SOCIETY OF ACTUARIES
Advanced Topics in General Insurance

Exam GIADV

Date: Wednesday, October 29, 2014
Time: 2:00 p.m. – 4:15 p.m.

INSTRUCTIONS TO CANDIDATES

General Instructions

1. This examination has a total of 40 points.

   This exam consists of 8 questions, numbered 1 through 8.

   The points for each question are indicated at the beginning of the question.

2. Failure to stop writing after time is called will result in the disqualification of your answers or further disciplinary action.

3. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions on the exam booklet.

Written-Answer Instructions

1. Write your candidate number at the top of each sheet. Your name must not appear.

2. Write on only one side of a sheet. Start each question on a fresh sheet. On each sheet, write the number of the question that you are answering. Do not answer more than one question on a single sheet.

3. The answer should be confined to the question as set.

4. When you are asked to calculate, show all your work including any applicable formulas.

5. When you finish, insert all your written-answer sheets into the Essay Answer Envelope. Be sure to hand in all your answer sheets because they cannot be accepted later. Seal the envelope and write your candidate number in the space provided on the outside of the envelope. Check the appropriate box to indicate Exam GIADV.

6. Be sure your written-answer envelope is signed because if it is not, your examination will not be graded.

Tournez le cahier d'examen pour la version française.

1. (*4 points*) Property R Us Reinsurance Company uses the following exposure factors to price its commercial property per risk excess treaties:

Percent of Insured Value	Exposure Factor
0%	0%
10%	37%
20%	49%
30%	57%
40%	64%
50%	70%
60%	76%
70%	81%
80%	85%
90%	89%
100%	93%
110%	97%
120%	100%

(a) (*0.5 points*) Explain why the exposure curve allows for exposure above 100% of the insured value.

Property R Us has received a submission requesting 400,000 excess of 100,000 property per risk coverage for insured values and subject premium as follows:

Insured Value	Subject Premium
200,000	1,000,000
500,000	1,000,000
1,000,000	1,000,000

(b) (*2 points*) Calculate the expected loss in the requested layer assuming an expected loss ratio of 60%.

(c) (*0.5 points*) State the key assumption underlying the use of a single exposure curve to price this treaty.

The submission is revised to reflect the purchase of surplus share reinsurance with a retained line of 500,000, which will inure to the benefit of the property per risk treaty.

(d) (*1 point*) Calculate the revised expected loss in the layer.

2. (*4 points*) You are calculating the underwriting profit margin for a one-year earthquake insurance policy using the Capital Asset Pricing Model and you determine that both the underwriting beta and the risk-free rate are zero.

    (a)   (*0.5 points*) Explain what it means for the underwriting beta to be zero.

    (b)   (*0.5 points*) Identify any other information you might need to calculate the underwriting profit margin for this policy.

    (c)   (*1 point*) Evaluate whether it would be appropriate to use the Capital Asset Pricing Model to calculate the underwriting profit margin for this policy.

As a possible alternative, you turn to the Risk Adjusted Discount Technique with the following assumptions:

- The premium will be collected when the policy is effective.
- Expenses of 20 will be paid when the policy is effective.
- Losses are expected to be 72 and will be paid at the end of the year.
- The risk-free rate is 0%.
- The risk-adjusted rate for losses is –10%.

    (d)   (*1.5 points*) Calculate the underwriting profit margin.

    (e)   (*0.5 points*) Identify two drawbacks to the Risk Adjusted Discount Technique.

3. (*4 points*) You are calculating a risk margin for claim liabilities using the methodology set out in "A Framework for Assessing Risk Margins." The following information is provided:

Line of Business	Claim Liabilities	Coefficients of Variation		
		Independent Risk	Internal Systemic Risk	External Systemic Risk
Motor	600	6.0%	5.0%	4.0%
Property	400	8.0%	7.0%	3.0%
Total	**1,000**			**2.9%**

The correlation between lines for internal systemic risk was assessed at 25%.

(a)   (*1.5 points*) Describe the following sources of internal systemic risk:

   (i)   Specification Error

   (ii)   Parameter Selection Error

   (iii)   Data Error

(b)   (*1 point*) Calculate the aggregate coefficient of variation for both lines combined.

(c)   (*1.5 points*) Explain the likely effect, if any, of each of the following items on each of independent risk, internal systemic risk, and external systemic risk:

   (i)   New legislation increasing the statute of limitations for motor claims

   (ii)   Reduced correlation between motor and property claims

   (iii)   More accurate underwriting systems

4. (*7 points*) You are interested in determining the variability of unpaid claim estimates. The triangle of data you are working with is presented below. The shaded cells have been completed using the standard chain ladder method. It is assumed that all claims are fully developed after six years.

Mack's method of estimating reserve variability has been applied to this triangle. The key results are provided in the table.

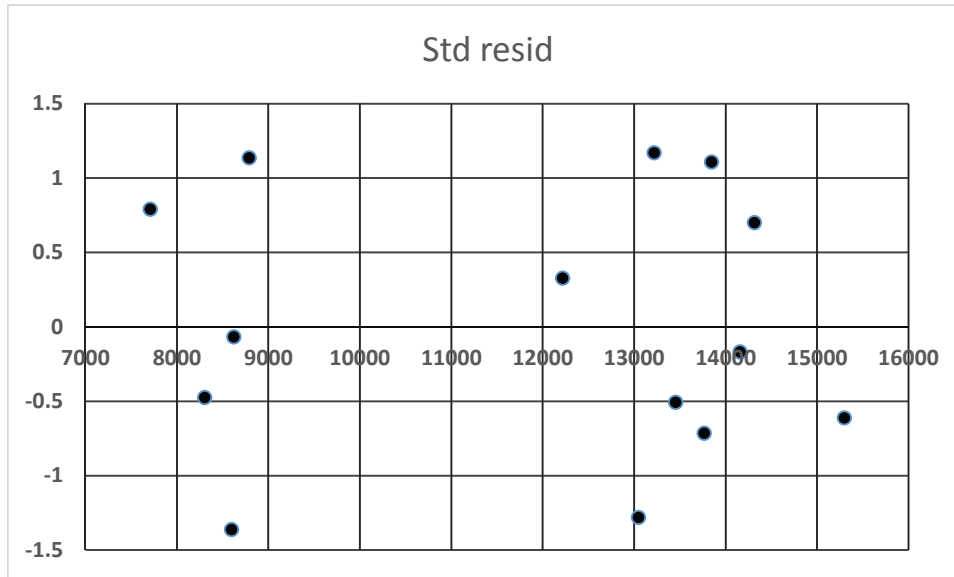| AY	1	2	3	4	5	6	Standard error
			Development Year				
1	8,600	12,221	13,221	14,317	14,784	14,815	0
2	8,306	13,049	13,455	13,768	14,034	14,063	50
3	7,709	13,847	15,300	15,619	16,027	16,060	183
4	8,623	14,159	15,096	15,717	16,128	16,161	632
5	8,791	16,224	17,380	18,095	18,568	18,607	888
6	9,021	14,917	15,980	16,638	17,072	17,108	2080
f_k	1.65362	1.07125	1.04117	1.02610	1.00210		
α_k^2	250.8709	12.8207	16.8267	1.2412	0.0916		

AY = accident year

(a) (*1.5 points*) Demonstrate that the value of α_4^2 was correctly calculated. (Your calculation need not match to all four decimal places.)

(b) (*1.5 points*) Demonstrate that the standard error for accident year 3 was correctly calculated.

(c) (*0.5 points*) Calculate the upper limit of a 95% confidence interval for outstanding claims for accident year 3 using a normal distribution. The 97.5th percentile of the standard normal distribution is at 1.96.

(d) (*1 point*) Propose a method for constructing an improved confidence interval. Justify your proposal.

The total developed claims over the six accident years is 96,815.

(e) (*0.5 points*) Explain why the variance of this estimate is greater than the sum of the six variances by accident year.

4.   Continued

Venter suggests that a test of the assumptions underlying Mack's method is to plot the residuals of the estimated versus actual increments. The following plot has the residuals standardized by dividing by the standard errors.

(f)   (*1 point*) Indicate whether this plot supports the Mack assumptions. Justify your answer.

Another suggestion of Venter is to calculate the correlation between the ratios of the incremental claims to the cumulative claims for successive development years. For this data, the estimated correlation coefficient of the ratios from development year 1 to 2 with the ratios from development year 2 to 3 is 0.391 based on four observations.

(g)   (*1 point*) Indicate whether this calculation supports the Mack assumptions. Justify your answer.

5. (*6 points*) You are given the following triangle of cumulative paid losses:

Accident Year	Months of Development		
	12	24	36
2011	4,000	7,000	8,000
2012	5,000	7,000	
2013	6,000		

On level premium for each year is 12,000.

The function $\sum_i c_i \ln(\mu_i) - \mu_i$ must be maximized to obtain maximum likelihood estimates of the parameters needed to apply Clark's stochastic reserving model.

(a) (*1 point*) Provide the term within this function corresponding to 0-12 months of development in accident year 2011, using Clark's Cape Cod method and an exponential distribution with cumulative distribution function $G(x) = 1 - e^{-x/\theta}$.

The maximum likelihood estimates of *ELR* and θ are 71.15% and 7.293, respectively.

(b) (*1.5 points*) Provide the fitted triangle of cumulative paid losses.

(c) (*0.5 points*) Estimate ultimate losses for accident year 2011.

(d) (*0.5 points*) Identify the number of degrees of freedom associated with the estimate of the scale factor, σ^2.

The estimate of σ^2 is 273.

(e) (*1 point*) Estimate the process standard deviation of the accident year 2011 reserve.

The covariance matrix of the estimates of *ELR* and θ, respectively, is:

$$\begin{pmatrix} 0.00770 & 0.0444 \\ 0.0444 & 1.75 \end{pmatrix}$$

(f) (*1 point*) Provide an expression for the estimate of the parameter variance of the accident year 2011 reserve using matrix notation. (Do not compute the result.)

(g) (*0.5 points*) Explain whether you would expect the parameter variance to be larger, smaller, or about the same as the parameter variance obtained using the LDF method.

6.   (*4 points*) Your company is renewing three accounts, X, Y, and Z. You are given the following information:

Account	X	Y	Z
Expected Losses	10,000	1,400	1,500
Variance of Losses	200,000	12,000	10,000
Correlation with X	1.0	0.8	0.4

- Losses from accounts Y and Z are uncorrelated.
- The risk load multiplier, λ, is 0.0025.

(a)   (*3 points*) Calculate the risk load for each account using the Shapley method.

(b)   (*1 point*) Explain how your answer to part (a) would have been different if the Covariance Share method had been used.

7. (*6 points*) You are an actuary at MEB Insurance Company and have just been given a new assignment. One of the company's actuaries had been working on an application of credibility before he abruptly quit to join the front office of a Major League Baseball team. All that he left were the following notes:

- The application is based on the Klugman study note.
- The application is to 50 large accounts, each with 10 years of experience data.
- For each account $(i = 1,...,50)$ and year $(t = 1,...,10)$ there is a measure of exposure $(w_{i,t})$ and trended observed pure premium $(Y_{i,t})$.
- For account i the vector of trended observed pure premiums is modeled as follows:

$$\begin{bmatrix} Y_{i,1} \\ Y_{i,2} \\ \vdots \\ Y_{i,10} \end{bmatrix} = \begin{bmatrix} \mu \\ \mu \\ \vdots \\ \mu \end{bmatrix} + \begin{bmatrix} \alpha_i \\ \alpha_i \\ \vdots \\ \alpha_i \end{bmatrix} + \begin{bmatrix} \gamma_{i,1} \\ \gamma_{i,2} \\ \vdots \\ \gamma_{i,10} \end{bmatrix} + \begin{bmatrix} \varepsilon_{i,1} \\ \varepsilon_{i,2} \\ \vdots \\ \varepsilon_{i,10} \end{bmatrix}$$

where μ is a constant, the vector of alphas has a normal distribution with mean 0 and a covariance matrix where each of the 100 elements is τ^2, the vector of gammas has a normal distribution with mean 0 and the covariance matrix

$$\begin{bmatrix} \delta & \delta\rho & \cdots & \delta\rho^9 \\ \delta\rho & \delta & \cdots & \delta\rho^8 \\ \vdots & \vdots & \ddots & \vdots \\ \delta\rho^9 & \delta\rho^8 & \cdots & \delta \end{bmatrix}$$

and the vector of epsilons has a normal distribution with mean 0 and a diagonal covariance matrix with diagonal elements $\sigma^2/w_{i,1}, \sigma^2/w_{i,2},...,\sigma^2/w_{i,10}$.

- Once REML estimates of the parameters are obtained, the credibility weights can be obtained in two ways. One is to minimize the mean squared error and the other is to minimize the mean squared error but subject to constraints on the credibility weights.

Before investing additional company time in developing this model, your supervisor has asked you to respond to some questions.

(a) (*1.5 points*) Describe each of the components in the model.

(b) (*0.5 points*) Explain how this model differs from the Bühlmann-Straub model.

7. **Continued**

The covariance matrix of the gammas implies something specific about the structure of the observations, possibly a standard time series relationship.

(c)   (*0.5 points*) Identify the time series model represented by this matrix.

(d)   (*1.5 points*) Explain what that model means with regard to the evolution of pure premiums over time.

(e)   (*1 point*) State the appropriate restriction to place upon the credibility weights that is consistent with the time series model in part (c).

(f)   (*1 point*) Explain the relative merits of the two approaches to obtaining the credibility weights.

8. (*5 points*) Casualty R Us Reinsurance Company has been presented with an opportunity to write a casualty per occurrence excess treaty covering the layer 500,000 excess of 500,000. The following limits profile has been provided:

Subject Premium	Underlying Limit	Policy Limit
5,000,000	0	500,000
5,000,000	0	1,000,000
4,000,000	0	1,500,000
2,000,000	500,000	500,000
3,000,000	500,000	1,000,000

(a) (*2 points*) Calculate the expected losses in the layer using an exposure rating approach with an expected loss ratio of 60% and the following increased limits factors:

Policy Limit	Increased Limit Factor
500,000	1.00
1,000,000	1.35
1,500,000	1.56

(b) (*1.5 points*) Identify the two methods for handling trend and policy limits when an experience rating approach is used to calculate the expected losses. State the assumption that underlies each method.

Individual losses of 600,000 and 750,000 are incurred.

(c) (*1 point*) Identify, for each loss, the range of ALAE amounts for which allocating ALAE to the layer in proportion to losses would result in Casualty R Us paying more than it would if ALAE were included with losses.

(d) (*0.5 points*) Identify any additional casualty per occurrence excess coverage that the ceding company should consider purchasing. Justify your answer.

END OF EXAMINATION

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