ, so using the parameter values from Set 1 results in D = (2% (10%)^2)/(2*0.7) = 0.014%.

(c) Explain how changes to each of the following affect the distribution of forward default intensity based on the results obtained in part (b).

(i) Volatility Coefficient (σ)

(ii) Mean rate of reversion to the long-run mean (κ)

Commentary on Question:
Candidates did relatively well on this part. Many candidates provided a correct explanation on how the changes in the terms affected the distribution, but did not specifically reference back the part (b) results as the question asked.

(i) Volatility Coefficient:
Shows the effect on the distribution of the forward default intensity on increasing the standard deviation.

Comparing parameter set 1 versus parameter set 2:
It shows a much higher variance at time 1.
The variance at time 9 increases sharply and is now closer to the long-term variance but not the same.
The long-term variance using parameter set 2 is much larger than the long-term variance using parameter set 1.

12. Continued

The long-term mean remains the same under both parameter sets.
So, both the variance and the long-term variance increase as the volatility coefficient increases.

(ii) Mean rate of reversion to the long-run mean:
Shows the effect on the distribution of the forward default intensity on increasing the mean reversion.

Comparing parameter set 1 versus parameter set 3:
It shows a much smaller variance at time 1.
Under parameter set 3, there is no change in variance at time 9 from time 1 as opposed to a higher variance at time 9 using parameter set 1.
The variance at time 9 is the same as the long-term variance when using both parameter sets.
The long-term variance is much smaller under parameter set 3 than under parameter set 1.
So, a higher mean reversion (κ) keeps the default rate close to the initial level.
Mean reversion has the effect of reducing the impact of volatility on the shape of the curve of forward default rates.

13. Learning Objectives:

2. The candidate will understand and be able to apply a variety of credit risk theories and models.

3. Candidate will understand the nature, measurement and management of liquidity risk in financial institutions.

Learning Outcomes:

(2h) Demonstrate an understanding of credit default swaps (CDS) and the bond-CDS basis, including the use of CDS in portfolio and trading contexts.

(3b) Measure and monitor liquidity risk, using various liquidity measurement tools and ratios.

Sources:

Portfolio Management – Ben Dor Ch. 1 page 31, 32 (5 exam point)

Portfolio Management – Ben Dor Ch. 5 page 88, 93, 130 (3 exam point)

Commentary on Question:

This question tests candidates understanding of duration times spread and liquidity cost score and applies these concepts to the investment policy of a portfolio.

Solution:

(a) Determine whether each asset in the portfolio complies with the liquidity risk constraint in the Investment Policy.

Commentary on Question:

Candidates performed well on this section. Many candidates were able to produce the correct formulas and provide the correct interpretations of the calculation results. A common mistake was to apply the methodology for Bonds 1 and 2 to the rest of the bonds when the methodology should change given the facts.

Bond 1 and Bond 2 LCS = (Bid spread – Ask spread) / 100 [quoted spread]
Bond 1 LCS over OASD = (520 -475)/100 = 0.45% < 1.5% (Complies with liquidity risk limit)
Bond 2 LCS over OASD = (380 -330)/100 = 0.50% < 1.5% (Complies with liquidity risk limit)

Bond 3 and Bond 4 LCS = (Bid spread – Ask spread) * adj factor / 100
Bond 3 LCS over OASD = (470-410)*1.5/100 = 0.90% < 1.5% (Complies with liquidity risk limit)
Bond 4 LCS over OASD = (490-436)*1.6/100 = 0.86% < 1.5% (Complies with liquidity risk limit)

13. Continued

Bond 5 LCS over OASD = (Ask price – Bid price) / Bid price /OASD/100
Bond 5 LCS over OASD = (108.4 – 99.2)/99.2/5.4/100 = 1.72% > 1.5% (Does not comply with the liquidity risk limit)

(b) Describe the advantages and disadvantages of establishing issuer limits based on spreads relative to ratings-based approach.

Commentary on Question:
Candidates performed poorly on this section. Candidates provided many advantages and disadvantages, however they often were not specific to the question. Credit was given only when an answer included elements that are provided in the solution below.

- Advantage: simple to specify a single uniform limit that requires increasing diversification with increasing risk
- Advantage: a spread-based constraint on a bond position is continuously updated as the bond spread changes, by contrast, a bond position based on a ratings-based approach will only change when the bond is downgraded.
- Disadvantage: strict enforcement of the investment policy requires forced sales to keep all issuer exposures within the limits. This may cause excessive transaction costs as spreads trade up and down
- Disadvantage: allows large positions in low spread issues and exposes the portfolio to credit torpedoes.

(c) Determine whether the current portfolio complies with the credit risks requirement of the Investment Policy.

Commentary on Question:
Candidates performed relatively well on this section. Many candidates determined the DTS for the entire portfolio rather than individual bonds. Other methodologies to arrive at the answer below were accepted.

Bond DTS Contribution = OADS*(Spread)*(Portfolio Weight)
Bond 1 DTS Contribution = (6.7*520/100*15%) = 5.2 < 6 (Complies with credit risk limit)
Bond 2: DTS Contribution = (5.8*380)/100*20%) = 4.5 < 6 (Complies with credit risk limit)
Bond 3: DTS Contribution = (6.3*470)/100*22%) = 6.5 > 6 (Does not comply with credit risk limit)
Bond 4: DTS Contribution = (4.6*490)/100*25%) = 5.6 < 6 (Complies with credit risk limit)
Bond 5: DTS Contribution = (5.4*600)/100*18%) = 5.8 < 6 (Complies with credit risk limit)

14. **Learning Objectives:**

2. The candidate will understand and be able to apply a variety of credit risk theories and models.

Learning Outcomes:

(2e) Demonstrate an understanding of the term structure of default probability.

Sources:

Introduction To Credit Risk Modeling, Bluhm, Christian, 2nd Edition Ch 2

Commentary on Question:

This question tests the knowledge of credit risk models and modeling approaches for correlated defaults specifically testing Credit Portfolio View(CPV), KMV and CreditRisk+.

Solution:

(a) Describe briefly how credit losses are modeled for each of the 4 main types of credit risk models.

Commentary on Question:

The candidates performed well on this section. Most candidates provided the complete list and explanations. Those that only provided a list received 25% of grading points were given.

Asset value models

- Default or survival of a firm depends on the state of the asset values at a certain planning horizon. If the process falls below a certain critical threshold, called the default point of the firm, then the firm has survived. Conversely if the process remains above the threshold, the firm survives. *(from Ch 1 Bluhm pg 38)*
- Bernoulli type model
- Mostly internal models

Macroeconomic models

- A Ratings-based portfolio model which incorporates the dependence of default and migration probabilities on the economic cycle. *(from Ch 2 Bluhm pg 71)*

Actuarial Models

- Model based on typical insurance mathematics with default probabilities defined by a particular probability distribution
- For example the CreditRisk+ model is typical example of a Poisson mixture model. *(from Ch 2 Bluhm pg 76)*

14. Continued

Intensity Models

- Focus is on default times, such that if a default time is less than the planning horizon, this triggers a default. Default times are triggered by an intensity process., or basic affine process. *(from Ch 2 Bluhm pg 78)*

(b) Explain the two modes of calibration that CreditPortfolio View (CPV) uses to simulate the segment-specific conditional default probabilities.

Commentary on Question:
The candidates performed relatively well on this section. Most candidates provided the two modes but did not provide supporting statements. Full credit was given for making two correct statements for each of CPV Macro and CPV Direct. No credit was given for merely listing the names CPV Macro and CPV Direct.

CPV Macro: default and rating migration shifts are explained by a macroeconomic regression model.
The macroeconomic model underlying systematic influences on the economic future of obligors is calibrated by means of time series of empirical data.
The calibration of CPV Macro is more complicated than the alternative CPV Direct.
The difficulties in calibrating CPV Macro are mainly due to the many parameters that have to be estimated.

CPV Direct: the segment-specific conditional default probabilities are directly drawn from a gamma distribution.
In other words, the conditional probability determining a segment's risk index is not implied by some underlying macroeconomic factor model
Working with CPV Direct, the user can avoid all the difficulties some macroeconomic regression model incorporates.
The effort in sector calibration is reduced to the calibration of two parameters of a gamma distribution for each risk segment.

(c) Compare and contrast the probability of default modeling approaches of KMV and CreditRisk+.

Commentary on Question:
The candidates performed relatively well on this section. Candidates overall identified the Bernoulli/Poisson difference but generally did not hit on the other distinguishing factors. Full credit was given for comparing and contrasting on at least four attributes of the modeling approaches. Partial credit was given for correct key words and complete statements of one model.

14. Continued

In the default only mode, KMV is of **Bernoulli** type, deciding about default or survival of a firm by comparing the firms's asset value at a certain horizon with some critical threshold. CreditRisk+ is a typical representative of the group of **Poisson** mixture models. The mixture distribution adopted by CreditRisk+ incorporates the gamma distribution.
KMV assumes that asset value process is dependent on underlying **factors** reflecting industrial and regional influences, thereby driving the economic future of the firm. CreditRisk+ implements a **sector** model. Sectors can be identified with industries, countries, or regions, or any other systematic influence on the economic performance of counterparties with a positive weight in this sector.
For KMV, asset correlations between counterparties are exclusively captured by the **correlation** between the respective composite factors. For CreditRisk+, two obligors are **correlated** if and only if there is at least one sector such that both obligors have a **positive** sector weight with respect to this sector.
One can think of a sector as a 'factor-inducing' entity, or every sector could be thought of as generated by a single underlying factor. In this way, sectors are factors are somehow comparable objects.
The conditional default probabilities of both models can be expressed explicitly in close forms.

(d) Recommend one of the two models for modeling mortgage defaults and explain the reason.

Commentary on Question:
The candidates performed well on this section. Most students did provide a recommendation <u>and</u> a justification. Partial credit was given for key words or simply recommending KMW without an explanation.

Recommend KMV. KMV is of **Bernoulli** type while CreditRisk+ is of **Poisson** type. Bernoulli model always induces a **higher default correlation** than the Possion model.
Higher default **correlations** result in **fatter tails** of the corresponding loss distributions. Thus KMV is better than CreditRisk+ in this case.

15. Learning Objectives:

2. The candidate will understand and be able to apply a variety of credit risk theories and models.

Learning Outcomes:

(2m) Understand the rationale, markets and risks of structured finance.

Sources:

QFIA-101-13: Managing Credit Risk, The Great Challenge for Global Financial Markets, Caouette, John B. et al., 2nd edition 2008, chapter 24

Commentary on Question:

This question tried to bring candidates out of the traditional examples by transposing an ABS transaction in an unexpected context. The traditional vocabulary of an ABS transaction was avoided as much as possible to let the candidate identify correctly the similarity between that deal and a more traditional ABS deal. If the candidate was able to correctly recognize the different components of an ABS deal, the question became pretty easy.

Note the exam committee spent a lot of time constructing the background to the question and as a result it is highly likely it will be used again for future exam questions.

Solution:

(a)

(i) Describe how asset securitization is being used in the elementary playground example provided above.

(ii) Sketch a diagram that displays the structure labeling all key participants and asset flows.

Commentary on Question:

The candidates performed well on this section. The most common mistake was to link Bruno with the investors, however he is only linked to Donald. In fact, investors may never be aware that Bruno is part of the transaction, although knowing it may make them feel more secure about this deal. Additionally many candidates identified the marbles in this deal as assets, however they are a currency. The assets are the loans that are secured by Donald.

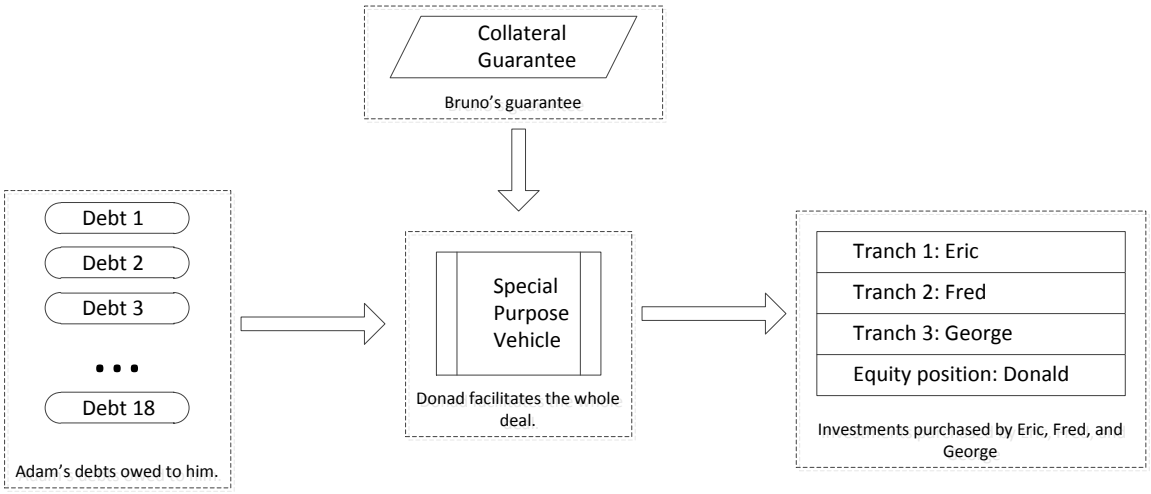
The diagram required the candidate not only to correctly identify all the participants in this transaction, but also to get the correct relationship among them.

15. Continued

(i)

- Students of ABC elementary are the investors.
- Donald is the SPV that make the whole thing work.
- Bruno acts as a guarantor on that deal.
- The assets are the loan and the marbles are the currency.

(ii)



(b)

(i) List the three key areas structured finance professionals focus on when evaluating an Asset-backed Security.

(ii) Critique the asset securitization represented in the elementary playground example provided above being sure to include each of the key areas.

Commentary on Question:
The candidates performed relatively well on this section. Some candidates provided answers common to every deal, however the question asked for specifics related to the case in the question. The point below missed by most candidates was overcollateralization. The list below is not exhaustive, other answers were also permissible providing they made sense.

(i)

- Originator or Seller/Servicer
- Underlying assets
- Structure of the deal

15. Continued

(ii)

- The loans, underlying assets, are to elementary school students. They are also at a different school. May make it hard to collect the marbles. (Underlying assets)
- Overcollateralization: there are 1100 marbles backing only 900 marbles of investment.(structure)
- The deal is broken into tranche offering different risk and return. (structure)
- Donald is retaining the equity tranche. (structure)
- There is no legal structure.
- Donald is a senior at school and presumably experienced at servicing loans. (Servicer)

(c) List and describe the benefits and motivations of the elementary playground asset securitization above for each of the different participants:

(i) OldSchool Elementary students that are indebted to Adam

(ii) NuSchool Elementary students that deal with Donald

(iii) Donald

(iv) Adam

Commentary on Question:
The candidates performed well on this section. Here again we expect the answer to be related to the school yard case. Most students missed the point of Old School students being indifferent to the deal, just like a borrower who is unaware that his loan got securitized. This case involves kids, their motivations are thus quite simple. They don't have the same motivations that banks have.

(i) Old school elementary students are indifferent. They have nothing to say, they just have to keep paying their debt. Except that they may have to deal with Donald instead of Adam.

(ii) Nu School Elementary students can earn some extra marbles by setting aside marbles that they are not using or needing right now. They can do so at different level of risk.

(iii) Donald can grow his marble collection. He also has some control on his risk given that he is in charge of servicing the loan.

(iv) Adam just wants to play marbles and needs some marbles to do so. This is an easier and faster way for him to get marbles in order to start playing.