GI ADV Model Solutions
Fall 2014

1. Learning Objectives:

4. The candidate will understand how to apply the fundamental techniques of reinsurance pricing.

Learning Outcomes:

(4b) Calculate the price for a property per risk excess treaty.

Sources:

Basics of Reinsurance Pricing, Clark

Solution:

(a) Explain why the exposure curve allows for exposure above 100% of the insured value.

Commentary on Question:
It was not necessary to specifically name business interruption coverage. A statement that there can be additional coverages earned some credit.

Insured values often do not include business interruption.

(b) Calculate the expected loss in the requested layer assuming an expected loss ratio of 60%.

The calculation is 60%(1,000,000)[100% – 70%] + 60%(1,000,000)[93% – 49%] + 60%(1,000,000)[70% – 37%] = 642,000.

(c) State the key assumption underlying the use of a single exposure curve to price this treaty.

Commentary on Question:
There is no alterative acceptable response.

The key assumption is that the same exposure curve applies regardless of the insured value.

1. **Continued**

(d) Calculate the revised expected loss in the layer.

Commentary on Question:
No credit was lost if an incorrect value from Part (b) was carried over.

For insured values of 1,000,000, the net insured value is 500,000 and the subject premium is reduced to 500,000. The third item in the solution to part (b) becomes (60%)(500,000)[93% – 49%]. The revised expected loss is 576,000.

2. Learning Objectives:

5. The candidate will understand methodologies for determining an underwriting profit margin.

Learning Outcomes:

(5b) Calculate an underwriting profit margin using the capital asset pricing model.

(5c) Calculate an underwriting profit margin using the risk adjusted discount technique.

Sources:

Ratemaking: A Financial Economics Approach, D'Arcy and Dyer

Solution:

(a) Explain what it means for the underwriting beta to be zero.

Commentary on Question:
Candidates earned partial credit for knowing that it relates to lack of correlation. However, the specific two uncorrelated items needed to be identified for full credit.

Beta is zero indicates that earthquake losses are uncorrelated with market returns.

(b) Identify any other information you might need to calculate the underwriting profit margin for this policy.

No additional information is needed because the underwriting profit margin is zero.

(c) Evaluate whether it would be appropriate to use the Capital Asset Pricing Model to calculate the underwriting profit margin for this policy.

CAPM is not appropriate because it does not take catastrophe risk into account.

(d) Calculate the underwriting profit margin.

P = 72/0.9 + 20 = 100. The underwriting profit margin is (100 – 20 – 72)/100 = 8%.

2. Continued

(e) Identify two drawbacks to the Risk Adjusted Discount Technique.

Commentary on Question:
The solution below lists five drawbacks. Any two were sufficient to earn credit.

- Expenses are assumed to not depend on premiums.
- Present value of premiums is based on the risk-free rate, but lag in premium collection is not equivalent to investing in a risk-free security.
- It is difficult to determine the risk adjusted discount rate.
- It is difficult to allocate equity to policies.
- It considers only one policy term.

3. Learning Objectives:

2. The candidate will understand the considerations in selecting a risk margin for unpaid claims.

Learning Outcomes:

(2a) Describe a risk margin analysis framework.

(2b) Identify the sources of uncertainty underlying an estimate of unpaid claims.

(2c) Describe methods to assess this uncertainty.

Sources:

A Framework for Assessing Risk Margins, Marshall, Collings, Hodson, and O'Dowd

Solution:

(a) Describe the following sources of internal systemic risk:

(i) Specification Error

(ii) Parameter Selection Error

(iii) Data Error

Commentary on Question:

Candidates did relatively poorly with regard to giving complete definitions.

(i) Specification error – The error from an inability to build a model that is fully representative of the underlying process.

(ii) Parameter selection error – The error from the model being unable to measure all predictors of claim cost outcomes or trends in these predictors.

(iii) Data error – The error due to poor data, unavailability of data and/or inadequate knowledge of the portfolio being analyzed.

(b) Calculate the aggregate coefficient of variation for both lines combined.

The independent risk is $\sqrt{6\%^2(600/1000)^2 + 8\%^2(400/1000)^2} = 4.8\%$.

The internal systemic risk is

$\sqrt{5\%^2(600/1000)^2 + 7\%^2(400/1000)^2 + 2(0.25)(5\%)(7\%)(6/10)(4/10)} = 4.6\%$.

The aggregate coefficient of variation is $\sqrt{4.8\%^2 + 4.6\%^2 + 2.9\%^2} = 7.3\%$.

3. Continued

(c) Explain the likely effect, if any, of each of the following items on each of independent risk, internal systemic risk, and external systemic risk:

(i) New legislation increasing the statute of limitations for motor claims

(ii) Reduced correlation between motor and property claims

(iii) More accurate underwriting systems

(i) New legislation – Will increase the uncertainty regarding settlements, which will affect all three risk types. There is more variability (independent), it will be harder to model (internal), and the effect of the change will not be immediately known (external).

(ii) Reduced correlation – By definition there is no correlation effect on independent risk. The other two will be decreased.

(iii) More accurate underwriting – More stable systems will reduce independent risk. Models should be more accurate, reducing internal risk. There is unlikely to be an effect on external risk.

4. **Learning Objectives:**

1. The candidate will understand how to use basic loss development models to estimate the standard deviation of an estimator of unpaid claims.

Learning Outcomes:

(1a) Identify the assumptions underlying the chain ladder estimation method.

(1b) Test for the validity of these assumptions.

(1d) Estimate the standard deviation of a chain ladder estimator of unpaid claims.

Sources:

Measuring the Variability of Chain Ladder Reserve Estimates, Mack
Testing the Assumptions of Age-to-Age Factors, Venter

Solution:

(a) Demonstrate that the value of α_4^2 was correctly calculated. (Your calculation need not match to all four decimal places.)

$$\alpha_4^2 = \frac{1}{6-4-1}\sum_{j=1}^{6-4} c_{j4}\left(\frac{c_{j5}}{c_{j4}} - f_4\right)^2 = c_{14}\left(\frac{c_{15}}{c_{14}} - f_4\right)^2 + c_{24}\left(\frac{c_{25}}{c_{24}} - f_4\right)^2$$

$$= 14,317\left(\frac{14,784}{14,317} - 1.02610\right)^2 + 13,768\left(\frac{14,034}{13,768} - 1.02610\right)^2 = 1.2412$$

(b) Demonstrate that the standard error for accident year 3 was correctly calculated.

The variance is

$$c_{36}^2 \sum_{k=6+1-3=4}^{6-1=5} \frac{\alpha_k^2}{f_k^2}\left(\frac{1}{c_{3k}} + \frac{1}{\sum_{j=1}^{6-k} c_{jk}}\right) = c_{36}^2\left[\frac{\alpha_4^2}{f_4^2}\left(\frac{1}{c_{34}} + \frac{1}{\sum_{j=1}^{6-4=2} c_{j4}}\right) + \frac{\alpha_5^2}{f_5^2}\left(\frac{1}{c_{35}} + \frac{1}{\sum_{j=1}^{6-5=1} c_{j5}}\right)\right]$$

$$= 16,060^2\left[\frac{1.2412}{1.02610^2}\left(\frac{1}{15,619} + \frac{1}{14,317 + 13,768}\right) + \frac{0.0916}{1.00210^2}\left(\frac{1}{16,027} + \frac{1}{14,784}\right)\right] = 33,353$$

and the standard error is the square root, 183.

4. Continued

(c) Calculate the upper limit of a 95% confidence interval for outstanding claims for accident year 3 using a normal distribution. The 97.5th percentile of the standard normal distribution is at 1.96.

The outstanding claims are 16,060 – 15,619 = 441. The standard error is 183 and so the upper limit is 441 + 1.96(183) = 800.

(d) Propose a method for constructing an improved confidence interval. Justify your proposal.

Commentary on Question:
Candidates did not fully recall the proposal from the Mack paper. An alternative proposal with sound justification would be acceptable, but none was offered by candidates.

Use a lognormal distribution rather than the normal distribution. Use the method of moments to estimate the parameters of the lognormal distribution. Determine a normal distribution-based interval centered at μ and then exponentiate the endpoints.

(e) Explain why the variance of this estimate is greater than the sum of the six variances by accident year.

Commentary on Question:
Candidates understood that the cause was correlation but did not explain that it is the fact that the correlations are positive that causes the variance to exceed the sum.

The individual estimates are positively correlated due to the common age-to-age factors. The variance of the sum includes covariance terms, which are all positive.

(f) Indicate whether this plot supports the Mack assumptions. Justify your answer.

Commentary on Question:
There are two things to look at; candidates tended to only look at one of them.

To support the assumptions, there should be no patterns. The variability as you scan from left to right should be constant and there should be no pattern, such as a curve or trend. Neither of these is apparent from the plot and thus the assumptions are supported.

4. Continued

(g) Indicate whether this calculation supports the Mack assumptions. Justify your answer.

Commentary on Question:
The first point to note is that a large value indicates the assumptions are not supported. To earn full credit (and most did not) there must be some indication regarding the significance of the correlation.

A large correlation indicates the assumptions are not supported. One way to evaluate the significance of the correlation is to standardize it. One such standardization is $T = r\sqrt{(n-2)/(1-r^2)} = 0.601$. This is not significant relative to a *t* distribution and so the assumptions are supported.

5. Learning Objectives:

1. The candidate will understand how to use basic loss development models to estimate the standard deviation of an estimator of unpaid claims.

Learning Outcomes:

(1e) Apply a parametric model of loss development.

(1f) Estimate the standard deviation of a parametric estimator of unpaid claims.

Sources:

LDF Curve Fitting and Stochastic Reserving: A Maximum Likelihood Approach, Clark

Solution:

(a) Provide the term within this function corresponding to 0-12 months of development in accident year 2011, using Clark's Cape Cod method and an exponential distribution with cumulative distribution function $G(x)=1-e^{-x/\theta}$.

The requested term is

$$4{,}000\ln\left(ELR\cdot 12{,}000\left(1-e^{-\frac{6}{\theta}}\right)\right)-ELR\cdot 12{,}000\left(1-e^{-\frac{6}{\theta}}\right).$$

(b) Provide the fitted triangle of cumulative paid losses.

The triangle is:
4788 7814 8398
4788 7814
4788
The calculations are:
12,000xELRxG(6) = 12,000(0.7115)(1 – exp(-6/7.293) = 4788
12,000xELRxG(18) = 12,000(0.7115)(1 – exp(-18/7.293) = 7814
12,000xELRxG(30) = 12,000(0.7115)(1 – exp(-30/7.293) = 8398

(c) Estimate ultimate losses for accident year 2011.

Estimated ultimate losses are 8000 + 12,000(0.7115) – 8398 = 8140.

(d) Identify the number of degrees of freedom associated with the estimate of the scale factor, σ^2.

Degrees of freedom are 6 (observations) – 2 (estimated parameters) = 4.

5. Continued

(e) Estimate the process standard deviation of the accident year 2011 reserve.

The estimated process standard deviation is sqrt(140x273) = 195.

(f) Provide an expression for the estimate of the parameter variance of the accident year 2011 reserve using matrix notation. (Do not compute the result.)

The expression is

$$\begin{pmatrix} 12000 \cdot e^{-\frac{30}{7.293}} & 8538 \cdot \frac{30}{7.293^2} e^{-\frac{30}{7.293}} \end{pmatrix} \begin{pmatrix} 0.00770 & 0.0444 \\ 0.0444 & 1.75 \end{pmatrix} \begin{pmatrix} 12000 \cdot e^{-\frac{30}{7.293}} \\ 8538 \cdot \frac{30}{7.293^2} e^{-\frac{30}{7.293}} \end{pmatrix}.$$

(g) Explain whether you would expect the parameter variance to be larger, smaller, or about the same as the parameter variance obtained using the LDF method.

The variance is expected to be smaller because the on-level premium information allows for a better estimate of the reserve.

6. Learning Objectives:

5. The candidate will understand methodologies for determining an underwriting profit margin.

Learning Outcomes:

(5d) Allocate an underwriting profit margin (risk load) among different accounts.

Sources:

An Application of Game Theory: Property Catastrophe Risk Load, Mango

Solution:

(a) Calculate the risk load for each account using the Shapley method.

Commentary on Question:
Not all candidates used the proper allocation of the covariances when calculating the Shapley values.

First, obtain the three covariances:
Cov(X,Y) = 0.8sqrt(200,000x12,000) = 39,192
Cov(X,Z) = 0.4sqrt(200,000x10,000) = 17,889
Cov(Y,Z) = 0
Second, obtain the Shapley values:
For X: 200,000 + 39,192 + 17,889 = 257,081
For Y: 12,000 + 39,192 + 0 = 51,192
For Z: 10,000 + 17,889 + 0 = 27,889
Third, multiply by the risk load multiplier, 0.0025:
X: 643
Y: 128
Z: 70

(b) Explain how your answer to part (a) would have been different if the Covariance Share method had been used.

The covariance share method allocates covariances proportionally rather than equally. With X having a larger expected loss, it will receive a greater share of the covariance and thus a higher risk load.

7. Learning Objectives:

3. The candidate will understand how to use a credibility model with parameters that shift over time.

Learning Outcomes:

(3a) Identify the components of a credibility model with shifting risk parameters.

(3c) Estimate the parameters of the model.

(3d) Compare various models that might be used.

Sources:

Credibility with Shifting Risk Parameters, Klugman

Solution:

(a) Describe each of the components in the model.

Mu: The overall mean across all accounts
Alpha: The amount by which account *i* differs from the overall mean
Gamma: The amount by which year *j* differs from that account's mean
Epsilon: The random error

(b) Explain how this model differs from the Bühlmann-Straub model.

The Bühlmann-Straub model does not have the gamma vector. Equivalently, setting delta = 0 yields the Bühlmann-Straub model. Alternatively, it could be noted that within an account, all years are independent with the same variance per observation.

(c) Identify the time series model represented by this matrix.

The geometric pattern of the correlations indicates an AR(1) model.

(d) Explain what that model means with regard to the evolution of pure premiums over time.

An AR(1) model indicates that the mean for the next year is a linear combination of the most recent observation and the long-term mean.

7. **Continued**

(e) State the appropriate restriction to place upon the credibility weights that is consistent with the time series model in part (c).

There will be three different weights:
One on the overall mean (mu)
A second on the most recent (time 10) observation
A third that applies to the other nine observations

(f) Explain the relative merits of the two approaches to obtaining the credibility weights.

Using restricted weights produces a pattern that is easy to explain and is consistent with the model being used. Using unrestricted weights minimizes squared error and thus provides a better estimate of future values.

8. **Learning Objectives:**

4. The candidate will understand how to apply the fundamental techniques of reinsurance pricing.

Learning Outcomes:

(4c) Calculate the price for a casualty per occurrence excess treaty.

Sources:

Basics of Reinsurance Pricing, Clark

Solution:

(a) Calculate the expected losses in the layer using an exposure rating approach with an expected loss ratio of 60% and the following increased limits factors:

The expected loss is 60% times the sum of the following terms:
5,000,000(1.35 – 1.00)/1.35
4,000,000(1.35 – 1.00)/1.56
3,000,000(1.56 – 1.35)/(1.56 – 1.00).
The answer is 1,991,239.

(b) Identify the two methods for handling trend and policy limits when an experience rating approach is used to calculate the expected losses. State the assumption that underlies each method.

One method is to trend losses but neither policy limits nor subject premiums. This method assumes that policy limits do not increase over time.

A second method is to trend losses and policy limits and adjust subject premiums to match the higher limits. This method assumes that policy limits increase over time.

(c) Identify, for each loss, the range of ALAE amounts for which allocating ALAE to the layer in proportion to losses would result in Casualty R Us paying more than it would if ALAE were included with losses.

For the 600,000 loss, if ALAE is included an ALAE of 400,000 will lead to the full 500,000 being paid. If allocated, one-sixth will be allocated and an ALAE of 400,000x6 = 2,400,000 will lead to the same result. Thus, the requested range is ALAE greater than 2,400,000.

For the 750,000 loss, if ALAE is included an ALAE of 250,000 will lead to the full 500,000 being paid. If allocated, one-third will be allocated and an ALAE of 250,000x3 = 750,000 will lead to the same result. Thus, the requested range is ALAE greater than 750,000.

8. **Continued**

(d) Identify any additional casualty per occurrence excess coverage that the ceding company should consider purchasing. Justify your answer.

Consider 500,000 excess of 1,000,000 because that layer is exposed by those policies that have a limit of 1,500,000. Another option is clash coverage.