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**SOCIETY OF ACTUARIES**  
**Foundations of CFE Exam**

# **Exam CFEFD**

## **MORNING SESSION**

**Date:** Tuesday, October 28, 2014

**Time:** 8:30 a.m. – 11:45 a.m.

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### **INSTRUCTIONS TO CANDIDATES**

#### **General Instructions**

1. This examination has a total of 100 points. It consists of a morning session (worth 60 points) and an afternoon session (worth 40 points).
  - a) The morning session consists of 8 questions numbered 1 through 8.
  - b) The afternoon session consists of 5 questions numbered 9 through 13.

The points for each question are indicated at the beginning of the question. There are no questions in the Morning Session that pertain to the Case Study.
2. Failure to stop writing after time is called will result in the disqualification of your answers or further disciplinary action.
3. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions on the exam booklet.

#### **Written-Answer Instructions**

1. Write your candidate number at the top of each sheet. Your name must not appear.
2. Write on only one side of a sheet. Start each question on a fresh sheet. On each sheet, write the number of the question that you are answering. Do not answer more than one question on a single sheet.
3. The answer should be confined to the question as set.
4. When you are asked to calculate, show all your work including any applicable formulas.
5. When you finish, insert all your written-answer sheets into the Essay Answer Envelope. Be sure to hand in all your answer sheets because they cannot be accepted later. Seal the envelope and write your candidate number in the space provided on the outside of the envelope. Check the appropriate box to indicate morning or afternoon session for Exam CFEFD.
6. Be sure your written-answer envelope is signed because if it is not, your examination will not be graded.

Tournez le cahier d'examen pour la version française.



**\*\*BEGINNING OF EXAMINATION\*\***  
**Morning Session**

**1.** (7 points)

(a) (1 point) Describe incremental risks and component risks of a portfolio.

You are given the following assumptions:

- The daily total returns of both Security A and Security B are assumed to follow normal distributions, each with  $\mu = 0$  and  $\sigma = 1$ .
  - The correlation coefficient between returns of Security A and Security B is 0.5.
  - The 95<sup>th</sup> percentile of the standard normal distribution is 1.645.
- (b) (2 points) Calculate the 95% daily VaR associated with a portfolio of \$1,250 of Security A and \$3,375 of Security B. Show your work.
- (c) (2 points)
- (i) Calculate the IVaR associated with adding \$10 of Security A to the portfolio using the delVaR approach. Show your work.
  - (ii) Interpret the result of (i) with regard to portfolio risk.
  - (iii) Describe the impact if the \$10 addition to the portfolio was of Security B instead of Security A.
- (d) (2 points) Explain two reasons why an analysis of daily IVaR may not predict actual changes to portfolio VaR.

2. (9 points) You work for Elbert Square, an investment company. Elbert has negotiated a deal with the popular franchise coffee shop, HypeD. Elbert may buy a HypeD franchise now for 10 million or delay the decision to purchase a franchise for exactly one year for the same amount.

(a) (1 point) Identify two advantages and two disadvantages of delaying the decision to buy a HypeD franchise for one year.

Elbert expects the HypeD franchise to generate 1 million in free cash flow in the next year and this amount will grow by 5.5% each year after that. Elbert's current cost of capital is 15%.

(b) (1 point) Calculate the Net Present Value (NPV) of purchasing a franchise now. Show your work.

The risk free rate is currently 4.5%. Elbert estimates the volatility (standard deviation) of the value of the franchise to be 60%. You are given the following formulas:

$$C = S^x N(d_1) - PV(K) N(d_2)$$

$$d_1 = \frac{\ln\left[\frac{S^x}{PV(K)}\right] + \frac{\sigma\sqrt{T}}{2}}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

The following excerpt from the Standard Normal Table should be used:

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.50000	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.52790	0.53188	0.53586
0.1	0.53983	0.54380	0.54776	0.55172	0.55567	0.55962	0.56356	0.56749	0.57142	0.57535
0.2	0.57926	0.58317	0.58706	0.59095	0.59483	0.59871	0.60257	0.60642	0.61026	0.61409
0.3	0.61791	0.62172	0.62552	0.62930	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173

## 2. Continued

(c) (3 points)

- (i) Calculate the value of the option to buy the franchise in one year. Show your work.
- (ii) Recommend whether Elbert should invest in the franchise now or delay the investment decision for a year. Support your recommendation.
- (iii) Explain whether your recommendation would change if the volatility of the franchise's value was 10% rather than 60%.

Elbert decides to purchase the HypeD franchise now. Elbert is also looking to invest in a dry cleaner franchise, StarcheD.

Assume:

- Each investment requires 10 million of standalone risk capital.
  - Elbert estimates:
    - The HypeD franchise will produce 2 million in profits per year.
    - The StarcheD franchise will produce 1 million in profits in the next year.
  - Deadweight cost of risk capital is 15%.
  - Profits of the two franchises are uncorrelated.
  - Total risk capital is proportional to the standard deviation.
- (d) (1 point) Calculate Elbert's estimated first-year profits after deadweight costs for each of the following:
- (i) HypeD franchise as stand-alone
  - (ii) StarcheD franchise as stand-alone
- (e) (3 points) Recommend whether Elbert should invest in the StarcheD franchise. Justify your recommendation.

3. (9 points) Treble Clef Insurance (TC) is a publically traded U.S. insurance company that is considering purchasing one of two insurers that offer piano insurance and harp insurance, respectively.

(a) (1 point) Contrast how TC should tailor its business plan to potential providers of debt financing versus equity financing.

The following information applies to TC:

Debt	\$205 million
Equity	\$425 million
Cash	\$30 million
Equity cost of capital	10%
Debt cost of capital	6%
Corporate tax rate	35%

Either purchase would require an up-front investment of \$80 million. The piano business will not alter TC's overall risk profile. TC assumes the harp business will require a 12% equity cost of capital and an 8% debt cost of capital. TC will maintain the same debt-equity ratio going forward.

Expected Return Information:

	First Year Free Cash Flow	Annual Growth Rate
Piano	\$4 million	5%
Harp	\$8 million	4%

(b) (3 points) Recommend which insurer TC should purchase. Justify your recommendation.

(c) (1 point) Calculate the amount of debt that must be used to fund the purchase recommended in part (b) to maintain TC's debt-equity ratio.

TC plans to issue bonds to fund part of the purchase.

(d) (2 points) Explain how each of the following provisions would impact TC's bond issue:

- (i) Restrictive covenants
- (ii) Convertibility
- (iii) Call provisions

### **3. Continued**

TC is also considering enhancing its statutory surplus position. A consultant has suggested securitizing future revenue from the purchase. The CEO suggests issuing more bonds than necessary for the purchase and using the excess to enhance TC's statutory surplus position.

(e) (2 points)

- (i) Assess the consultant's suggestion. Support your assessment.
- (ii) Assess the CEO's suggestion. Support your assessment.

4. (8 points) Securities regulators in the nation of Collard Greenes (CG) have just allowed a securities market to open. Due to CG's strict privacy laws, current market prices of securities and the risk-free rate are unobservable.

There are three securities available in CG's securities market: A, B, and C. Assume one-period market equilibrium conditions exist.

A survey of CG's market participants reveals the following homogeneous beliefs about the three possible future states,  $\omega_1$ ,  $\omega_2$ , and  $\omega_3$ , one period into the future:

- Subjective probabilities of each future state  $p(\omega_i)$ ;
- Marginal rates of substitution (MRS); and
- Expected prices,  $X_j$ , corresponding with each future state:

	$p(\omega_i)$	MRS	$X_A$	$X_B$	$X_C$
$\omega_1$	1/3	0.90	2	8	10
$\omega_2$	1/6	0.90	5	7	2
$\omega_3$	1/2	0.95	6	9	5

- (a) (2 points) Calculate current market prices of the three securities, A, B, and C.
- (b) (4 points) Assume there is a complete market of one-period Arrow-Debreu securities.
- (i) State the definition of an Arrow-Debreu security.
  - (ii) Calculate prices for the three Arrow-Debreu securities,  $\psi_\omega$ .
  - (iii) Calculate the one-period risk-free rate for CG.

Insurance regulators for CG have proposed a new dynamic solvency test. For financial assets and liabilities that contain embedded options, it requires an analysis of future cash flows using prescribed risk neutral scenarios.

- (c) (2 points) Critique the regulator's proposal.



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5. (7 points) Your firm has been hired to model Lucky Tom's retirement portfolio performance using Monte Carlo simulation. His retirement portfolio consists of three investments: a stock fund, a fixed-income fund, and an energy fund.

(a) (1 point) Identify an advantage and disadvantage of using Monte Carlo simulation to model the funds in Lucky Tom's portfolio.

You are given the following variance-covariance matrix for Lucky Tom's three investments:

	Stock	Fixed Income	Energy
Stock	0.160	0.072	-0.080
Fixed Income	0.072	0.090	-0.030
Energy	-0.080	-0.030	0.250

Your firm uses a random number generator to produce independent Gaussian values with zero mean and unit variance. It then creates correlated random values from the independent Gaussian values.

The correlated values,  $\varphi$ , are used in a standard geometric Brownian motion process to model Lucky Tom's investments:  $dS / S = \mu dt + \sigma \varphi \sqrt{dt}$ .

You have the following Cholesky decomposition formulas for elements of the lower triangular square-root matrix,  $L$ :

$$L_{1,1} = \sqrt{A_{1,1}} \qquad L_{j,1} = A_{j,1} / L_{1,1}$$

$$L_{j,j} = \sqrt{A_{j,j} - \sum_{k=1}^{j-1} L_{j,k}^2}$$

where  $A$  is the variance-covariance matrix.

## 5. Continued

(b) (2 points)

(i) Determine the missing values for  $L$ :

$$L = \begin{bmatrix} 0.400 & 0.000 & 0.000 \\ 0.180 & ? & ? \\ -0.200 & 0.025 & ? \end{bmatrix}$$

(ii) Verify  $L$  is the lower triangular square-root matrix of  $A$ .

The first three values produced by your firm's Gaussian generator are:  $\begin{bmatrix} 0.9 \\ -1.1 \\ 0.2 \end{bmatrix}$ .

(c) (2 points) Calculate the first three correlated values,  $\varphi$ , for the Monte Carlo simulation using the first three Gaussian values.

Lucky Tom's friend believes your firm should not use Brownian motion for all three investments. She has researched two other random processes:

I.  $dx = \eta_E (\bar{x} - x) dt + \sigma_E dz_E + dq_E$ , where

- $x$  is the natural log of the spot price
- $dz_E$  is a Wiener diffusion process
- $dq_E$  is a Poisson process

II.  $dr = k(\mu - r)dt + \sigma\sqrt{r} dz$

(d) (2 points)

(i) Describe the two processes, I and II, above.

(ii) Recommend a process, I or II above or geometric Brownian motion, as the most appropriate for each of Lucky Tom's investments.

6. (7 points) The five TV stations in the city of Hannavas all advertise the most accurate weather forecasts.

Station A just uses today's high temperature as the estimate for tomorrow's high temperature.

Station W assumes that days are divided into three states: cold, temperate, hot. They created a transition matrix that relates the probabilities of moving from one state to another based on a day's high temperature:

Type of Day	Average High Temperature	Likelihood of Conditions Tomorrow		
		Cold	Temperate	Hot
Cold	55	70%	30%	0%
Temperate	70	25%	50%	25%
Hot	90	0%	40%	60%

Yesterday's high temperature was 66 and today's high is 72.

- (a) (1 point) For Station A:
- (i) Estimate the high temperature for two days from today.
  - (ii) Identify the type of stochastic process its daily high temperature forecast assumes.
- (b) (2 points) For Station W:
- (i) Describe the type of model its daily high temperature forecast assumes.
  - (ii) Briefly discuss what other parameters might be necessary to make an estimate.
  - (iii) Suggest a model distribution for temperatures within the various daily states. Defend your suggestion.

The Hannavas newspaper has published the following chart, calling into question the TV stations' forecasting ability:

	Station A	Station C	Station F	Station N	Station W
Forecasted Monthly Average Temperature	67	80	75	86	77
% of Time Station is Most Accurate	17%	19%	1%	13%	50%

**6. Continued**

(c) (1 point)

- (i) Calculate the maximum entropy for this set of observations.
- (ii) Estimate the monthly average temperature using maximum entropy.

(d) (1 point)

- (i) Calculate minimum entropy for the set of observations, not including Station F.
- (ii) Estimate the monthly average temperature using minimum entropy.

Your own view, as a citizen of Hannavas, is that tomorrow's high temperature will be 74. You wish to use your own view of tomorrow's forecasted high temperature to refine your estimate of the monthly average temperature. You are given the following:

	Station A	Station C	Station F	Station N	Station W
Tomorrow's Forecasted High Temperature	72	75	90	73	70

(e) (2 points)

- (i) Develop an estimate of the monthly average temperature by weighting multiple stations' observations to match your own view of tomorrow's high temperature with entropy of at least 0.94.
- (ii) Explain why the entropy methodology cannot be used to refine your estimate of the monthly average temperature if your own view is that tomorrow's high temperature will be 69.

7. (6 points) You are assisting the CRO of Oglethorpe, Inc., with a report on historical U.S. Treasury yield curve movement.

You suggest using a principal component analysis (PCA) to explain U.S. Treasury yield curve changes. Results are below:

	PC1	PC2	PC3	PC4	PC5
<b>Eigen Value</b>	2.4765	0.0664	0.0123	0.0032	0.0016
<b>Factor Loadings</b>					
<b>3 month</b>	0.0148	-0.1064	0.3433	-0.2836	-0.4284
<b>1 year</b>	0.0596	-0.3270	0.5251	0.0272	0.0177
<b>5 year</b>	0.4007	-0.3267	-0.3051	-0.0837	0.0741
<b>10 year</b>	0.4422	0.1221	-0.1709	0.0279	-0.4066
<b>30 year</b>	0.3878	0.5908	0.1966	-0.5259	0.4134

The CRO states that convexity is not as important as trend and tilt in explaining the variation in U.S. Treasury yields.

- (a) (2 points) Critique the CRO's statement. Justify your critique.

Oglethorpe is prohibited from trading derivatives. Therefore, a key challenge for Oglethorpe is the valuation of pension liability cash flows beyond 30 years.

- (b) (2 points) Describe three methods that can be used to derive discount factors beyond 30 years.

Recent economic data indicate the trend of the U.S. Treasury yield curve will drop to historically low levels and remain low for the foreseeable future.

- (c) (2 points) Recommend a method from your answer to (b) to derive discount factors in this scenario. Support your recommendation.

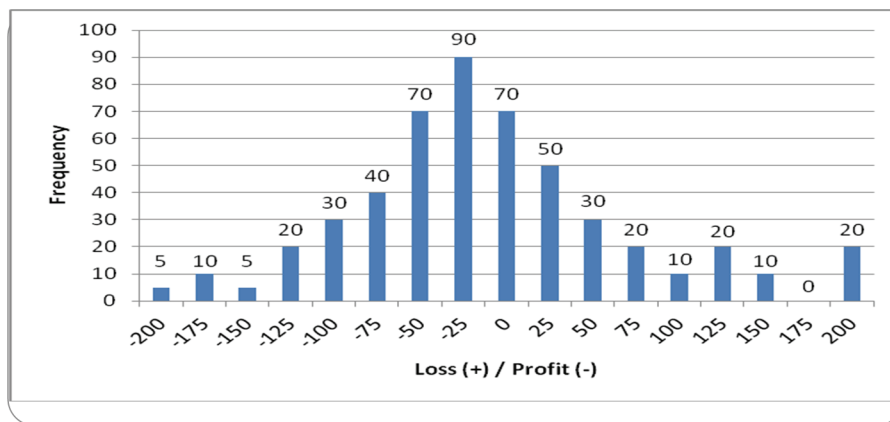
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8. (7 points) Reynolds-Warren Company (RWC), a U.S. insurer, is determining financial risk measures for its subsidiary in Risktopia, an emerging market. RWC has little control over data reliability or delivery for its subsidiary.

RWC is deciding between parametric and non-parametric approaches to calculate financial risk measures for its Risktopia subsidiary.

- (a) (1 point) List 2 advantages and 2 disadvantages of non-parametric methods.

RWC received the following histogram of 500 historical simulation loss/profit data for its Risktopia subsidiary from a consultant:



- (b) (1 point) Calculate the 95% VaR and 95% expected shortfall (ES) based on the histogram.
- (c) (2 points) Explain how to improve the results in (b) using the following:
- Bootstrapped historical simulation.
  - Weighted historical simulation.

RWC estimated confidence intervals for VaR and ES. Within the data, it was noted the most recent time period has twice the volatility of the older time period.

- (d) (1 point)
- Describe the impact of ignoring the change in volatility.
  - Explain how to correct for the change in volatility.



**8. Continued**

After several quarters of unexpectedly volatile results, RWC questions the appropriateness of its model.

- (e) (2 points) Recommend a better approach for calculating risk measures for RWC's subsidiary. Justify your recommendation.

**\*\*END OF EXAMINATION\*\***  
**Morning Session**

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