SOCIETY OF ACTUARIES
Quantitative Finance and Investment Advanced Exam

Exam QFIADV

MORNING SESSION

Date: Thursday, October 30, 2014
Time: 8:30 a.m. – 11:45 a.m.

INSTRUCTIONS TO CANDIDATES

General Instructions

1. This examination has a total of 100 points. It consists of a morning session (worth 60 points) and an afternoon session (worth 40 points).

   a) The morning session consists of 9 questions numbered 1 through 9.

   b) The afternoon session consists of 6 questions numbered 10 through 15.

   The points for each question are indicated at the beginning of the question.

2. Failure to stop writing after time is called will result in the disqualification of your answers or further disciplinary action.

3. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions on the exam booklet.

Written-Answer Instructions

1. Write your candidate number at the top of each sheet. Your name must not appear.

2. Write on only one side of a sheet. Start each question on a fresh sheet. On each sheet, write the number of the question that you are answering. Do not answer more than one question on a single sheet.

3. The answer should be confined to the question as set.

4. When you are asked to calculate, show all your work including any applicable formulas. When you are asked to recommend, provide proper justification supporting your recommendation.

5. When you finish, insert all your written-answer sheets into the Essay Answer Envelope. Be sure to hand in all your answer sheets because they cannot be accepted later. Seal the envelope and write your candidate number in the space provided on the outside of the envelope. Check the appropriate box to indicate morning or afternoon session for Exam QFIADV.

6. Be sure your written-answer envelope is signed because if it is not, your examination will not be graded.

Tournez le cahier d'examen pour la version française.

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Exam QFIADV-Front Cover

****BEGINNING OF EXAMINATION****
Morning Session

1. (*5 points*) You are the Chief Risk Officer of an insurance company which sells a variety of life insurance products. Your company uses OTC derivatives to hedge and transfer various risks. You are currently in the process of identifying and measuring counterparty risk.

(a) (*1 point*) Define:

(i) Counterparty risk

(ii) Counterparty exposure

(iii) Potential future exposure (PFE)

(b) (*1 point*) Describe two mitigants that reduce counterparty risk, and explain how each reduces this risk.

(c) (*2 points*) Describe four main specifications of a PFE measurement model.

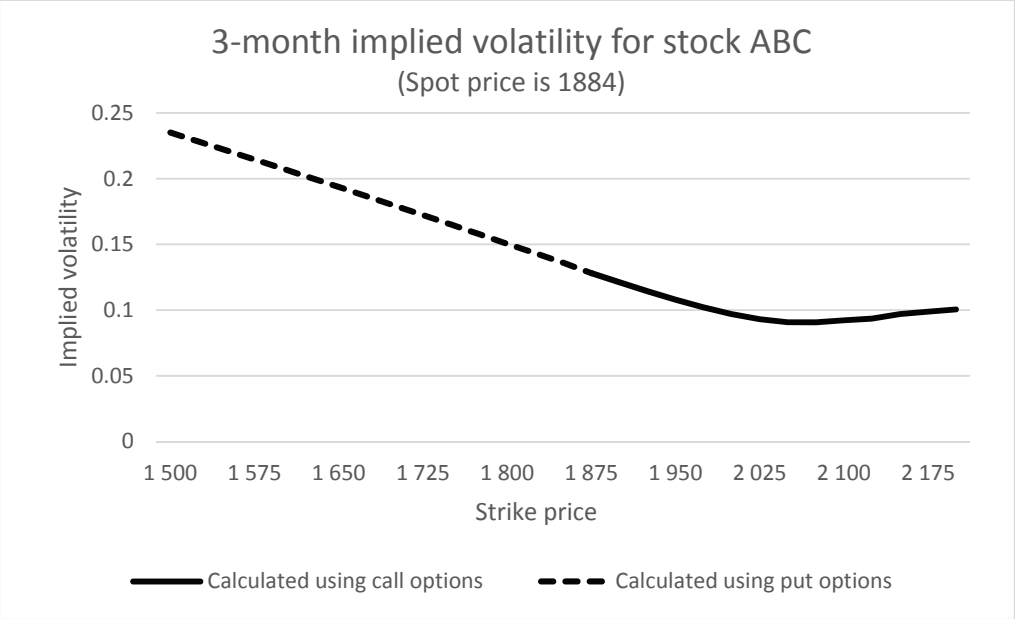
(d) (*1 point*) Describe two main uses of a PFE model.

2. (*7 points*) Your company has been using the Black-Scholes model to price variable annuities. Your boss thinks this model gives a good approximation of market prices, but you disagree. He wants you to explain why the constant volatility term in the Black-Scholes model is not appropriate to model the volatility of a stock price.

(a) (*1 point*) Explain the concept of volatility smile.

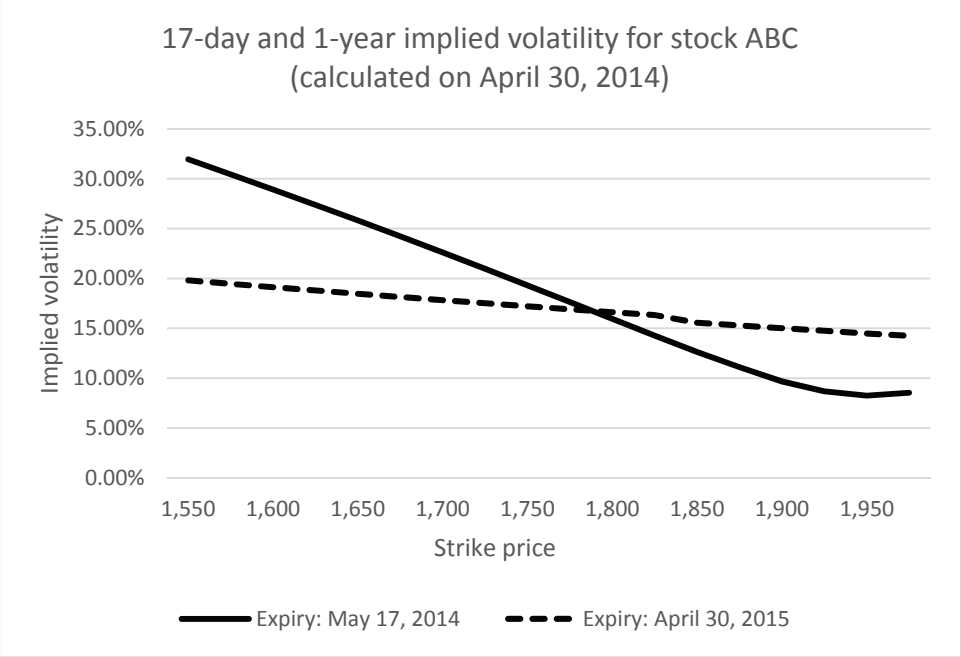
(b)

(i) (*1 point*) Identify the empirical fact about equity volatility smiles that is well illustrated in the graph below.



2. Continued

(ii) (*1 point*) Identify the empirical fact about equity volatility smiles that is well illustrated in the graph below.



Your team is using the Black-Scholes model to hedge their portfolio of variable annuities invested in an index with value S_t at time *t*. Since you raised concerns about the model's constant volatility assumption, they hesitate between hedging portfolios I and II presented below.

For a given variable annuity with embedded option value ψ_t at time *t*, the hedging portfolio $\prod_t$ is given by

$$\prod_t = \Delta_S S_t + \Delta_B B_t + \Delta_X X_t ,$$

Where Δ_S, Δ_B and Δ_X are the quantities invested in the index S_t, the money market B_t and the straddle option X_t on the index, respectively. In both portfolios, $\Delta_B = \psi_t - \Delta_S S_t - \Delta_X X_t$. The other quantities are indicated in the table.

QUESTION CONTINUED ON NEXT PAGE

2. Continued

Portfolio	Δ_S	Δ_X
I	$\frac{\partial \psi_t}{\partial S_t}$	0
II	$\frac{\partial \psi_t}{\partial S_t} - \Delta_X \frac{\partial X_t}{\partial S_t}$	$\frac{\partial \psi_t}{\partial \sigma} \Big/ \frac{\partial X_t}{\partial \sigma}$

(c) (*2 points*) Describe the hedging strategy of each portfolio, highlighting potential problems related to volatility hedging, if any.

(d) (*1 point*) Recommend one of the hedging portfolios.

(e) (*1 point*) Suggest improvements to Portfolio II.

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3. (*7 points*) You are the CEO of the ABC Life Insurance Company (ABC). Although ABC has traditionally been a market leader in product design, its new products have been poorly received repeatedly over the last three years.

You believe that a reason for this may be the Product Design Committee (PDC), which designs ABC's new products. For the past three years, the PDC has consisted of ten actuaries who have been with ABC for their entire career. During that time they were asked to provide expected ranges of new sales and only provided an accurate range (meaning that actual sales were within the specified range) about 20% of the time. This information was not shared with the committee.

(a) (*2 points*) Identify and explain how committee bias applies to the PDC and causes the PDC to repeatedly design poor products.

One of your underwriters, who has a background in behavioral finance, suggests that crowds should be used instead of committees.

(b) (*2 points*)

(i) (*1 point*) Identify and explain characteristics of crowds that differ from the PDC.

(ii) (*1 point*) Recommend changes to the PDC to take advantage of these crowd characteristics.

You decide to implement your underwriter's suggestion by allowing employees to anonymously make predictions about sales, rewarding them with bonuses based on the accuracy of their prediction.

You observe the following:

1. Employees getting numerous small bonuses are happier than those employees receiving one large bonus, even if the large bonus is greater than the sum of the small bonuses.
2. Employees often predict the occurrence of extremely unlikely sales outcomes.
3. Employees often deposit the earned bonuses into bank accounts instead of investing them into the equity market.

(c) (*3 points*) Explain each of the above observations in the context of Prospect Theory.

4. (*6 points*) You are an investment actuary at Happy Life, an insurance company in the U.S. Recently, the CFO has become more interested in adding alternative assets to the company's investment portfolio.

(a) (*2 points*) Describe briefly the four major features of alternative investments.

The CFO's main goals in adding an alternative asset class to the investment portfolio are the following:

(i) Higher overall returns

(ii) Willingness to transfer a larger proportion of the portfolio to long-term investments

(iii) Moderate increase to risk

(iv) Good background and supportive information about the asset class

An analyst in your investment department has suggested private equity funds as he believes this asset class is not as well-established as some of the others, and the potential benefits may be worth exploring.

(b) (*2 points*) Evaluate whether an investment in private equities meets each of the four goals.

(c) (*2 points*) Propose a different alternative asset class that aligns with the CFO goals.

5. (*7 points*) You are considering the following interest rate models for implementation at your life insurance company:

- Lognormal Forward-LIBOR Model (LFM)
- A G2++ short rate model

(a) (*2 points*) Compare and contrast the two models on:

(i) How they model interest rate dynamics

(ii) How they can be calibrated to market instruments

You are planning to hedge interest sensitive insurance liabilities using swaptions with various expirations and underlying maturities.

(b) (*1 point*) Recommend one of the two models and justify the choice.

Your manager has decided to use the Lognormal Forward-LIBOR Model. The following table shows the tenor structure and the associated ATM caplet lognormal implied volatilities:

Forward rates	Fixing in Years (T_i)	ATM caplet lognormal implied volatility
$F_0(t)$	0	-----
$F_1(t)$	1	12%
$F_2(t)$	2	13%
$F_3(t)$	3	16%
$F_4(t)$	4	15%

Your manager would like you to parameterize the volatility surface by using a piecewise constant function that is defined in the following table:

Instant. Vols	$t \in [0, T_1]$	$t \in [T_1, T_2]$	$t \in [T_2, T_3]$
$F_1(t)$	η_1	-----	-----
$F_2(t)$	η_2	η_1	-----
$F_3(t)$	η_3	η_2	η_1

(c) (*1 point*) Calculate η_3 based on the above tables.

5. Continued

Now your manager would like you to consider a parametric approach for the volatility surface. The following equation will be used to parameterize the volatility surface:

$$\sigma_i(t) = \Phi_i\left[\left[\left(a\left(T_{i-1} - t\right) + d\right)e^{-b(T_{i-1}-t)} + c\right]\right]$$

(d) (*1 point*) Describe the features of this parameterization.

(e) (*2 points*) Describe how you would calibrate this model to caplet prices.

6. (*8 points*) You are a corporate investment actuary at Trading Life Co. researching fixed-income investment strategies in the Energy sector. You are considering trades on bonds issued by Sauce Petroleum Co. and Pai Solar Co.

(a) (*1 point*) List and explain two reasons why investors enter negative basis trade.

You are given the following information about four bonds issued by Sauce Petroleum Co.:

Maturity (years)	1	2	3	4
Bond spread (bps)	125	130	135	140
Default probability	3%	4%	4%	5%
Expected recovery rate	70%	70%	60%	60%

(b) (*1 point*) Calculate the CDS Spread (assuming a simplified one-step time period approach) and Bond-CDS basis for each bond.

(c) (*1 point*) Identify the bond that offers the best *negative* basis trade arbitrage opportunity and describe the strategy.

(d) (*1 point*) Identify the bond that offers the best *positive* basis trade arbitrage opportunity and describe the strategy.

You are given the following information for a Pai Solar Co. bond and the risk-free yield curve:

Maturity (years)	3
Face value	1000
Annual coupon	4%
1-year spot risk-free rate	1%
2-year spot risk-free rate	2%
3-year spot risk-free rate	2%
Z-spread	280bps

(e) (*2 points*) Calculate the market price of the 3-year Pai Solar Co. bond.

(f) (*2 points*) Calculate the par asset swap spread of the 3-year Pai Solar Co. bond.

7. (*7 points*) You are the Chief Investment Officer of ABC Life. You manage the long duration investment grade bond portfolio of the company. Your mandate is to minimize the Tracking Error Volatility (TEV) compared to the U.S. Investment Grade (IG) credit index. The following bonds are listed in your portfolio with best estimate bid-ask spreads:

Bond	Option Adjusted Spread Duration	Bid-Ask spread (basis points)	Benchmark Bond (Yes\No)
A	5	50	No
B	5	37	Yes
C	10	20	No

Bonds A and B have been consistently quoted by dealers every business day for the last year while Bond C has not been quoted for three months. You are also given the following:

- Adjustment Factor = 1.6
- Non-quoted Adjustment Factor = 1.08

(a) (*1 point*) Calculate the Liquidity Cost Score™ (LCS) for each bond.

You have been alerted by the Chief Risk Officer of a potential sovereign crisis which may cause credit and liquidity concerns. You anticipate market illiquidity and that trading activity will significantly slow down for the next few months.

(b) (*2 points*) Assess the potential impact of these market conditions on the bonds' LCS.

You have been asked to establish a strategy to survive the predicted adverse market conditions while staying true to your mandate and considering cost as a key component. You have the following choices on how to structure your portfolio:

(i) A portfolio rebalanced to manage liquidity on the basis of issue size, volume and time of issue.

(ii) A portfolio rebalanced such that the targeted portfolio LCS is lower than that of the Investment Grade index LCS.

(iii) A portfolio rebalanced such that the targeted portfolio LCS equals that of the Investment Grade index LCS.

(c) (*4 points*) Assess the appropriateness of each strategy and recommend which one to use.

8. (*6 points*) The portfolio you manage has exposure to Canadian interest rates. You have decided to use principal component analysis (PCA) to assess the interest rate risk.

You are using random variables $x_1,\ldots,x_5$ to represent changes in the 6-month, 1-year, 5-year, 10-year, and 20-year rates, respectively.

For purposes of the PCA, you have tranformed the random variables x_i into standardized data as follows:

$$z_i = \frac{x_i - \mu_{x_i}}{\sigma_{x_i}}$$

You are denoting the principal components by $y_i = \beta_{i1}z_1 + \ldots + \beta_{i5}z_5$, where $z_1,...,z_5$ are determined as above.

You are given the eigenvectors of the estimated covariance matrix of *z*, and the corresponding eigenvalues, in the table below:

i		1	2	3	4	5
Eigenvalues	λ_i	1.11	0.06	0.03	3.63	0.18
Eigenvectors	β_{i1}	0.60	-0.55	0.15	0.39	0.40
	β_{i2}	0.48	0.69	-0.31	0.44	0.08
	β_{i3}	-0.12	-0.02	0.63	0.50	-0.57
	β_{i4}	-0.36	-0.39	-0.66	0.48	-0.23
	β_{i5}	-0.52	0.27	0.19	0.41	0.67

(a) (*1 point*) Determine which of the principal components above are necessary in order to explain at least 90% of the variability of the term structure of interest rates.

(b) (*2 points*) Compute the factor structure for each of the principal components you determined in (a).

You have determined that the most adverse scenario for your portfolio is a downward parallel shift in the term structure.

(c) (*1 point*) Identify which component from your principal component analysis corresponds to your portfolio's adverse scenario.

8. Continued

You are given the following information:

Maturity i	6-Month	1-Year	5-Year	10-Year	20-Year
Current Interest Rate	0.93%	1.56%	2.51%	3.00%	3.54%
Standard Deviation of x_i	0.43%	0.41%	0.25%	0.17%	0.12%
Average of x_i	0	0	0	0	0

(d) (*2 points*) Compute the two-standard-deviation term structure movement related to your portfolio's adverse scenario.

9. (*7 points*) You are an investment actuary at Longevity Life and are considering interest rate models for pricing interest sensitive products.

(a) (*1 point*) Describe the primary shortfall associated with using a one-factor model.

(b) (*1 point*) Describe the components of the G2++ model and explain why they lead to efficient procedures for pricing zero coupon bonds and caplets.

(c) (*3 points*) Identify three potential approaches that can be used to calibrate a two-factor G2++ model to real-market volatility data. For each, outline key considerations.

You decide to develop a G2++ model, however your manager is concerned about using such a model due to the possibility of negative rates. The unconditional expectation of the time 10 instantaneous short rate is 2% and the corresponding standard deviation of the instantaneous short rate is 1%.

(d) (*1 point*) Calculate the risk-neutral probability of a negative short rate at time 10.

(e) (*1 point*) Outline a brief response to your manager on the possibility of negative rates.

****END OF EXAMINATION****
Morning Session

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